ADAM MICKIEWICZ UNIVERSITY, POZNAŃ
Faculty of Educational Studies

Maha Abu Hatoum

Online Collaborative Discourse as Formative Feedback for the Improvement of the Achievements of Students with Difficulties in Mathematics in the Seventh Grade in the Urban Arab Sector

Doctorate supervised by

Prof. Dr Hab. Hanna Krauze-Sikorska

Co-supervisor Dr Tomasz Przybyła

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Abstract

The changes in pedagogical academic terms and the integration of technology in the schools did not change much of the nature of the teaching of the subject of mathematics, especially for students with difficulties. Classroom teaching has remained frontal and traditional, where the teacher is the main source of knowledge, and the learner receives the information. The passive learner receives the information and learns it through memorization for the solving of questions or as a part of the preparation for a test. In mathematics, the inculcation of the knowledge in this format is very common; it makes it easier for the teacher since it allows the teacher to convey the information widely and rapidly. The transmission of the information in this manner enables the teacher to represent knowledge in the way easiest for her and preferred by her, without the consideration of the learner's preferences. Good students can follow the teachers, and sometimes with a little help from the parents they can surmount the difficulties, but students with difficulties very quickly fall into the cracks, lose interest in the topic, and feel they are foreigners in the mathematics lessons. One of the main challenges in teaching mathematics to students with difficulties is to actively involve them in the building of mathematical knowledge from understanding and to avoid routine learning of procedures (Haylock, 1991).

The use of digital environments has become an essential part of learning in the domain of mathematical knowledge. Current standard digital environments in mathematics nowadays are GeoGebra and Desmos. The environments offer two digital spaces, the first dedicated to the field of algebra and the second to the field of geometry. Both environments allow the immediate and rapid transition between the various mathematical representations.

The online mathematical discourse is a model of collaborative learning that is based on the sociocultural theory of Lev Vygotsky, which holds that learning is a social process through which the learner develops abilities and high cognitive functions. Social reciprocal relationships and mediation are two basic elements in developing high mental functions, such as perception, attention, memory, and thinking (Ilam, 2003). Collaborative mathematical discourse that occurs through the use of digital learning environments is rich

in mental representations that enable the mediation of the learning material in a dynamically and tangibly. In research, the mediation of the learning material is undertaken in small groups and through goal-oriented collaborative mathematical discourse. The collaborative mathematical discourse is performed between the teacher and the student and between the student and the rest of the group members. The model of the online collaborative mathematical discourse that uses digital learning environments as collaborative environments includes a method of interactive activities:

- inquiry activity for the learner's life,
- dynamic transition between the different mathematical representations,
- sharing of the knowledge and ideas for possible solutions,
- carrying out synchronous and asynchronous tasks.

Every student share with the members of his group his manner of solution through discourse and thus exposes them to different levels and forms of representations. The participation in the group allows its members to develop their understanding at a pace suited to them and according to their ability.

This research study focuses on the influence of online collaborative discourse on the students with difficulties in the field of knowledge of mathematics and learning achievements. A central assumption of the present research study is that the collaborative mathematical discourse based on digital environments advances students with difficulties in academic and achievement terms and increases the effectiveness of the teacher's work. Accordingly, the research study attempts to define the characteristics of the online collaborative discourse and to examine its influence on the students' achievements, the perception of self-efficacy, the motivation to learn mathematics, and the change that occurs in the learning strategies.

To further the depth of the analysis of the data, it was decided to hold the research study according to the quantitative-qualitative approach. In the research process based on the quasi-experimental method, various mixed methods research techniques were used to obtain a consistent and comprehensive answer to the research questions. In the framework of the research study, two state middle schools of equal socioeconomic status and similar technological infrastructure were chosen. From the two schools, a group of students with difficulties in mathematics was chosen after a mapping test that was held at the start of the process. The students were divided into a quasi-experimental group and a control group of students with difficulties that did not join the quasi-experiment. At the end of the research study, a comparison was performed between the two groups. As background for the data collection, a process was performed to analyze the interviews that were held with the teachers. The data analysis facilitated the understanding of the process of the development of the students' learning abilities, the identification of the types of difficulties among them, as well as the ways of thinking and the quality of the process of solution.

The collection of the data in the schools was performed during the whole school year 2020-2021 (school year in Israel begins in September and ends in June). Twelve mathematics teachers (Six teachers that participated in the quasi-experiment and six teachers were on the control group) and 30 seventh-grade students from two middle schools, students with difficulties in mathematics that participated in the quasi-experiment and, another 28 seventh-grade students from two middle schools, students with difficulties who did not participate in the research study (control group). The students were examined through quantitative research instruments – attitudes questionnaire, mapping test, and summative test. The questionnaire examined three elements: self-efficacy, learning strategies, and intrinsic motivation. The mapping test and the summative test were intended to map the students' level in terms of achievement and the way of building the process of solution. Between the mapping test and the summative test, the students participated in discourse-based mathematical activities in digital environments in small groups with the teacher's accompaniment. The transcription of some of the lessons enabled us to learn about their information behavior and the solution process of a mathematical solving problem. The staff of teachers, which underwent special training for the participation in the experiment, was examined using a quantitative instrument, a questionnaire that examines the mathematics teachers' attitudes regarding the effectiveness of the use of the online environment as a means that increases the effectiveness of their work and the learning dimensions in the learner with difficulties, and a qualitative instrument – structured interviews, in order to learn about the effectiveness/ difficulties that were when implementing the teaching methods in this field.

The analysis of the research findings indicated that the integration of online collaborative discourse as an essential part of the curriculum promoted students with difficulties, in cognitive academic terms and in emotional social terms. The online collaborative mathematical discourse advanced students in terms of achievement so that the achievements in the subject of mathematics rose significantly. In social-emotional terms, the intrinsic motivation and self-efficacy among students with difficulties also improved significantly. The online collaborative mathematical discourse supported and improved learning strategies and coping with memory problems. In cognitive terms, the availability of technology is one of the ways that we can use to provide a solution for students with difficulties by reducing the load from the mental burden; namely, the difficulties in the active memory and the long-term memory influence the students' ability to solve problems and to examine thinking processes. The reduction of the load of complicated calculations by using technologies and the transfer of the mental load – offloading – from the person to a tool, encourage the performance of high thinking processes (Solomon, 2000). In addition, visual representations in learning advanced the learning both in terms of the memory and in terms of the understanding (Mayer, 2009). Essential illustration means in teaching mathematics are most important, since they bridge between the abstract idea and its representation and the concrete tangible level (Linchevsky & Tuval, 1993). Pappert (1980) emphasizes the importance of the student's visual-motor experience with objects learned in the outside reality. He maintains that children build their knowledge structures as a result of concrete experiences. Furthermore, the qualitative analysis of the teachers' interviews emphasizes that the program's success depends on the manner and degree of the teachers' exposure to the types of the discourse, the components of the discourse, and the types of the learners' difficulties and mediation strategies that improve the learners' understanding. The teacher's understanding of the importance of the discourse and its implementation in the correct way in the classroom significantly advances students with difficulties in mathematics. The research findings support the perception of the proposed model, in which the teachers who integrate online collaborative mathematical discourse on their teaching are more effective in their work with students with difficulties and the students who participate in discourse develop academic and achievement ability and have higher order mathematical abilities.

Therefore, it is believed that the Israeli education system must integrate online mathematical discourse into the mathematical curriculum as an essential and central part that includes mathematical problems from the learner's real world and mathematical activities that have the potential of different representations. In addition, the research study recommends broadening the scope of online collaborative teaching to include all students with difficulties, a greater number of teachers, and the other study levels in the school teaching system in the middle school.

Introduction

The present research study grew from the mathematics teacher's teaching experience and the difficulties that they tackle for students with difficulties in the acquisition of the field of knowledge of mathematics in the middle school. From a social-cultural perspective, students with difficulties in mathematics are students whose scholastic achievements are low (Denvir, Stolz, & Brown, 1982) and experience a sense of frustration, failure, and lack of self-confidence which significantly hinders their ability to learn math and to participate in mathematical activities (Haylock, 1991). Every year, teachers of mathmathics, are required to help students with difficulties in order to advance them in terms of learning and achievement and the thought always was that the promotion of the students is carried out through the building of worksheets, additional practice of the material, and questions of illustration/inquiry, but the end of the year tests did not show a change in the results.

With the development of technology and digital means and the beginning of their use as a part of the teaching of all students in the subject of mathematics, teachers, began to search for ideas and advice through the integration of online teaching for the advancement of students with difficulties in mathematics. Until now, our teaching in the school was frontal, occurring through the teacher's monologue in the classroom. The learning occurred through the transmission of the learning material from the teacher to the learners (Weiss, 2010), and the main discourse was on the level of the questions asked by the teachers, questions of clarification, or questions that volunteer information so that the material the teacher explained during the lesson is understood, or general questions on the teacher's part to ascertain that the learners understood everything. Integrating technology and digital means opened new educational horizons and led to the situation in which many teachers attempted to integrate these environments into their teaching. However, despite the goodwill, frontal teaching in the classroom remained prevalent. The teachers who integrated digital environments did this as a part of the frontal lesson and only to illustrate a topic. The development of an online mathematical discourse arose as a part of the form of teaching. Nevertheless, deciding what to learn and how to teach to develop a learning discourse was complicated and required considerable effort on the teachers' part. This quasi-experimental research proposed the use of collaborative mathematical discourse with the integration of digital learning environments in order to cope with the underachievement of students with difficulties in mathematics. The advancement of the learning of the students with difficulties in mathematics is based on the use of digital mathematical learning environments through the use of in-depth, purposeful, and online mathematical discourse. A class in which discourse enables learning that is "collective, reciprocal, supportive, cumulative, and purposeful" (Boyd & Markarian, 2011, p. 529). The discourse was carried out in small groups, collaboratively, since research shows that collaborative learning in small groups positively influences the interaction between the students and increases the effectiveness of the teachers' work (Heggart & Yoo, 2018; Jonassen, 1996). Online environments allow teachers to adopt collaborative instruction approaches that suit learners' learning needs and advance work individually (Horizon Report, 2014). Work in small groups does not always ensure scholastic progress; the advancement of learning also depends on other factors, such as the students' manner of involvement and their participation in group learning tasks. Therefore, promoting involvement and cooperation is accomplished through purposeful student discourse in which the learners share information, build ideas together, while providing demonstrations in different ways, and justifying their work processes. The aim of mathematical discourse is the development of learning strategies, such as recall of the information, processing of the information, and control ability. The goal of using ICT as a collaborative learning environment is to support and improve learning strategies and deal with memory problems (Drigas, Kokkalia, & Lytras, 2015; Ilam, 2003). Technology offers students who have difficulties to reduce the load from the mental burden. Difficulties in active memory and long-term memory influence the students' ability to solve problems and examine thinking processes (Geary, 2004). Advancing learning discourse is discourse that is accompanied by immediate feedback and formative feedback. The potential of this discourse is not utilized in the classes because of the teachers' difficulty following after all the students in the group simultaneously. The technological environment enables immediate feedback, formative assessment, and self-correction by the learner, plays an active part in building his knowledge and supports in-depth reflective thinking (Solomon, 2000).

In the research conducted as part of the doctoral dissertation, the mathematical discourse occurred during the use of the digital environments that provided a response for the different representations of mathematical problems. Visual representations and means of illustration are essential in the teaching of mathematics and bridge between the abstract idea and its representation on the concrete tangible level and the influence is on the level of the understanding and recall of the learned material (Linchevsky & Tuval, 1993; Mayer, 2009). The research study examined the conditions under which the collaborative mathematical discourse that makes use of digital environments advances the learning strategies of students with difficulties and how the discourse that occurs can facilitate the advancement of the positions of students with difficulties towards the subject of mathematics and the raising of the achievements of the learner who has difficulties in mathematics.

The practical contribution of the research study is the identification of the nature and components of the discourse in the online environment that promotes the thinking of the learner who has difficulties and helps the teacher with the identification of the learner's mathematical difficulties. The intervention program is unique and may be a basis for the continuation of the discussion of the role of the online mathematical discourse as a main part in the curriculum, which facilitates the development of learner with difficulties and the effectiveness of the teacher's work. The theoretical contribution of the research can be the promotion of the understanding of the importance of the mathematical discourse in terms of the character, principles, and types, with the integration of the digital environments, regarding the promotion of students who have difficulties in terms of their learning and achievements.

