Summary of the PhD Thesis

Group Actions on Acyclic Manifolds and Real Projective Spaces

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Among important problems concerning actions of compact Lie groups G on smooth manifolds is the description of the occurring fixed point sets. Such a set is a smooth manifold, assuming the action of the group G is smooth. Therefore, one may ask what are the necessary and sufficient conditions for a smooth manifold to be diffeomorphic to the fixed point set of a smooth action of G on a smooth manifold with specific properties, e.g, on a contractible manifold (such as a disk or Euclidean space). In this case, the answer to the posed question goes back to Lowell Jones (when G is a finite p-group), Krzysztof Pawałowski (when G is a torus or G is an extension of a finite p-group by a torus) and Robert Oliver (when G is a finite group not of prime power order).

In this thesis, one considers smooth actions of a group G on smooth manifolds which are pseudo-equivalent to a given G-template (i.e., a finite connected G-CW complex with non-empty and connected fixed point set). The results of authors mentioned above concern group actions on contractible manifolds, i.e., pseudo-equivalent to one point. In this thesis, the results are extended to actions of G on manifolds pseudo-equivalent to a mod-p acyclic G-template (Theorem 0.1) and an acyclic G-template (Theorem 0.2). Under the additional assumption that the fixed point sets in question are stably parallelizable manifolds, the description of the fixed point sets is obtained without any additional restriction on the G-template. Moreover, a necessary and sufficient condition is given for the existence of a smooth fixed point free action of a finite group G (not of prime power order) on a compact smooth manifold pseudo-equivalent to any given G-template (Theorem 0.4). In particular, there exists a smooth fixed point free action of a finite group G on a compact smooth manifold pseudo-equivalent to an even-dimensional real projective space with the trivial action of G, if and only if G is an Oliver group. Also, it is shown that each finite Oliver group G has a smooth fixed point free action on an even-dimensional real projective space (Theorem 0.5).