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Stochastic quantization and Osterwalder-Schrader
axioms for quantum field theory models

Summary

The purpose of this thesis is to construct the $P(\Phi)_2$ Euclidean quantum field theory, extending our discussion from [3]. We first construct the finite volume measure of the $P(\Phi)_2$ model on $\mathcal{D}'(\mathbb{S}_R)$, where \mathbb{S}_R is a 2-sphere of the radius $R \in \mathbb{N}_+$. To this end, we rely on the Nelson hypercontractivity estimate. Then, we utilize the parabolic stochastic quantization and the energy method with the help of the pull-back map $j_R^* : \mathcal{D}'(\mathbb{S}_R) \rightarrow \mathcal{S}'(\mathbb{R}^2)$, induced by the stereographic projection j_R , to obtain the infinite volume measure on $\mathcal{S}'(\mathbb{R}^2)$. To this end, we verify the global existence in time of solutions for the corresponding stochastic partial differential equations with fixed cutoffs, a topic that was only briefly addressed in [3].

The rest of the thesis is devoted to verifying the Osterwalder-Schrader axioms: regularity, reflection positivity and invariance under translations, rotations, and reflections. Such a combination of axioms, which is crucial for obtaining a non-trivial local relativistic QFT on Minkowski space-time, was not obtained in earlier constructions based on the stochastic quantization, such as [1, 2, 4].

Bibliography

- [1] S. Albeverio, S. Kusuoka, *The invariant measure and the flow associated to the Φ_3^4 quantum field model*, Ann. Sc. Norm. Super. Pisa Cl. Sci., (5), 20, 1359–1427, [arXiv:1711.07108].
- [2] S. Albeverio, S. Kusuoka, *Construction of a non-Gaussian and rotation-invariant Φ^4 measure and associated flow on \mathbb{R}^3 through stochastic quantization*, [arXiv:2102.08040].
- [3] P. Duch, W. Dybalski, A. Jahandideh, *Stochastic quantization of two-dimensional $P(\Phi)$ quantum field theory*, Ann. Henri Poincaré **26**, 1055–1086, <https://doi.org/10.1007/s00023-024-01447-w>.
- [4] M. Gubinelli, M. Hofmanová, *A PDE construction of the Euclidean Φ_3^4 QFT*, Commun. Math. Phys., **382**, 1–75, [arXiv:1810.01700].

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