Chapter 1: Mathematics in the Development of Civilization

Mathematics is a system that develops all the time through organized and logical thinking. This development is undertaken in the context of the understanding of the development of cultures (Imhausen, 2016). Mathematics developed over hundreds of years and under the influence of human development (Stewart & Tall, 2015). The science of mathematics developed in parallel to the development of the different human cultures, from the time of the ancient world until the modern era of today. Today mathematics is a dominant international domain with logical foundations that grant mathematics the status of certainty, and it also serves as a tool for the investigation of the unknown (Joyce, 1998). According to the history of the different cultures, mathematics is considered to be the foundation of every field of study (Court, 2006). Today it is possible to see the influence of mathematics in every aspect of human activity and every subject area (Smith, 2004; Unguru, 1989).

Advanced mathematical tools are a vital component in all areas of modern science, including physics, biology, medicine, space studies, robotics, communication, finance, defense, and of course technology and computers (Smith, 2004). Mathematics had an important role in the development and advancement of medicine. Mathematical theories helped with the development of medical devices and led to novel innovations in lifesaving technologies. The use of probability and statistics helped verify the effectiveness of medicines and examine the lifespan of patients (Zayed, 2019). Today, the understanding of complex systems and structures such as high-technology systems depend on mathematical inputs and outputs (Smith, 2004).

The contribution of mathematics to humanity lies in the construction of manpower with high mathematical skill and knowledge in the field of science, technology, and engineering and will lead to innovation and productivity strategy (Smith, 2004). Therefore, “since the 17^{th} century, mathematics has been an indispensable adjunct to the physical sciences and technology, and in more recent times it has assumed a similar role in the quantitative aspects of the life sciences” (Gray, Folkerts, Knorr, Fraser, & Berggren, 2020, p. 1). The science of mathematics contributes to modern society in the building of the knowledge economy and in the physical sciences, technology, business, financial services, and many areas of ICT (Smith, 2004). Mathematics has a basic role in the development of the rules of sports. In each area of sport there are mathematical considerations that include "classical mechanics; linear and angular momentum, work, impulse, kinetic and gravitational energy, simple harmonic motion, friction, rolling, rotating frames, Euler angles and projectiles" (Crothers, 1992, p. 117). These are considerations that enrich the field and enable it to constantly develop and lead to better athlete performances. It is possible to see that the contribution of the science of mathematics to humanity is found in all areas of life. The contribution is significant to the construction of analytical thinking abilities. On the one hand, it contributes to the development of original and creative thinking, and on the other hand, it contributes to problem solving in the "real" world and the understanding and analysis of the information obtained from different sources.

1.1 Genesis of Mathematics

Mathematics organized the life of people hundreds of thousands of years ago. Historically, the start of the development of mathematics was from the need to count, measure quantities, and deal with situations in nature (Arbel, 2005; Stewart & Tall, 2015). The humanity of today began to develop one hundred thousand years ago. The human species called *homo sapiens* was considered the only one with the ability to develop sophisticated mathematical ideas through the development of language and symbolism (Shriki, 2011a; Tall, 2013). The use of symbols and mathematical operations has been found in many cultures, and therefore every culture contributed to the development of mathematics in a different way and according to its needs, with the intercultural transference in knowledge (Arbel, 2005; Shriki, 2011a).

Mathematics of the ancient world began to develop in Babylon and Egypt (Arbel, 2005; Shriki, 2014a). In ancient Egypt, because of the environmental conditions, Egyptian civilization developed with a high quality of life. It was necessary to predict the seasons of the year and to drive the development of scientific research in the field of astronomy and the development of arithmetic and the creation of the Egyptian calendar. The pyramids were the proof of complex engineering knowledge, indicating the developed engineering and arithmetic capabilities of ancient Egyptians (see Figure Number 1) (Shriki, 2012a). Egyptian mathematics was characterized by two traits: the use of fractions when the numerator is 1 and arithmetic. The system of numeration employed by the Egyptians was according to base 10 (see Figure Number 2) (Arbel, 2005). They knew actions of addition, multiplication, and division. Egyptian geometry was practical, lacking in proofs, and established on the basis of trial and error (Unguru, 1989; Zayed, 2019).

Figure Number 1: Egyptian Counting System in Hieroglyphics (Adopted from Arbel, 2004, p. 57)

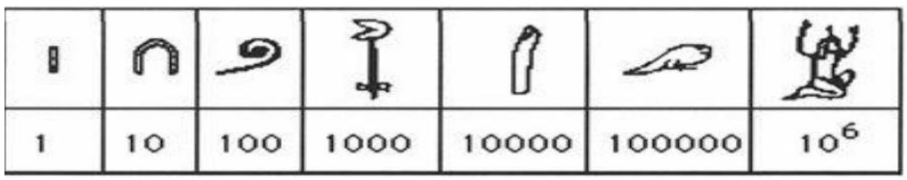
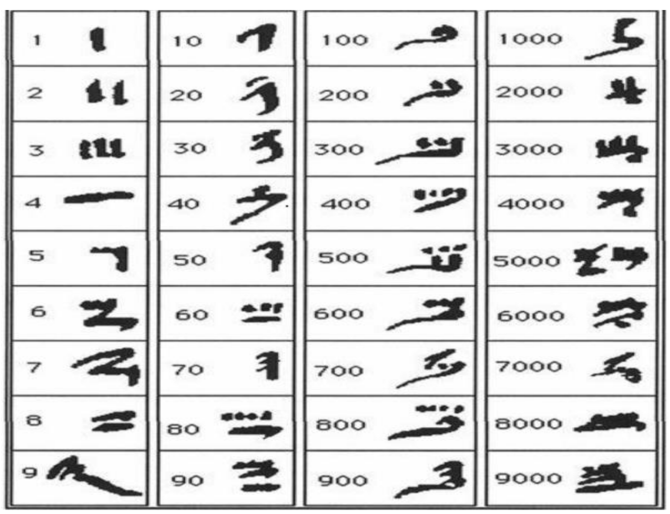


Figure Number 2: Egyptian Counting System in Hieratic (Adopted from Arbel, 2005, p. 57)



In Babylonian culture, tablets were found that indicate that the Babylonians had arithmetic knowledge. They knew methods of the measurement of land, the weighing of the measurements of liquids, the calculation of interest, and the use of fractions (Arbel, 2005; Stewart, 2008). They had knowledge in algebra, and they developed a way to solve second and third order equations (Shriki, 2012a). Measurement units were devised to quantify distance, area, volume, and time. In Iraq, people counted in sets of 60 (see Figure

Number 3). We still divide hours into 60 minutes and minutes into 60 seconds. Calendars were used to predict seasons and astronomical events, and geometric reasoning was used to measure distance indirectly. The Babylonians used geometric shapes in their buildings (Stewart, 2008). They recognized and used theorems related to the relationships between the sides of similar triangles (Shriki, 2011b).

Figure Number 3: Drawings (Symbols) Representing Words of Numbers, through Vertical or Horizontal Cuneiform (Adopted from Shriki, 2011b, pp. 15-23)

While the mathematics of the ancient world developed in Babylon and Egypt, the greatest mathematicians and scientists of the ancient world were the Greeks (Shriki, 2014a). The Greek mathematicians differentiated between the science of numbers (arithmetic) and the wisdom of calculation (logistics) (Savion & Seri, 2016). In addition, Greek mathematics presented for the first time the concept of the proof; this developed the idea of drawing conclusions while using logical tools and its influence led to discoveries, not only in the mathematics but also in the fields of physics, biology, medicine, and politics (Shriki, 2014a). The Babylonians and the Egyptians used mathematics as a tool for everyday needs, in contrast. The Greeks used this knowledge for the discovery and proof of new mathematical ideas (Shriki, 2014b). They adopted the geometric approach, which used proofs without reference to numbers (Unguru, 1989). Thales, called the "father of geometry", was the first to prove an argument logically. His approach, which was based on the formulation of general arguments, constituted in essence the start of deductive geometry (inference from the general to the particular) (Shriki, 2014b). In the period of the Greeks, mathematics developed into four main areas: geometry, absolute numbers – arithmetic, applied mathematics, and mathematical analysis (Seri & Savion, 2016; Shriki, 2014a). Greek mathematics developed in an accelerated manner within a relatively short period of time of about three hundred years. The Greek mathematicians attained such significant discoveries that mathematics barely developed beyond this level until the sixth century A.D., nearly more than five hundred years later. In this period, the main research and engagement in mathematics occurred in India and China and in the Islamic countries, which broadened the achievements of different cultures and developed algebra and trigonometry (Hila for Mathematics: The History of Mathematics, n.d.(no date)). Then came the period of the Renaissance, in which mathematical engagement was renewed.

In the 15^{th} century, the wars ended, and with the invention of printing the era of the transmission of information between cultures commenced (Arbel, 2005). Mathematics became a modern doctrine, through René Descartes (Tall, 2013), who is called the "father of modern philosophy" and is considered a key figure of the scientific revolution of the 17^{th} century (Mastin, 2020b). Descartes determined the purely logical nature of modern mathematics and made it the foundation stone in the research of reality in a scientific manner (Hila for Mathematics: The History of Mathematics, n.d.). He invented the idea of analytical geometry (Arbel, 2005) and presented infinitesimal calculus, which is the basis for differential and integral mathematics developed by Isaac Newton and Gottfried Wilhelm Leibniz (1646-1716) (Seri & Savion, 2016; Tall, 2013). The connection between algebra based on symbolic calculations and geometry based on visual embodiment added considerably to mathematical evaluation. The development enabled "to plan the paths of communication satellites above the earth, spaceships travelling to the moon and back and probes travelling to the outermost reaches of the solar system" (Tall, 2013, p. 235).

In the 20^{th} century, mathematical research led to the development and extension of specific mathematical fields (Stewart & Tall, 2015) and new branches such as computer sciences and game theory (Hila for Mathematics: The History of Mathematics, n.d.). The invention of the computer during the 20^{th} century complemented the abilities of the human mind in terms of complicated and immediate calculations. The person focused more on the concept and the computer on the process, enabling mathematical problems that had been open for hundreds of years to be solved (Tall, 2013). The development of mathematics is a process that has lasted for thousands of years, a process in which only people from *homo sapiens* can develop advanced mathematical thinking. The development of mathematics contributed to and was the basis for the development of most of the essential areas of life, such as philosophy, medicine, astronomy, physics, and others.

1.1.1 The Historical Evolution of Mathematics

The cultural progress and the development of mathematics went hand in hand for the past four thousand years (Stewart, 2008). The importance of mathematics went hand in hand with human development (Imhausen, 2016). The history of mathematics emphasizes four main periods in the development of mathematics. Every period was characterized by significant progress in the science of mathematics. The four periods are: the ancient era, ancient Greece, new mathematics, and modern mathematics(Unguru, 1989).

The first period, the ancient era, addresses the mathematics that developed in the cultures of the ancient world, Babylon and Egypt. The development of mathematics began from the engagement in everyday life. Some assert that mathematics began to develop because of the trade among the different tribes that led to the need to carry out calculations and catalyzed the advancement of the calculation process. There is evidence that emphasizes the need for counting from about 2500 B.C. (Arbel, 2005). Some maintain that the Egyptians invented geometry to overcome the need to re-measure the fields that the Nile floods every year (Arbel, 2005). Others maintain that the invention of mathematics was at the hands of priests of religion and was intended to glorify religion (Unguru, 1989). What is known today is that the engagement in mathematics that includes methods of counting, arithmetic operations, processes of calculation, and geometric calculations began in Babylon and Egypt about 4000 years B.C. and were the basis for the development of an entire theory engaging in numbers (Gazit, 2004).

The second period is Ancient Greece, in which Euclidean geometry developed. The invention of the alphabet in this period contributed to human development and to the spread of the cumulative knowledge about cultures (Arbel, 2005). Schools of logic, science, and mathematics were established. Mathematics was viewed as more than a tool to solve particular problems; it was seen as a means to understand divine laws. In this period, the first mathematical system was established based on postulates, theorems, and proofs (Tall, 2013). A numeration system using base 10, positional notation, the zero symbol, and powerful arithmetic techniques was developed. Mathematicians found methods of solution for quadratic, cubic, and higher degree polynomial equations. The English word "algebra" was derived from title of an Arabic book "Hisab al-jabr wa al-muqabala"; al-jabr became algebra (Unguru, 1989).

