Referee Report on the Doctoral Thesis

Title: *Stochastic quantization and Osterwalder-Schrader axioms for quantum field theory models*
Candidate:*Azam Jahandideh*

The thesis *Stochastic quantization and Osterwalder-Schrader axioms for quantum field theory models* is concerned with *constructive Quantum Field Theory.* The aim of this field, situated at the intersection of probability theory, analysis, and mathematical physics, is to construct probability measures ν on $\mathcal{S}'(\mathbb{R}^d)$, the space of Schwartz distributions on $\mathbb{R}^d$, which satisfy a set of conditions known as the *Osterwalder–Schrader axioms.* These axioms ensure that such Euclidean field theories give rise to a relativistic quantum field theory in the sense of Wightman.

One of the Osterwalder–Schrader axioms requires invariance of the measure under *Euclidean transformations* of $\mathbb{R}^d$, namely translations and rotations. This axiom is notoriously difficult to verify in concrete constructions. In practice, one constructs the measure by approximating $\mathbb{R}^d$ by a compact domain—typically a torus $L\mathbb{T}^d$ with $L \to \infty$—and then passing to the infinite-volume limit. While such approximations preserve translation invariance, they necessarily break rotation invariance due to the geometry of the torus, which can only be hoped to be restored in the limit. In the few cases where full Euclidean invariance has been established, this is usually achieved by employing two different approximation schemes—one preserving rotation invariance and the other preserving translation invariance—and then exploiting a uniqueness property of the limiting measure.

The central contribution of the thesis is the introduction and implementation of a new method for establishing full Euclidean invariance in constructive quantum field theory. This method is based on a simple yet original and elegant idea which, remarkably, does not appear to have been previously proposed in the literature. Instead of approximating $\mathbb{R}^d$ by a torus (preserving translations) or by a disk (preserving rotations), the author approximates $\mathbb{R}^d$ by a *sphere of diverging radius.* The key insight is that the symmetry group of the sphere has the same dimension as the Euclidean group and converges to it in the infinite-radius limit. The beauty of this approach lies not only in its conceptual simplicity and versatility, but also in the fact that it applies in situations where uniqueness of the limiting measure fails, a major limitation of previously known techniques.

The thesis implements this idea in the setting of the $P(\phi)_2$ quantum field theory, a scalar field theory with polynomial interactions in two dimensions, using the framework of *stochastic quantisation.* Stochastic quantisation was introduced by Parisi and Wu and has been significantly developed over the last decade through contributions by, among others, Da Prato–Debussche, Hairer, Mourrat, Weber, and others.

In the thesis, Nelson's method is first used to construct the $P(\phi)_2$ measure on a sphere of finite radius; this is carried out in Section 3. Subsequently, in Section 5, the author employs the energy method from the stochastic quantisation literature, developed by Weber and Mourrat, to obtain estimates that are uniform in the volume of the sphere. In order to apply this method, it is first necessary to establish that the invariant measure of the stochastic quantisation equation coincides with the desired $P(\phi)_2$ measure; this identification is rigorously proved in Section 4.

Finally, Section 6 is devoted to proving full Euclidean invariance and the additional Osterwalder–Schrader axiom of *reflection positivity.* The proof of Euclidean invariance is a direct implementation of the new spherical approximation method outlined above. The argument establishing reflection positivity is likewise original and, to the best of my knowledge, has not appeared previously in the literature.

Sections 1 and 2 provide a comprehensive literature review, contextual background for the thesis, and the necessary preliminary material from probability theory and partial differential equations.

In conclusion, this thesis provides a beautiful and conceptually new solution to a long-standing open problem in constructive quantum field theory. Moreover, the technically demanding implementation of advanced methods from the literature demonstrates the author's ability to conduct independent, high-level mathematical research. I therefore conclude that the candidate's doctoral dissertation fully justifies the award of a doctoral degree in mathematics.

Nikolay Barashkov
Max Planck Institute for Mathematics in the Sciences
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