The third period is the era of new mathematics, which includes algebra and differential and integral mathematics. This period saw the publication of precise trigonometry tables, improvement of surveying methods using trigonometry, and mathematical analysis of trigonometry relationships. Logarithms were introduced by John Napier in 1614 as a calculation aid. This advanced science in a manner similar to the introduction of computer. Symbolic algebra was developed, principally by the French mathematicians François Viète and René Descartes. The Cartesian coordinate system and analytic geometry were developed by René Descartes and Pierre Fermat. Calculus was co-invented by Isaac Newton and Gottfried Leibniz (Eves, 1990). A powerful tool to solve scientific and engineering problems, calculus opened the door to the scientific and mathematical revolution (Unguru, 1989).

The fourth period is modern mathematics, in which an attempt is made to simplify mathematics and establish its foundations. The new geometries inspired modern theories of higher dimensional spaces, gravitation, space curvature, and nuclear physics. Set theory was used as a theoretical foundation for all mathematics. Theories of probability and statistics were developed to solve numerous practical applications, such as weather prediction, polls, medical studies; they are also used basis for nuclear physics (Arbel, 2005).

Today the use of computers and technology facilitates the development of the science of mathematics in all its branches, through the examination of possible solutions for complex mathematical problems (Mastin, 2020a). Computer development of electronic computer hardware and software solves many previously unsolvable problems and opens new directions of mathematical research. The human operator can specify the concept in terms of the known formulae, and the computer can do the calculation and manipulation. The combination of *homo sapiens* and computer provides a new twist to the tale of human evolution in mathematics (Tall, 2013).

1.1.2 Philosophy of Mathematics – Is Mathematics Discovered or Invented?

In the seventh and sixth centuries B.C. Thales of Miletus stated that mathematics includes not only the investigation of geometric shapes but also proof of geometric theorems, and thus mathematics emerged in Ancient Greece as a science based on reflective thinking. This method united between philosophy and mathematics (Kasdan, 2007) and discussed the questions of whether mathematics is a discovery or an invention (Lessel, 2016), whether mathematics is universally valid or depends on human thinking, and why scientists use mathematical formulae to describe and explain events and phenomena, or in other words, why math works (Livio, 2011). The foundations of mathematics and the source of its certainty developed with the start of the 20th century into three schools: intuitionism, formalism, and logicism.

Intuitionism. The school of intuitionism began with the Dutch mathematician Luitzen Egbertus Jan Brouwer[1]. The position came following the argument of the philosopher Immanuel Kant[2] that mathematics developed from our intuition of space, time, and number. He asserted that mathematical knowledge derives from human thinking, and therefore mathematics is an outcome of human activity of mental constructs and the existence of mathematical objects depends on the person's ability to build it clearly (Snapper, 1979; Tall, 2013).

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[1] Luitzen Egbertus Jan Brouwer (1881-1966), a Dutch mathematician, the father of the intuitionism school of philosophy in mathematics.

[2] Immanuel Kant (1724 –1804) an influential German philosopher in the Age of Enlightenment

Formalism. David Hilbert[3] is the father of the school of formalism. This method is based on the use of axioms, definitions, and proofs of theorems through a limited number of logical steps. The explanation of the arguments of the theory of mathematics is through a system of equations, formulae, symbols, and inferential rules (Tall, 2013).

Logicism. The school began with the German philosopher Gottlob Frege[4]. His book, *Foundations of Arithmetic,* is the basic text of the logicism project. According to logicism, logic is the basis of the classic mathematics (Carnap, 1999), and mathematical theorems are necessary logical truths (Snapper, 1979). Logicism is associated with arithmetic and real analysis (Stanford Encyclopedia of Philosophy, 2017). According to Rudolf Carnap (1999), the logicism thesis separated into two parts: the first part maintains that the *concepts* of mathematics are derived from logical concepts into clearer definition. The second part is the *theorems* of mathematics, Frege maintained that the axioms of arithmetic can be proved from the definitions, using logic, and not from logic using the definitions. Ernst Snapper (1979) described the argument in the following sentence: “Crises in classical philosophy reveal doubts about mathematical and philosophical criteria for a satisfactory foundation for mathematics” (Snapper, 1979, p. 207).

Beside the different schools, an argument developed about the nature of mathematics as an invention or a discovery. Those who identified with the argument that mathematics is an *invention* emphasized that there is no truth in mathematics but that it is a product of the human brain and does not exist without it (Lessel, 2016; Livio, 2011; Rowlands & Davies, 2011). In contrast, mathematical Platonism, or realism, identified with the argument that mathematics is a *discovery*. They maintained that mathematics, even if it is not written or known to us, has existed from the beginning and needs to be discovered and to be phrased in a universal language. This is human activity that was born from the ability to collect objects, to build relationships, and to create theoretical objects (Lessel, 2016; Rowlands & Davies, 2011). The importance of the understanding of the nature of mathematics is so that we can maneuver to improve and implement mathematics in a

[3] David Hilbert (1862-1943), a German mathematician, one of the most influential mathematicians of the 19th and early 20th centuries.

[4] Friedrich Ludwig Gottlob Frege (1848 –1925) a German philosopher, logician, and mathematician

conscious and intelligent manner (Abbott, 2013). Mario Livio (2009) maintained in his book *Is God a Mathematician?* that mathematics is a part of nature and the universe in which we live. Consequently, this topic has occupied many researchers. Albert Einstein mused, "how is it possible that mathematics, a product of human thought that is independent of experience, fits so excellently the objects of physical reality?" (Livio, 2011, p. 81).

The discussion on the nature of mathematics establishes the position and importance of mathematics and the conclusions that arise from the argument between the schools that the universe is subordinate to the laws of mathematics or at least it is possible to decipher it through mathematics (Livio, 2009). Today, mathematics as a science contributes significantly to the natural sciences in particular and the general description of the phenomena in the world. It is derived because "mathematics is a useful tool in describing regularities we see in the universe" (Abbott, 2013, p. 2153). When all the technological innovations that have flooded the world – television, computers, mobile phones, planes, and satellite navigation systems – would not exist without mathematics (Stewart, 2008).

1.2 Foundations of Mathematical Thinking

The development of mathematics and the nature and origin of mathematics are important in order to understand how the brain absorbs, processes, and prepares phenomena in the mathematical context.

1.2.1 Mathematical Thinking

Thinking is considered a mental process in which the brain develops ideas, declarations, and proposals. Most of the thinking is expressed verbally through language, when it is also possible nonverbally through pictures, sounds, and emotions (Byers, 2015). Brain development and experience increase the precision of the non-symbolically numerical representations (Izard, Sann, Spelke, & Streri, 2009). Mathematical thinking begins with simple numerical and geometrical understanding and develops on the basis of existing knowledge, when appropriate activities are designed and develop it anew (Wille, 2009). According to the constant rules of logic (Ministry of Education, 2010a), this is the ability to draw accurate conclusions from data.

Mathematics is a science that addresses logical thinking and quantitative thinking. The development of mathematics began from methods of counting, measurement, and geometric shapes (Gray et al., 2020). The young child thinks mathematics and has the ability to accumulate informal mathematical knowledge that includes differentiation of quantities and comparison between them before he enters into the school (Fischer, Suggate, & Stoeger 2020). Children build these abilities during observations and interactions with their natural environment. For thousands of years, mathematical thinking developed from the needs of survival and everyday needs, ranging from the sensory-motor perception to the development of symbols and language (Tall, 2013). Among children, the use of the fingers is a means for learning to count and supports the process of counting and both help to develop conceptual understanding (Siegler, 1991) and to increase young children's numerical performance (Klichowski & Przybyla, 2017). "Fine motor skills are closely related to early numerical skill development through finger-based numerical counting that aids the acquisition of mathematical mental representations" (Suggate et al., 2017, p. 1085). The repetition of finger counting procedures supports internalization and automatization. It helps children to represent numbers as finger-based number representations (Fischer et al., 2020; Suggate et al., 2017). Fine motor skills among children significantly predict future mathematical ability (Pitchford, Papini, Outhwaite, & Gulliford, 2016). From the perspective of cognitive neuroscience, finger training has become an important element of the process of teaching young children math (Klichowski & Przybyla, 2017). Children possess neural mechanisms for representing numbers of objects, and they improve abstract numerical representations using language (Izard et al., 2009).

The development of mathematical language of children about quantification skills is spontaneous and is carried out in two parallel dimensions: the verbal and the nonverbal (Li & Baroody, 2014). The ability to absorb and understand quantities is innate (Christodoulou, 2017; Tall, 2013). This natural tendency shows that children absorb the quantitative value of the number presented before them automatically. The automatic nature of the processing of quantities among adults and children is a product of an innate brain mechanism of learning experiences that begin from the age of the preschool (Suggate, 2017). The neural link between numbers and practice advances the process of acquiring mathematical competences and the representations of language (Klichowski & Przybyla, 2017). The child develops for himself and in himself strategic and conceptual knowledge and good ability to solve problems. The child builds these abilities through observations and interaction with his natural environment. Therefore, it is important to cultivate these skills through exposure to stimuli and mental situations, so that he can transform his knowledge into formal knowledge that is the basis of learning mathematics in the continuation (Stewart &Tall, 2015). Basic elements in the development of mathematical thinking are the development of the language, the development of the concept of the number, and the development of the arithmetic ability and the perception of the shape.

1.2.2 Elements of Mathematical Thinking

1.2.2.1 Language

Mathematics is defined today as a language. The argument is that mathematics and language have similar structural properties. Both include concepts, words, sentences, symbols, and special rules (Danesi, 2016; Miura & Okamoto, 2003). “Children are born with an innate number mechanism that enables infants to construct non-verbal number concepts that precede and provide a basis for verbal number concepts and skills” (Li & Baroody, 2014, p. 610). Therefore, in parallel to the acquisition of spoken language, children acquire also mathematical language (Danesi, 2016; Miura & Okamoto, 2003).

The beginning of the development of language skills occurs through gesturing with the hand and pointing at something (Tall, 2013). According to Michal Klichowski, & Tomasz Przybyla (2017), the neural link between the motor representation and the numerical representation is strong and influences the representations of language. The strategy of finger training increases young children’s numerical performance and the process of the acquisition of mathematical competences. Children at a very young age begin to display mathematical knowledge. They naturally use mathematical concepts in everyday discourse in the home, in preschool, and in the playground. Thus, when they reach preschool/kindergarten age (four-five), most children know mathematical concepts in an in-depth manner (Ginsburg, Lee, & Boyd, 2008; Miura & Okamoto, 2003). They differentiate between quantities and comparison according to the quantity and size of items at the time of play and also giving explanations (Gopnik, Sobel, Schulz, & Glymour 2001). Many children in preschool and kindergarten know to count and understand the decimal structure even before they have started to learn arithmetic. They perform arithmetic actions and reduce quantities and even divide equally and understand the concept of zero. They perform these actions through the use of mathematical language (Baroody, 1987; Tall, 2013; Danesi, 2016). Jerome Seymour Bruner (Shanker & Bakhurst, 2001) maintained that the social environment constitutes a supportive mechanism for the acquisition of language. Stuart G. Shanker and David Bakhurst (2001) emphasized the great importance of the social context regarding cognitive development and regarding the linguistic development.

The building of language and the ability to communicate verbally develops from interaction with those around the child, from nonverbal actions or objects and characteristics (Shanker & Bakhurst, 2001). Lev Vygotsky (1994) asserted that children develop language through experiences. They develop, clarify, and generalize the meaning of words through the learning of words as symbols of the concepts they experienced. The generalization of the ideas is performed through communication that is vital when building a mathematical language. The mathematical language is important to the understanding of mathematical structures, the assimilation of new concepts, and the retrieval of the knowledge (Chard, 2003).

1.2.2.2 Concept of the Number

The concept of the number is acquired among different children at a different pace and in a different way, depending on the developmental stage in which the child is found, on the child’s exposure to the concept, and on the child’s activities and experiences. According to Daniel B. Berch (2005) “number sense reputedly constitutes an awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or mental number line” (p. 333). The sense of numbers and the development of the concept of the number begin in early childhood-preschool and kindergarten age. The ability to count is the basis for the development of the understanding of the concept of the number. To develop this ability, Rochel Gelman and Charles Randy Gallistel (1986, p. 243) listed five counting principles that they assert are innate:

1. The one-one principle. Every item in a group has only one number from the continuum of the counting words, and the reverse is true as well. Every number has only one item.
2. The stable order principle. The counting of numbers is performed in a regular order.
3. The cardinal principle. The last number that was counted represents the quantitative value of the counted objects.
4. The abstraction principle. It is possible to list objects in a heterogeneous group.
5. The order irrelevance principle. The order of the objects that are counted in a group is not relevant.

The assimilation of a number sense and the understanding of the meaning of numbers lead the learners to develop strategies of solving complicated mathematical problems, to create a sense of quantity, and to create a conceptual sense in mathematics (Berch, 2005). The sense of numbers is composed of three areas: number knowledge, counting, and arithmetic operations (Yilmaz, 2017). These are fundamental skills in arithmetic that are considered the basis of more advanced studies in mathematics and success in the work world: the ability to calculate calculations and understand ideas and mathematical principles. Stanislas Dehaene and Laurent Cohen (1995) proposed a "triple code model". They stated that the child understands the concept of the number only when he creates a relationship between the name of the number, the quantity, and the graphic symbol and understands that they represent the same mathematical idea. Pirjo Aunio and Pekka Räsänen (2016), proposed a model of the inculcation of four main skills that need to be developed among children:

1. Symbolic and non-symbolic number sense.
2. Understanding of mathematical relations: Advanced mathematical-logical principles, arithmetic principles, accounting operation signs (for instance + or -), place value, and decimal system.
3. Counting skills: Knowledge of number symbols verbal continua of numbers (for instance, from '1' to '10'), and counting of concrete objects.

4. Basic skills in arithmetic: arithmetic combinations, skills of addition and subtraction with number symbols.

The number sense is the key to mathematical knowledge (Clarke et al., 2011). Research studies show that the children who have difficulties with number sense develop when they are of school age learning difficulties in mathematics (Berch, 2005). Difficulties with the concept of the number make the actions of addition and subtraction difficult, as well as the view of the relationship between the whole and its parts. The background for the difficulty of the mental schema upon which it is necessary to build the representation and its meaning is not developed well among these children, and they do not succeed in building mental representations of quantities and associating between the symbols and the processes (Ben Yehuda & Ilam, 2008). Berch (2005) added that this harms automatization and assessment ability and quantity discriminations and develops difficulty in numerical information processing. There is importance in the work on the concept of the number and the sense of the numbers already from the preschool and kindergarten. Therefore, we need to focus on the cultivation of two elements, the number sense and fact fluency, which encourage the learners to build patterns relations and develop reasoning strategies (Baroody, 2008). Work on this topic improves the children's abilities and makes their learning more effective in the continuation and influences their achievements (Aunioa & Räsänenb, 2016; Baker, 2019; Baroody, 2008).

1.2.2.3 Space and Shape

The development of the perception of shapes and space is a main topic among children that influences their mathematical thinking in the continuation. The category of space and shape is a large category that encompasses properties and shapes of objects, visual information about shapes, position, and direction through interaction with shapes from the real world, and geometry is one of the elements of this category. According to David Tall (2013), the learner during these years learns to coordinate between the eye, hand, and brain, with the use of language to describe the processes of action, properties of figures, which entails structural abstraction. Structural abstraction plays an important role in the field of geometry and the development of mathematical thinking.

Pierre Van Hiele (1986) developed a theory about the stages of the development of thinking and understanding of geometric shapes. The theory at its basis is developmental, and it is possible to discern five levels in the development of geometric thinking and the learner cannot reach an advanced level before mastering the previous levels. The five levels are:

1. Level 0. Visualization. The learner identifies, compares visual perception and nonverbal thinking. The shapes are perceived as wholes, and there still is no understanding of the properties of the shape but only identification and naming.
2. Level 1: Analysis. In this stage the learner researches the shapes in terms of their properties empirically and the shape is linked with a constellation of traits. The learner knows to measure, to fold, and cut paper while using geometric properties.
3. Level 2: Abstraction. In this stage, the learner knows to link logically between the shape properties and is aware of the relationships between them. The learner understands relationships between properties, knows to give simple nonformal explanations (use of illustration and diagrams). He understands the role of the verbal definition.
4. Level 3: Deduction. In this stage, the learner understands the role of the assumptions and definitions and can prove by himself certain arguments. The learner understands the role of definitions, theorems, axioms, and proofs
5. Level 4: Rigor. On this level, the learner understands the formal aspect of geometry. The learner uses all sorts of proofs and can understand other geometries.

In the continuation, the theory of Van Hiele (1999) used three levels – the visual level, the descriptive level, and the deductive level.

1.2.2.4 Arithmetic

The mastery of basic arithmetic skills is an important goal in early education (Göbel, Watson, Lervåg, & Hulme, 2014). The counting technique is the first action in the learning of arithmetic. The human mind builds the different skills for the performance of appropriate calculations on the basis of different activities (Tall, 2013). According to Arthur J. Baroody (2006), there are three stages that characterize the progress of children on the way to the knowledge of basic facts: counting strategies, reasoning strategies, and mastery. Knowledge of the combinations of basic factors means reaching the third stage, automation in giving answers. Xia Lia and Arthur J. Baroody (2014) add that meaningful object counting is one of the counting strategies which the learners use, verbally or non-verbally, to check sums and differences.

Many students have difficulties achieving the mastery of the basic facts in mathematics, and this influences the automation in the providing of an answer. Difficulties can derive either from impairments of cognitive processes or deficient teaching (Baroody, 2006). The symptoms of children who have learning difficulties include inefficiency, because of the use of learning by heart and not by creation of meaning; inappropriate applications, repetition of facts instead of building contexts between the existing knowledge and new knowledge, and inflexibility, when the manner of the teaching does not encourage children to construct concepts or create reciprocal relations between strategies. The awareness of the learners' difficulties and of the importance of computational fluency to advance students in terms of mathematical achievements helps develop new teaching strategies. Based on Vygotsky's socio-cultural theory, to improve the students` understanding, "it's important to determine the area (zone) between what a student can accomplish unaided and what that same student can accomplish with assistance" (Caruana, 2012, p. 1). Edward C. Rathmell (1978) found that the teaching of thinking strategies to the students helps the children organize the facts into a system with meaning and thus improves their learning and the recall of the basic facts.

The best way to advance children is to use more effective strategies. One of the key skills that teachers need to use is scaffolding the student`s learning process. Scaffolding is a strategy that advances the students' learning in the zone of proximal development. The development of this learning process occurs in a collaborative manner between the teacher and the students (Shepard, 2005). The encouragement of the learners to share and invent informal strategies, learning of the facts in a way meaningful to the learner, cultivation of abilities of looking for patterns and relationships, and emphasis of teaching that encourages the building of knowledge through more structured discovery learning activities (Baroody, 2006).

1.2.3 Importance of Mathematical Thinking

Mathematics is a human activity that is based on centuries of human experience using our brains and our strength. In mathematics there are different kinds of understanding, when one of them is to think logically or build mathematical models – pattern of mathematics (Stewart & Tall, 2015). There are differences between the children in the development of mathematical understanding. Some of the children focus on graduated processes of learning of structured steps, as opposed to other children who develop flexible knowledge structures that enable them to advance and develop the mathematical knowledge through the use of different representations (Tall, 2013). Tall added that mathematical thinking ability depends on three dimensions that are common among people: “recognition” to build patterns, “repetition” of actions until they perform them automatically, and “language” that includes the use of symbols. Therefore, in educational terms, it is very important to use and develop mathematical thinking ability. This importance is emphasized in three ways: as a goal of schooling, as a way of learning mathematics, and for teaching mathematics (Stacey, 2006).

1.2.3.1 A Goal of Schooling

The development of mathematical thinking is necessary in academic life and is useful in the modern world, and it is the basis that supports science, technology, and economic development (Isoda & Katagiri, 2012; Stacey, 2006). It is the way to understand mathematical problems through the use of different mathematical learning sources (Mustafa, Sari, & Baharullah, 2019). Governments that have economic welfare cultivate strong levels of what is called mathematical literacy (De Lange, Blum, Dossey, Marciniak, Niss, & Shimizu, 2006), which reflects the ability to use mathematics to solve problems in the real world. The conceptual framework of the PISA test (Programme for International Student Assessment) shows that mathematical literacy entails components of mathematical thinking, including reasoning, modelling and making connections between ideas (Stacey, 2006). In addition, mathematical thinking contributes to the development of 21st century skills. The development of these skills is important to the learner’s real life and judgment regarding the validity of the information from quantitative and logical considerations

(Organisation for Economic Co-operation and Development – OECD, 2018a). The development of mathematical thinking is accomplished through the development of a suitable curriculum through which all learners can maximize their potential (Tall, 2013).

1.2.3.2 A Way of Learning Mathematics

In the learning process, thinking activity involves the whole person, feeling and will of the students (Mustafa & Sari, 2017). Mathematical thinking is the outcome of the goal of learning, using the problem-solving approach. Therefore, mathematical thinking needs to appear and be a part of the lessons at the time of the learning. Learning mathematics is undertaken actively, out of understanding, use of previous knowledge and experience (National Council of Teachers of Mathematics – NCTM, 2000). Solving mathematical problems constitutes a mean of the development of mathematical thinking, which is carried out through mental compression into "thinkable concepts" and "connection" between them to build knowledge structures that can "blended" together to offer more sophisticated ways of thinking" (Tall, 2013, p. 50).

The manner of solving of mathematical problems enables the drawing of conclusions about the mathematical thinking of students (Sapti, Purwanto, Mulyati, & Irawan, 2016). The manner of mathematical thinking is reflected in the strategies of mathematical understanding of students (Sherin, Jacobs, & Philipp, 2011). Arthur L. Costa & Bena Kallick (2000) maintained that as students grow in their problem-solving ability, they become more flexible in their thinking. The development of this type of thinking enables us to understand the need for the use of knowledge and abilities and how to learn independently (Isoda & Katagiri, 2012).

1.2.3.3 A Way of Teaching Mathematics

Mathematical thinking is the most important ability required to develop independent thinking among the learners. It is important that every learner in the school have the ability to work according to his level and ability independently. The learning methods must be personal. This begins with the correct planning on the teachers' part for the lesson through the "improvement" of the lesson according to the learners, so as to enable support and development of the learners (Isoda & Katagiri, 2012). Effective teaching of mathematics requires the understanding of the learners' knowledge and what they need to learn and accordingly to support them and to challenge them (NCTM, 2000). The nature of the teaching needs to provide the learners with opportunities to develop their mathematical thinking. The outcome is that the teachers are the pillars, and they have an important part and role in the development of the mathematical thinking (Stacey, 2007). The attitudes, beliefs, and perceptions of the students regarding mathematics greatly influence the learners' success, and the teachers perform a mapping for the learner and create accordingly a curriculum that will make the classroom into an exciting place and the learning of mathematics into a joyous, meaningful, challenging, and interesting experience (Katz, 2014).

1.2.4 Competences of Mathematical Thinking

Mathematical thinking as a process is a very complicated activity (Mason, Burton, & Stacey, 2010; Stacey, 2006). To build appropriate mathematical thinking ability, it is necessary for the students to assimilate skills based on abilities of mathematical thinking that include reasoning, problems solving, and making connections between ideas (Stacey, 2006; NCTM, 2000). The abilities and competencies required of the student for this purpose are based on the article of Mogen Niss (2015) and detailed in the accepted requirements in the OECD member states (De Lange et al., 2006).

Reasoning and proof. "People who reason and think analytically tend to note patterns, structure, or regularities in both real-world and mathematical situations" (NCTM, 2000). The learners build mathematical conjectures and develop and evaluate arguments and proofs in all content areas and at all levels. The success in this process depends on the development of the meta-cognitive skills that are important to the development of their mathematical thinking ability (Katz, 2014).

Mathematical communication. This is the way that the learners share ideas and clarify understandings. Through the discussion of ideas and providing explanations that include mathematical arguments and rationales (NCTM, 2010). The communication about topics with mathematical content is carried out orally or in writing through the use of mathematical language and care to be clear and persuasive (Katz, 2014; NCTM, 2000). The cooperation among the learners creates an opportunity to develop their understanding through the focus of their thinking.

Ability to present problems and their solution. According to Paul Richard Halmos (1980), problem solving is "the heart of mathematics", and it is the essence of mathematics. In solving mathematical problems, mathematical thinking according to Masami Isoda and Shigeo Katagiri (2012) is the ability to think and to make decisions independently. This is scholastic ability, which includes goals such as understanding mathematical actions and how to use them, functional thinking, analogical thinking, and the writing and reading of a mathematical expression to a problem more effectively.

Representations. Mathematical ideas are represented in a number of ways: pictures, graphs, tables, etc. The learner is required to know the mutual relations between these representations (NCTM, 2000) and to develop the ability to move between the different representations regarding the problem that needs to be solved (Katz, 2014).

Connections. This is the learners' ability to link between the different mathematical ideas so that their understanding will become more meaningful and profound. The linkage is in the interaction between the learners and the mutual relations between the different mathematical topics and their areas of interest and experience (NCTM, 2000).

Mathematical aids and tools competency. Mathematics includes a variety of physical objects through which mathematical entities are represented, and therefore in its framework the student is required to know about a range of tools and aids that include visual aids, tangible aids, and electronic information that may benefit the mathematical activity and make use of them through the insight about the properties and limitations of these tools and aids (Niss, 2015).

Mathematical symbols and formalism. This includes the deciphering and interpretation of formal mathematical language and the understanding of its relationship to everyday life. The student is required to mutually translate between regular spoken language and mathematical language, to address statements and expressions that include symbols and formulae, to use variables, to solve equations, and to perform arithmetic actions (Niss, 2015).

Mathematical modeling. This is the ability to translate the reality into a mathematical model, to interpret and to understand mathematical models in the solving of problems through the evaluation of the model and the critical analysis of the model and its results, and the construction of new models on the basis of existing models (Niss, 2015).

These elements of mathematical thinking “reveal how humans build ideas through *perception*, *operation* and increasingly subtle *reasoning* using mathematical symbols and subtle developments in language” (Tall, 2013, p. xv). In other words, mathematics is a logical subject, and therefore the development of understanding is carried out through the building of mathematical models – pattern of mathematics – and the ability to build contexts with other areas. The importance of the building of these relationships is the creation of a sense of mathematics. This develops the learners’ mathematical thinking and enables effective and flexible use of the mathematical knowledge (Stewart & Tall, 2015).

1.2.5 Mathematical Reasoning

“Reasoning is the process of manipulating and analyzing objects, representations, diagrams, symbols, or statements to draw conclusions based on evidence or assumptions” (Battista, 2016, p.1). Thus, people who reason and think analytically tend to build patterns, structures, or regularities in both real-world situations and symbolic objects; they ask if those patterns are accidental or if they occur for a reason; and they conjecture and prove (NCTM, 2000).

Mathematical reasoning competencies include abilities like following and assessing chains of arguments, knowing what a proof is and how it differs from other kinds of reasoning, uncovering the basic ideas in a given line of argument, and devising formal and informal arguments (Niss, 2003). The improving of mathematical reasoning is entailing the ability to think and to solve mathematical problems logically through the justification of the solution. It is defined as the process of reaching a decision by using critical, creative, and logical thinking (Erdem & Gürbüz, 2015). The process of solving the task or mathematical problem is undertaken through the documenting of the process and form of calculations in any visual way. It is necessary to visually process the given problem and to perform the calculation steps before reaching a decision and selecting the appropriate response (Vansteensel et al., 2014). Therefore, mathematical reasoning (both deductive and inductive) includes making judgments about the validity of information and their quantitative and logical implications. This process contributes to the development of a select set of 21st century skills. It is a way of evaluating and making arguments and evaluating interpretations and inferences related to statements and problem solutions (OECD, 2018a).

1.2.6 Mathematical Thinking Patterns

The person's activity until he reaches a solution is called *problem solving* (Tall, 2013). In the mathematical thinking process while problem solving, there is a clear pattern that includes four categories that are divided into two main parts, the first part is exploring possibilities and the second part is proving theorems (Tall, 2013). The four categories were proposed by John Mason, Leone Burton, and Kaye Stacey (1982) and described in their book *Thinking Mathematically*. The first part includes: specialize particular cases and specific examples and generalize building patterns and relationships, and the second part includes conjecturing, predicting relationships and results, convincing and seeking proof.

In the process of mathematical thinking, the learner builds patterns when solving the problem. The process of the building of a pattern in the mathematical thinking depends on the strategy of the mathematical problem solving. The process goes through three stages: entry, attack, and review (Mason, Burton, & Stacey, 1982; Stacey, 2006; Tall, 2013). In the solving process of entry, the learner examines possibilities of the solution. In the stage of attack, the learner chooses a possible solution, and then the learner goes to the review stage. In this stage, the solution is examined, as well as its effectiveness, and the possibility of tailoring the solution to other problems and building new insights is examined. The broadening and building are undertaken in the stage of the generalizing. In this stage, the learner shifts from specializing thinking of particular cases and specific examples to broader solution and determines techniques for the continuation of a solution (Tall, 2013).

The development of the first stage is an important basis for an effective transition to the second stage of generalization. Amy B. Ellis (2007) defines generalization as “a process which engages learners in at least one of three activities: (a) identifying a commonality across cases, (b) extending one’s reasoning beyond the range in which it originated, or (c) deriving broader results from particular cases” (p. 197). We could also say that this type of thinking broadens the dimension of the concept and the conclusion of general characteristics during the solving of the problem (Isoda & Katagiri, 2012). The importance of the building of generalization in mathematics lies in the development of the learner’s ability to solve mathematical problems. A generalizing task and using patterns is a strategy for mathematical problem solving, which can be solved by “examining special cases, organising the results systematically, finding a pattern and using it to get the answer” (Stacey, 1989, p. 147). Maybe that’s why most of the students search for patterns when solving problems but find it difficult to decide how and when to use this technique as a strategy. It is an important skill in mathematical thinking, and therefore students need to practice searching for patterns, conjecturing, and generalizing (Thompson, 1985).

1.2.7 The Expanded Mind – Linear and Multithreaded Thinking

Human beings are rational, and a main component of rationality is the ability to reason (Baron, 2008). Reasoning depends on thinking over consistently about the possibilities of the beginning point – a perception of the world, a set of assertions, a memory, or mix of them (Johnson-Laird, 2010). Deductive reasoning is based on a three-step process of mental models’ theory (Barrouillet, Grosset, & Lecas, 2000). The first step is producing a process for the construction of a representation (a mental model) (Srinivasan, Tripathi, & Tandon, 2011). The second step is producing a putative conclusion from the resulting model. These two steps do not embody any reasoning process; thus in the third step, we examine the applicability of the conclusion, If there is no alternative model, then it is necessarily true, but if there is such a model, the conclusion must be judged to be uncertain. Peter Senge (1990, p. 160) remarked that “our mental models determine not only how we make sense of the world, but how we take action". During the process of constructing mental models, we derive a conclusion from them (Johnson-Laird, 2010). "Mental models are deeply ingrained assumptions, generalizations, or even pictures or images that influence how we understand the world and how we take action" (Senge, 1990, p. 11). The thinking is shaped according to the mental models that people build (Groves & Vance, 2009; Senge, 1990). "Thinking style has been defined as one's preferred manner of using mental abilities to govern daily activities, including understanding and solving problems, and challenges" (Groves, Vance, & Paik, 2008, pp. 308-309).

Human thinking is based on two separate cognitive strategies: the first is characterized as intuitive, experience-based, or holistic, whereas the second can be characterized as formal, rule-based, or analytic (Sloman, 1996; Norenzayan et al., 2002). Charles M. Vance, Kevin S. Groves, Yongsun Paik, and Herb Kindler (2007) posited two concepts on the basis of human thinking strategies: the linear thinking style and the nonlinear thinking style. This division of the thinking style is inherent on the definition of every term: individuals who have linear thinking style prefer to attend to external, tangible data and facts and are processing this information by conscious logic and rational thinking. Julie Spencer-Rodgers, Melissa J. Williams, and Kaiping Peng (2010) argue that linear thinking has been formed in the tradition of Ancient Greek philosophy (e.g., Aristotle's logic). Individuals who have a nonlinear thinking style prefer to attend to internal feelings, impressions, and sensations and to process this information by intuition, creativity, and insight (Groves & Vance, 2009; Groves et al., 2008). Nonlinear thinking models include elaborating strategies that are supposed to deal with complex nonlinear phenomena where linear thinking cannot (Bolisani & Bratianu, 2018). It is important to create a balance between nonlinear thought and linear thought of logic and reason (Groves et al., 2008).

The solving of mathematical problems depends on the learner. To solve mathematical problems, the learner builds representations in several ways and forms and builds the ability to shift between them (Katz, 2014; NCTM, 2000; Srinivasan, Tripathi, & Tandon, 2011). The learner independently determines the form and style of mathematical thinking by understanding the mathematical actions and the manner of use (Isoda &

Katagiri, 2012). It is necessary to remember that people with a linear thinking style, for the most part, are identified as having cognitive intelligence, while in contrast, people with nonlinear thinking are characterized as having emotional intelligence (Brătianu & Vasilache, 2009).

1.3 Pedagogy in Mathematical Education among Adolescents

The subject of mathematics today is considered a basis for technological and scientific development and its influence is apparent in our professional and social lives and in everyday activity (Wahyudi & Waluya, 2017; Maasz & Schloeglmann, 2006). Learning theories are essential in the building of the curriculum and the teaching-learning of the subject of mathematics. The two main theories in teaching mathematics in this context are the cognitive learning theory and the constructivist theory. There are methods based on these theories and led by the teachers. The teachers' success in teaching depends on improving their understanding, the students' learning abilities, experiences, reasoning, and logical abilities (Lessani, Yunus, Abu Bakar, & Khameneh, 2016). We also know that success in learning mathematics depends on several components, such as thinking ability and social interaction (Wahyudi & Waluya, 2017).

Thinking and understanding are central to the learning and teaching processes. Douglas P. Newton (2000) in his book *Teaching for Understanding* emphasized that the understanding is an essential characteristic of the quality of teaching and encourages the correct organization of the mental world, which increases the retrieval of items more rapidly with the adjustment to new situations. This gives cognitive autonomy that separates what is important from what is less important and builds a knowledge structure meaningful to the learner through the use of diverse sources of information. The mathematical model proposed in Israel in the curriculum of the subject of mathematics is the spiral structure that is built on the idea that all science has temporary validity and that every scientific action conveys uncertainty from stage to stage. The reduction of the uncertainty in a certain sense creates uncertainty in another sense. The solving of existing problems creates in parallel new questions (Khun, 1970; Schwab, 1978).

The penetration of the technological resources that appeared in the first decade of the 21st century supported a variety of new learning patterns that developed in the educational institutions in Israel. The development of the idea of the spiral, which is considered to be the basis of the Israeli curriculum in mathematics, has become easier in the digital era. Today the learning in the computerized online environment is broad and diverse, since technology is available for the learner at all times and in a place convenient for him (Zhang, Zhao, Zhou, & Nunamaker, 2004). In Israel, the model of the penetration of technology into teaching the subject of mathematics was undertaken as a continuation of the class environment, by giving classroom/home computerized assignments and learning films that are used to continue the learning outside of the school walls. The contribution of the integration of digital learning environments is that as the students are more involved in their learning processes, their learning is more effective (Wang & Reeves, 2007) and there is an increase in the learner's ability to build new knowledge and ability to transfer knowledge in the learning process (Jacobson & Archodidou, 2000). The building of the new knowledge through the use of technology challenges the learners according to their level and gives them learning tailored to the learner in terms of both the pace and the manner of the learning (Zucker & King, 2009).

1.3.1 Cognitivism and Mathematics

Cognitive theory is very important in the explanation of learning processes. Jean Piaget (1972) maintained that it is impossible to convey knowledge; rather this is a process in which the learner builds for himself the knowledge, which he called a process of knowledge structures. Mental structures are built on the basis of the learner's experience with bodies of knowledge. This learning is an active process in which every learner builds actively through processing, comparison, organization, and interpretation of the absorbed information. Benjamin S. Bloom, based on the 1956 work, *The Handbook I-Cognitive Domain*, described a taxonomy that deals with cognition. He divided the cognitive skills into subsets that included knowledge, comprehension, application, analysis, synthesis, and evaluation. Later, in 2000-2001, one of his students, Lorin W. Anderson, and Bloom's partner in the development of the model, David Krathwohl, updated the six stages of the taxonomy. The newer steps of the taxonomy were arranged as knowing, understanding, applying, analyzing, evaluating, and the last and highest function, creating (Anderson & Krathwohl, 2001).

The cognitive psychology has an important role in mathematics and the cognitive development of students. The assimilation and accommodation process provide insight into how students gain their knowledge (Zevenbergen, Dole, & Wright, 2004). Today, in the digital era, it is possible to interpret this with the help of the cognitive theory of multimedia learning. The idea of this theory is that students build relationships with their knowledge in a more meaningful and in-depth manner with the use of words and pictures (Mayer, 2009). The use of multimedia encourages the learners to understand the material in greater depth through the building of connections between mental representations of the material that was learned and the new material, in order to build new knowledge (Sorden, 2013). According to Roxana Moreno and Richard Mayer (2007), the use of interactive learning environments can influence the learners' own learning processes by responsiveness to the learner's action during learning.

The implementation of this theory in the field of the teaching of mathematics can be seen in the skill of problem solving. Problem solving is a meaningful element of mathematics education. According to the report of the NCTM (2011), ways of teaching were recommended for the development of learner skills suitable for the 21st century and teaching that includes cognitive processes and strategies for the development of the learner who is skilled in solving problems. According to the report of the Trends in International Mathematics and Science Study (TIMSS) (International Association for the Evaluation of Educational Achievement (IEA), 2007), while students are solving problems in math, they need to make use of a range of cognitive skills that ensure covering the content domain which they already outlined: the first domain *Knowing*, the second domain *Applying* and the third domain *Reasoning*.

Problem solving is a major part of learning mathematics. Students can reflect on their thinking during the problem-solving process that demands to have opportunities to formulate, tackle, and solve complex problems. Solving mathematical problems helps students to acquire ways of thinking, perseverance, and curiosity. Learners can improve and adapt the strategies they develop to other problems in their lives and in other unexpected contexts (NCTM, 2011). Problem solving in mathematics emphasizes and improves the ability of students to develop and adjust more complex, abstract, and powerful mathematics structures in order to solve a variety of real-life problems (Tarmizi & Bayat, 2012). Therefore, learning environments must support the development of future-ready skills that include: collaboration, critical thinking, creativity and innovation, communication, and problem solving (Becker, Brown, Dahlstrom, Davis, DePaul, Diaz, & Pomerantz, 2018).

1.3.2 Constructivism in Teaching Mathematics

The constant change and innovation in the 21st century led to the multiplicity of information sources. The building of the knowledge was undertaken through the use of diverse information sources, in which the knowledge is no longer perceived as final and absolute. The availability, and frequency of information raised the need to train graduates with new skills. These are learners who can adjust to the new and sophisticated technological world and who have flexible skills for handling and using this information. Therefore, the objective of the teaching of today is not the transference and learning by rote of knowledge but the development of an independent learner, who can think about the solving of new problems through the integration between existing knowledge and new knowledge, the creation of linkages, and reasoning ability. When transferring the information, the person is active cognitively and emotionally, in order to build knowledge. The learner links in an active way the knowledge in logical and meaningful connections to the knowledge and experience already existing in him in order to make sense in the presented material (Glaserfeld, 1998; Mayer, 2009; Narayan, Rodriguez, Araujo, Shaqlaih, & Moss, 2013).

The constructivist approach is a learner-centered approach that encourages the learners to actively construct their own knowledge based on their experiences and real-world problem solving (Elliot, Kratochwill, Littlefield, Cook, & Travers, 2000). Based on this, the role of the teacher and the school is:

1. To become a facilitator of the activities, help students to assess their understanding (Narayan et al., 2013), and support processes that encourage learning (Lessani, Yunus, Abu Bakar, & Khamaneh, 2016; Libman, 2013).
2. To strengthen the learners' motivation and to help the learners develop their ability to learn, while disputing the existing knowledge and supporting the learner when building his learning (Glaserfeld, 1995; Oliver, 2000; Tam, 2000).
3. The teacher develops a rich and diverse learning environment that includes the development of cognitive skills such as critical thinking, logical thinking, asking questions, and effective use of information that includes the analysis of the data and the drawing of conclusions (Brooks & Brooks, 1997; Libman, 2013).

In the teaching/learning process the teachers choose a constructivist activity that provides complex learning (Narayan et al., 2013) and challenge the learners to use the existing mental structures in order to integrate new outside information into the personal framework (Glynn & Duit, 1995). According to Maureen Tam (2000), in the constructivist perspective the learning depends on the existing knowledge of the learners, the social context, and the problem to be solved. She claimed that there are two characteristics that are central to the constructivist learning process: "good" problems rely on using the learners' knowledge to solve problems that are meaningful and realistically complex, and collaboration supports the learners to learn through interaction with others. Vygotsky (1978) emphasized that cognitive development derives from social interaction from guided learning within the zone of proximal development as children and their partners co-construct knowledge. Despite actions of mediation of awareness, the person does not respond spontaneously and reflexively versus a situation, but his response is mediated by the awareness that re-shapes the situation. The assumption is to provide authentic activity, a social negotiation with support, and use a multiple mode of representation (Narayan et al., 2013).

In the constructivist learning environment, a class is a community of learners who are involved in the construction of their learning through activity, discourse, and collective thinking (Tam, 2000). The idea of the learning in which the learner is involved in a mental and personal process, which is called active learning, is an outcome of the Information

Processing Theory. The theory sees the learning as a process comprised of a number of stages: receiving a stimulus, personal search for meaning for the stimulus, retrieval of the information that was built, ability to remember the information when necessary, transforming the knowledge into a part of the behavior, and perception of the learner (Caliskan & Sunbul, 2011). In other words, the learning environment needs to allow concrete experience that contributes to the learner's development and to the learning process (Vygotsky, 1978). Martin G. Brooks and Jacqueline G. Brooks (1997) presented teaching strategies that promote a process of the building of knowledge, such as the presentation of problems with relevance to the students, the building of learning around the basic concepts, and in parallel the building of contexts between the learning contents and giving value to the learners' ideas that will have influence on the shaping of the curriculum in the continuation. Robyn Zevenbergen, Shelley Dole, and Robert Wright (2004) maintained that learning mathematics is a process that includes social activity, dialogue, and interaction, during the building of the mathematical knowledge. They added that according to the sociocultural theory of Vygotsky, a learning environment built on learning in groups emphasizes the importance of the teacher's use of proper language and appropriate communication that encourages mathematical understanding. In constructivist teaching, the mathematical knowledge is built on the basis of knowledge that is learned. According to this approach, teachers will offer assignments that encourage the associating and bridging between mathematical topics learned in the past (Libman, 2013). One of the characteristics of constructivist teaching is that it is context-dependent and authentic and makes knowledge meaningful for the learner.

Learning mathematics by using realistic problems from student life allows mathematics to be easier to remember, imagine, represented, and manipulated (Wahyudi & Waluya, 2017). According to Abdolreza Lessani, Aida Suraya, Yunus, Kamariah Abu Bakar, and Azadeh Zahedi Khameneh (2016), one of the teaching methods in mathematics built on the constructivist approach is the discovery learning method. Discovery learning requires the students to discover something by themselves. Discovery learning is undertaken through the use of diverse activities related to the discovery and construction of knowledge. Teachers use discovery learning to ensure three educational goals: to develop the learner's independent thinking without dependence on the teacher, to encourage the learners to learn by gathering, organizing, and analyzing information to reach their own conclusion, and to develop higher order thinking skills in the learner. Problem-solving and discovery-learning skills are enhancing students' creativity to cope with life challenges. These methods encourage students to think in their daily life by enhancing their thinking and reasoning ability (Lessani, Yunus, Abu Bakar, & Khameneh, 2016).

1.3.3 Old and Modern Concepts of Mathematics Education

After the Industrial Revolution, the world has experienced essential changes in most areas of life. The status of traditional education has eroded as a result of technological, economic, and political developments (Lavi, 1990). This trend led to the shift from the education of a minority to education for all and the obligation of the education systems in the countries around the world to help students realize the personal potential of each one was emphasized. In parallel, the development of the multiple intelligences theory of Howard Gardner (1996) presented the education systems with a challenge for the development of new ways to bridge the differences among the students and lead to successful, personally tailored learning.

The concept of personal accommodation, which developed in the 1970s, paved the way to the development of new educational approaches in the education system, approaches that emphasize the training of graduates with skills that will suit the new reality (Lavi, 1990) through the adjustment of the learning contents, the learning aims, and the pace of learning to the needs, abilities, and learning style of each learner (Gardner, 1996; Volansky, in Teachers' Union, 2013). John Dewey (1938) maintained that the role of education is to teach the child and to direct him to think but not to teach him what to think. Accordingly, the importance and role of education are the assurance of the person's wellbeing through education and learning. Nowadays, the education systems of the 21st century are facing a reality called the Era of Information. This era is characterized by the accelerated development of the means of technology and the spread of information and communication technologies, which have had an essential impact on the social, political, and economic structures.

In the Information Era, the education systems have been forced to undergo a constant process of shaping with the intention to strengthen the relevance of education to, the learner, to create a new graduate with the ability to cope with the requirements of the new and changing world. According to the *2019 Horizon Report*, the assimilation of technology in the educational field led to the renewed internalization of the education system about the form of the teaching and learning. The emphasis in the assimilation of the technology is on the inculcation and development of new skills of digital fluency, which is a new skill defined as: "the ability to leverage digital tools and platforms to communicate critically, design creatively, make informed decisions, and solve wicked problems while anticipating new ones" (Horizon Report, 2019, p. 14). Digital fluency obligates the in-depth understanding of digital learning environments that enable the creation and sharing of content in order to adapt it to the new context. This is the ability to use digital tools to leverage the technology strategies in the sense of support in critical thinking and complex problem solving.

The use of technological instruments enables the development of personally tailored learning, according to the *2018 Horizon Report*, the argument is that the digital learning environments can monitor the learners' progress through the analysis of the data, thus enabling the teachers to re-shape their manner of teaching according to the learners' requirements and form of progress. The report proposes using technological environments, networks, and online learning instruments adapted to the school curriculum, and thus every student will receive a learning response tailored to his ability and pace. The emphasis on the shaping of a pedagogical future that will provide a response to the challenges and opportunities derived from the future trends (Nachmias & Mioduser, 2001).

1.3.4 Model of Future-Oriented Pedagogy in Education

The reality that is changing and the rapid progress in the different areas of life in the world following the development of technology led to a future that is unexpected and ambiguous. The first area that was influenced was the field of education, when it has the responsibility to tackle the challenge of the preparation of the graduates of the future (Melamed, 2008; Nachmias & Mioduser, 2001). Uzi Melamed (2008) cautioned against the risks of the “flight of the students” from the school following the unsuitability of the system and the learning patterns to the requirements, expectations, and new lifestyle of the learners in the technological era. Therefore, the challenge today is to shape education for the learner in two dimensions. The first is the relevance of education to the learner, education that is suited to the learner’s abilities and aspirations, while the second dimension is the learner’s relevance to the world, or in other words, the training of the learner who can contribute to himself and his environment (Morgenstern, Pinto, Vagerhoff, Hoffman, & Lotati, 2019).

Nir Michaeli (2016) raised the question about the nature of contemporary education that will prepare the learner/citizen for the future. He identified three approaches:

- The first approach is the present-oriented approach. This approach gives up predicting the future and holds onto the known and clear present. This approach includes two streams. One stream maintains that technological changes are mainly technical and are not considered “revolutions”. There is no rejection of the old and this is added to the contents and human essence. The second stream believes that innovation threatens the fundamental values of society and therefore it is necessary to protect the known and old world and to preserve the cultural assets in it. According to this approach, it is necessary to shape education on the basis of the values of the past and the channeling of the educational act will be through the acquisition of language and skills of the past or the present.
- The second approach is the futurist approach. According to this approach, the shaping of the education system is in connection to the challenges of the future. This shaping includes the inculcation of skills, knowledge, abilities, and languages with dominance and meaning in the future. On this basis, collaborations and initiatives between economic, business, social, and economic organizations and education organizations on the local, national, and global level are established. These initiatives train the learner to function in the virtual digital world.
- The third approach is the Utopian approach. This approach sees education to be the basis for the shaping of a future reality. According to this, the future is not disconnected from the reality and there is no need to plan for it, but there is a constant need to correct and improve.

The future-oriented pedagogy model is composed of challenges that were derived from future trends in the areas of society, technology, economics, education, environment, and politics, or in short, the STEEEP model. The model is composed of pedagogical aspects (content, curricula, teaching and assessment), organizational aspects (technological and physical infrastructures), and the principles of future-oriented pedagogy (which will be discussed at greater length in the next chapter). The future-oriented pedagogy unit in the Ministry of Education built a model that relies on the identification of the future trends that influence education, the definition of challenges of education, and the formation of recommendations and published it in a book of 410 pages (Morgenstern et al., 2019). The book defined six principles in the model of future-oriented pedagogy that represent the fields on which the education system must focus to maintain relevance in a changing reality.

1. *The Principle of Personalization*. The differences between people do not need to be an annoyance for the people of education, so that the elimination of the personal differences between the students is not an educational goal. The role of education is to identify, encourage, and cultivate the differences and accordingly to organize the learning processes according to the learner's abilities, needs, and desires (Lavi, 1990; Volansky, 2013). The implementation of personalization in the field of education shifted the reference to the learner from standardized to personally tailored by the teacher, the learner, and the use of the appropriate technology that adjusts the learning to the learner's personal style automatically (Birenboim & Gazit, 2005).

2. *The Principle of Cooperation*. The process of collaborative learning occurs when the learners build their knowledge through interaction with the group members. Each one of the learners constitutes an integral component in the process of social, academic, and cognitive development of the group members. Cooperative learning develops the learner's cognitive constructs and increases the desire and motivation for the performance of tasks (O'Donnell & Kelly, 1994). Mutual social relationships and social mediation are two basic elements in development of the high mental functions such as perception, attention, memory, and thinking (Ilam,

2003). Cooperative learning is based on four principles: the learner at the center, the use of authentic problems, social interactions, and the development of solutions by the group members. There are three levels of cooperation in learning: sharing, cooperating, and collaborating (Matoof, 2014). The application of the principle of cooperation is undertaken through the use of a range of advanced technologies that enable flexibility and speed in the collaborations, through the bridging of gaps of space, time, and culture and will allow the learner to broaden his circle of peers through the holding of decentralized, interactive, and dynamic cooperative learning.

3. *The Principle of Non-Formality*. This is learning outside of formal organizational and educational frameworks through the utilization of the many common learning opportunities throughout life. The technological development enabled people to easily and effectively carry out non-formal activities that enrich the individual and provide a learning experience regardless of place and time. Open learning environments allow the learner to be free of the traditional learning model, strengthen the individual's learning ability, and cause the learner to encounter the challenges of the changing reality in an independent manner.

4. *The Principle of Glocality*. These are two contradictory trends, globalization and localization. Glocality in education is the formation of awareness, identity, and global and local abilities in the learner through the creation of balance between them. The technology gives the learner exposure to broad circles in terms of geography and culture, and this dynamic world enables experience under the conditions of the changing reality. The learner's exposure to contradictions between the two worlds creates contrasts, thus developing in him the ability to cope with the challenges of the changing reality and with the relevance of education.

5. *The Principle of Changeability.* The increase of the ability of the learner and the system to shape for themselves a desired future, adjustment to changes in the complex reality with a nature of uncertainty rapidly and effectively. Skills the learner requires include cognitive flexibility, adjustment to conditions of

uncertainty, awareness of the self and the environment, and personality attributes such as belief and self-confidence and personal motivation (Dede, 2007).

6. *The Principle of Inclusion*. The integration of the identity and personal destiny, the formation of the whole self, identity, and purpose of personality, which will help the learner navigate in the changing reality. The stability and internal self-confidence, which is formed independently, will give the learner the ability to cope with a dynamic world and uncertainty.

1.3.5 Pedagogical Aspects – Implications on the Field of the Teaching of the Learning of Mathematics

The world of today created a new reality, which is dynamic and changing. Siemens George (2006) developed the **theory of connectivism**, which emphasized the importance of the use of information and communication technology as a network that helps the individual constantly update and enables the processes of information processing. He adds that the learning in the digital spaces creates a network that connects between concepts and nodes of knowledge or information sources, so that the transition from information to knowledge, in the new era, is considered a more important step in learning than the knowledge of specific content areas. The establishment of the learning on the online and digital infrastructures has forced every graduate of the education systems to have the ability to organize the thinking, to organize information, and to make it into knowledge (Dede, 2007; Solomon, 2000).

Technological educational development and mainly digital learning environments have influenced the education system in two described in the literature dimensions. The first dimension is that it offers new possibilities in the field of teaching-learning and presents the education system with a challenge to attempt new pedagogical approaches. The learning has shifted from learning focused on memory and facts to learning that emphasizes creativity, problem solving, analysis, and evaluation (Nachmias & Mioduser, 2001). According to Graham P. McMahon (2007), using digital environments positively influences the ability to think critically and creatively. The report of the National Council of Teachers of Mathematics (NCTM, 2000) proposed to integrate technology in teaching and learning of mathematics, because it improves the students' mathematical thinking and supports the students' development of a deeper understanding of mathematics. The second dimension is that digital learning environments have increased the students' motivation and positive attitude towards the learning and the learners' involvement in their process of learning and in their academic achievement (Rosen, 2011). The use of digital learning environments supports meaningful learning and advances effective learning that influences the learner's self-efficacy (Garrison & Kanuka, 2004).

1.3.6 Teaching and Learning Mathematics in Israel in the Era of Technology and ICT

At the beginning of the 21st century, the different types of computer technologies began to expand, take control, and be assimilated in every corner in our lives. The revolution in information technology changed modern communication by using interactive visual elements. The new media impacts our culture and education and creates a new generation that is addicted to technological devices (Przybyla & Przybyla, 2015). Therefore, many countries, including Israel, have invested considerable effort in order to meet the challenges of learning that is significant and relevant to the 21st century. Learning that emphasizes skills such as critical thinking, complex problem solving, and collaboration is important to transform the students into skilled learners who can adjust to changes (U.S. Department of Education, 2010).

The adjustment of the education system to the 21st century has the goal of the assimilation of information and communication technology (ICT) and the construction of a new pedagogy. The changes proposed by the use of technologies obligate the education system to change (Solomon, 2000). There is a change from the traditional approach where the basic learning techniques were the absorption of knowledge from the teacher, the learning by rote of knowledge, and individual learning, when the dominant instrument of learning is the book and the notebook. In the traditional system of studies, the teacher will find it difficult to turn effectively and equally to the students of the class (Katz & Ophir, 1996). This traditional approach contradicts the constructivist perception, which is the basis of the new learning environments, which are computer-supported collaborative learning environments, when the learning techniques in these environments are based on meaningful learning through individual experience in which the learner accumulates information through interaction between human and technological partners and through active involvement over time and solving of real problems. A computerized learning environment emphasizes the learner's personal pace, through feedback and peer support (Dunleavy, Dexter, & Heinecke, 2007).

In Israel, the model of the teaching-learning of mathematics is more like a partial technological innovation. The researchers David Mioduser, Rafi Nachmias, and Alona Forkosh-Baruch (2009) found that the mathematics teachers in Israel do not often use computerization (22%) relative to their peers in the world (50%). This positioned Israel in the one before last place among all the countries participating in the research. Israel was ranked among the three countries at the bottom of the list in the use of computerization among mathematics teachers. The use of computerization in the mathematics teachers' teaching in actuality fits the descriptions of the traditional teaching paradigm, such as presentation of information, illustrations, and directions for work in the classroom and evaluation of the learning through tests.

Today, to achieve the objectives of effective assimilation of the use of technology among mathematics teachers, two things have been done. The first course of action is institutional action plans to help teachers integrate the technology as a part of the curriculum (Oliver & Stallings, 2014). In Israel, there is the deployment of computerized literacy assignments in mathematics on the part of the Supervision/Coordination of Mathematics in the Ministry of Education (n.d.-h). Every month, for the academic year, it is necessary to use at least one assignment. The second thing is the deployment of programs for professional development of teachers (Kramarski & Kohen, 2016; Oliver & Stallings, 2014). The professional development ensures the effective integration of technology in the process of teaching, with conditions that will satisfy three skills – contextual, instructional, and technological, which are known to the teachers and include goals, innovative teaching strategies, and specific development of the use of technology (Oliver & Stallings, 2014). The empowerment of the teachers in the use of different forms of technology through professional development emphasizes the teacher's importance as an active partner in the assimilation of the technology as a part of the teaching-learning process in the subject of mathematics, for instance the use of the curricula and open tools (Givon, 1996) that enable the presentation of mathematical information through the use of visual aids and simulations, such as the widespread use of the **GeoGebra and Desmos** software.

1.3.6.1 Open Tools in Mathematics – GeoGebra and Desmos Software

The use of technological means in the teaching of mathematics is important to the development of the knowledge and skills that are necessary for optimal functioning in the 21st century (National Mathematics Advisory Panel, 2008). The integration of these tools was determined as an important standard by the National Council of Teachers of Mathematics (NCTM, 2000). In recent years, the integration of interactive technological tools as a part of the lessons plans in mathematics is a main objective of the supervision of mathematics in the State of Israel. The goal of the supervision is to establish meaningful and relevant learning for the 21st century and to emphasize skills of problem solving and intelligent use of technology for the illustration of the learning and for its adjustment to every learner. In the teaching-learning of mathematics, there are two software programs that are very useful for and well known among mathematics teachers in Israel: GeoGebra and Desmos.

GeoGebra Software

Figure Number 4: GeoGebra Software Application Symbol

GeoGebra is software for the teaching and learning of the subject of mathematics. GeoGebra is freeware, developed both for teachers and for students. GeoGebra is a visual and dynamic software program intended for the learning and teaching of mathematics at all levels of study (elementary school, middle school, high school, and academic/post-high school). “GeoGebra is dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus” (Winkowska-Nowak, n.d.). GeoGebra was programmed by Markus Hohenwater and allows easy and effective use for purposes for learning and teaching. Today the software has been translated into forty languages and can be downloaded from the official GeoGebra website at https://www.geogebra.org/ (Royati et al., 2010).

Desmos Software

Figure Number 5: Desmos Software Application Symbol

Desmos is a free suite of math software tools that include the Desmos Graphing Calculator and Scientific Calculator. Desmos is used by more than forty million teachers and students around the world (Desmos Website, n.d.). The software tools support a number of languages, including English, French, Italian, and in part even Hebrew. Desmos as a software not only is used for the illustration of equations and inequalities but also includes mathematical possibilities of lists, regressions, drawing of graphs with intersection points in the same dimension, graphing a polar function, and two types of axis systems (Desmos, n.d.). It also enables multiple functions to be graphed on the same screen. The output of the graphing is interactive, and the user can save the graph and share the link with other users who in turn can make changes in it and carry out commands in it. Desmos can be downloaded by users and used online either on laptops, desktops, or mobile phones (Liang, 2016). The Desmos graphing calculator is an available application and can be downloaded from the website: https://teacher.desmos.com.

1.3.6.2 Models of Teaching-Learning in the Subject of Mathematics in Israel in the Digital Era

In the Ministry of Education in Israel lesson plans in the subject of mathematics are prepared on the basis of one of two digital models of teaching: digital models of teaching for blended learning in the class and digital models for hybrid teaching.

The first model is digital models of teaching for blended learning in the class. These are class models for blended learning in the secondary school. Michael B. Horn and Heather Staker (2011) defined blended learning as follow: "Blended learning is any time a student learns at least in part at a supervised brick-and-mortar location away from home and at least in part through online delivery with some element of student control over time, place, path, and/or pace" (p. 3). This is a dual learning experience that combines between the experience of online learning and face-to-face learning (Garrison & Kanuka, 2004; Zou, 2005).

The advantages of blended learning include more consistent and personalized pedagogy. On the one hand, this allows the learner a flexible learning experience in terms of time and place and in terms of pace and style of learning that is personally adjusted to the learner. On the other hand, the teachers focus more on activity that advance higher order thinking skills, such as critical thinking, writing, and project-based learning (Garrison & Kanuka, 2004; Horn & Staker, 2011). Therefore, D. Randy Garrison and Heather Kanuka (2004) found that blended learning has the potential to advance effectiveness and efficiency of the meaningful learning experience. Elizabeth A. Ashburn and Robert E. Floden (2006) emphasize in their book *Meaningful Learning with Technology* that technology is for the learners an intellectual partner that helps them advance thinking and promotes meaningful learning as long as the learning is based on the learners' involvement in the building of knowledge, on conversation, on self-expression of the acquired knowledge, and on the deployment of reflective thinking.

The second model is digital models of teaching for distance learning built on synchronous and asynchronous learning – hybrid learning/teaching (Ministry of Education, n.d.-g). This is the combination of online and face-to-face learning in a virtual class. This is considered one of the models of blended learning. This learning advances the learner's independence and is personally tailored to every learner in terms of time, place, and learning style. Hybrid learning enables the teacher to create a personal relationship with the students and plan a variety of learning materials according to the learners while providing immediate and personal feedback. Hybrid learning developed greatly and gained momentum in the world and in the State of Israel during the Covid-19 crisis and the end of the studies within the school walls in the 2020-2021 school year.

The use of technological tools contributes to the learning of mathematics (Roschelle, Patton, & Tatar, 2007). The use of technology enables the students to hold independent inquiry-based learning and to develop ideas and concepts. The technology enables the teachers to carry out personal learning in which every student progresses according to his abilities and degree of understanding and enables the teacher to follow up in real time after the students and their difficulties in the understanding of mathematical principles. Two models of teaching were developed in order to help the teachers build lesson plans that enable the development of digital literacy. Every model of digital teaching must include (Ministry of Education, n.d.-e):

- A series of actions that make use of interactive digital tools.
- A detailed lesson plan based on a time axis for the effective management of the lesson. The time axis is important for the organization of the stages of teaching-learning in terms of the contents, variety, and products and enables meaningful learning.

The main goal of the two models of teaching is to develop literacy and digital skills in the following way (Ministry of Education, n.d.-d):

- The development of skills in a tailored and personalized manner for every learning, 'personal development'.
- Fulfillment of the utmost potential of each learner requires the characterization of the skills and the building of a personal plan for every learner.
- The learning continuum that is built from the lowest to the highest level, with integration between the different skills in one task.

- The integration of the skills in the context of the curriculum and the learning contents, with the advancement of the ability to transfer the skill from one context to another in a flexible manner and in new contexts.

Chapter 2: Information and Communication Technologies and Collaborative Learning as an Environment Supporting the Teaching of Mathematics

2.1 Mathematical Education – From Theoretical Assumptions to Educational Practice in Israel

At the beginning of the 21st century, the different types of computer technologies began to expand, take control, and be assimilated in every corner in our lives. "The rapid proliferation of information and communications technology (ICT) is an unstoppable force, touching virtually every sphere of modern life, from economies to societies to cultures (…) and shaping everyday life" (United Nations Children's Emergency Fund - UNICEF, 2017b, p.16). Technological development influenced the development of products, the ways of communication, the multiplicity of information sources, the availability of information sources. It led to a change in all areas of life, for better and for worse. The main influence was on the children, for whom in one respect the use of ICT facilitated the sharing of knowledge, cooperation, spurred tremendous creativity, expanded children's access to a wealth of enriching and entertaining content, and in another respect opened for the children a dangerous world of uncontrolled communication, online bullying, digital dependency, and screen addiction (UNICEF, 2017b).

The revolution of ICT requires the development of a new person, a person who uses technology and has the skill of personal and interpersonal communication, collaboration, and ability to converse online. However, the development of the education system, which lagged behind, became a main objective, with the aim of adjusting the system of the new challenges and the learners to the skills of the 21st century. Skills of the 21st century, such as digital literacy (Fraillon, Ainley, Schulz, Duckworth, & Friedman, 2018), that include effective use of digital environments and safe communication, the ability to critically evaluate information and create digital content (UNICEF, 2017b), critical thinking, complex problem solving, and collaboration, are important in order to transform the students into skilled learners who can adjust to changes (U.S. Department of Education, 2010).

2.1.1 Technological Development and Economic Influence

Information and communication technologies play an important role today in our lives both in the professional lives and in the personal lives (Fraillon et al., 2018; NCTM, 2000; UNICEF, 2017b). The global economic development, which paralleled the rapid development in ICT, led to a change in people's style and quality of life, type of work, and manner of learning (Voogt & Roblin, 2012). One of the results of the development of ICT was the control of computers over our lives; people began to rely on the computers to perform different tasks. According to Frank Levy and Richard J. Murnane (2004), computers do the hard and routine work and therefore the workers will need to preserve their place in the job market, to acquire new work skills and high order skills that require expert thinking and complex communication. Computers may do the work according to certain standards and rules, but they cannot solve new problems when the solution obligates new strategies and dictates to us to leave standards and rules (Dede, 2009). Alvin Toffler (1980, p. 189) maintains that computers are not superhuman since they are on the list of human achievements that increase our mind power and deepen our form of thinking.

At the end of the 20th century, information and communication technology (ICT) entered our lives and provided us with tools for the creation, collection, and storage of knowledge and also created the ability to communicate and share information (Kozma, 2003). The rapid development of information technologies hastened their assimilation in our lives and provided means for the creation, collection, and storage of knowledge and the creation of communication and collaboration. In the digital age, information and communication technology made possible the exchange of information around the world in a rapid manner, thus influencing the life of citizens in all areas – the community, the workplace, and the schools (Cheung & Slavin, 2013; Fraillon, Ainley, Schulz, Friedman, & Gebhardt, 2014). In addition, the concept of globalization appeared, expressing a process of the increasing power of large multinational corporations and the decline of the importance of geographical and national boundaries (Ates & Alsal, 2012; Avis, Fisher, &

Thompson, 2018). According to the OECD (1992, in Aspin & Chapman, 2001), the citizens of the future need to be equipped with skills of the "knowledge economy". "The worker of the 21st century must have science and mathematics skills, creativity, fluency in information and communication technologies, and the ability to solve complex problems; these are also important capabilities for citizenship at the local, national, and global level" (Dede, 2009, pp. 2-3). The International Computer and Information Literacy Study (Fraillon et al., 2018) added ICT literacy and digital literacy skills. Digital literacy is defined as the ability of individuals to use technical skills needed to use the technologies that are important for effective participation and communication processes of people at home, at school, in the workplace, and in society (Fraillon et al., 2018). Joke Voogt and Natalie Pareja Roblin (2012) maintain that the change in the requirements sets before the education system a serious challenge to prepare the learners for the new world and to rebuild the curriculum so as to adjust to the requirements of the 21st century. It is important to instill skills in the learners in the 21st century so they can integrate in the continuation in a way suited to the job market. Ester van Laar, Alexander J.A.M.van Deursen, Jan A.G.M.van Dijk, and Jan de Haan (2017) indicate two groups of skills. The first group is core digital skills: technical knowledge, management of information, communication, cooperation, creativity, critical thinking, and problem solving. The second group is contextual skills: ethical awareness, cultural awareness, flexibility, self-direction, and life-long learning.

The processes of globalization and technologies developing in the 21st century led to governments around the world to establish strategies of the assimilation of these skills and their mode of realization. The aim of the strategies is to achieve change at all levels of life and the educational sector, while providing learning opportunities throughout the lives of the citizens (Aspin & Chapman, 2001). Learning in the "information society" is a lifelong process that is not limited to education institutions and to a certain age. Rather, this is an action that continues throughout life. Lifelong learning is an important tool for the development and realization of personal abilities, unique talents that develops responsibility in the citizen and serves as a meaningful means for the advancement of a developed economy, a strong society, and solidarity (Aspin & Chapman, 2001; Ates &

Alsal, 2012; Natividad, Mayes, Choi, & Spector, 2015), which can meet the challenges of the future (Ministry of Education, 2006a).

2.1.2 Technology Impact: Long Term Implications of ICT

The rapid development of technology in our world has led to confusion and a lot of questions about the importance of using it, how to deal with it, and the outcomes. Information and communication technology has penetrated into all areas of life around us and has essentially influenced the manner of life and communication of people and has led to revolutionary changes in the manner of their work, their attachment, and the learning of people (Jorgenson & Vu, 2016). It is a powerful revolution in all areas of life (Melamed & Goldstein, 2017). According to Alexander I. Stingl (2018), information is available in all places and at all times. The possibility of communicating with others are no longer depends on time or place when the geographic distances have become irrelevant, and communication is fast and reaches all places. The needs that arose led to the rapid development of the use of technology, and computerization today has become an inseparable part of the different areas of life. According to Ofer Kenig (2002), the era of technology is a new era that should not be weighed in concepts of good or evil, since this is a period from which we only advance onward and human society is responsible for determining and shaping the manner of use of technology.

Globalization and the rapid development of ICT have greatly changed the way in which we live, work, and learn. Toffler (1980) maintains that the implication of technological development is not only on the economy but also on the culture; civilization becomes more diverse, and people and organizations become filled with the flows of information important to the activity of the social system. Stingle (2018) holds that since the year 1990 the use of the Internet and technology has fundamentally changed the world, in terms of the reciprocal relations and the social interaction, and thus requires the re-design of modern society, in which a digital divide has been created that derives from social inequality in the use of technology between the sexes, between the social classes, and between developed societies and less developed societies, and between the different cultures. Toffler (1980) asserts that the main impact of the computerization and technology that have conquered our lives is in the field of economy and culture. Technology and digital culture have reshaped the structure of the employment and social world, through the flood of information, the development of personal skills, and the transformation of the individual into more active and independent. Roi Yozevitz (2017) notes the revolution of education in the technological, entertainment, and business fields and maintained that the modern world changes significantly and rapidly and therefore we must learn and be updated all the time, or our abilities will become irrelevant.

In the field of education, the coping with the technological means is still at the beginning, and there is lack of uncertainty regarding the appropriate way and type of pedagogy correct for the production of the necessary benefit from the technological means. According to Uzi Melamed and Olsen Goldstein (2017), there are three types of learning: one-way learning, independent learning, and cooperative learning, when digital technology empowers and maneuvers between them. Nevertheless, this is not the central part of learning, since the era of the computer and the Internet awakens in the learners' awareness an "entertainment orientation". Therefore, in education, we need to rebuild the learner, a new type of orientation, a learner who addresses the computer and the Internet as a means of learning in which the researchers put forth effort but also take risks and become frustrated.

2.1.3 Skills Required in the Technological Digital Era

Lifelong learning, flexibility, ability to adjust and adapt, self-direction, cross-cultural social ability, creativity, leadership, and responsibility to take initiative – all are skills necessary for the learner in the ICT era. Therefore, it is necessary to change and rebuild the curricula in the schools and adjust them to the era of technology, while establishing and emphasizing skills of the 21st century (Voogt & Roblin, 2012).

The success in the assimilation of the skills of the 21st century among the learners will be expressed in the learner's ability to cope with the rapid and continuous changes and the ability to adjust themselves to new situations, and this will be the basis in the continuation for the person's development in professional and economic terms (Ongardwanich et al., 2015). They divided the skills of the 21st century into three categories: learning skills and innovation, the ability to cope with the flood of information in the era of the development of media and technology, and personal and interpersonal skills of lifelong learning and collaborative learning ability. Van Laar, van Deursen, van Dijk, and de Haan (2017) emphasize that the job market in the technological era requires skilled workers who can cope effectively with the flood of information both in the personal facet and in the professional facet in the world around them and with complex tasks. According to the report of the Center for Digital Education (2018), the skills required for the students' success in the future job market are critical thinking, communication, creativity, and collaboration. According to the National Research Council (1999, p. 2), to adjust to the rapid technological changes and to master effectively the information technologies that are spreading in an uncontrolled manner we need three types of knowledge: contemporary skills, foundational concepts, and intellectual capabilities.

2.1.4 Development of the 21st Century Skills in the Learner

The topic of the skills of the 21st century is a topic that greatly occupies those who address the curriculum. One of the tasks of the future school is to give the students skills that will prepare them well for life in the 21st century. The question that arises today regarding the use of technology is now whether the technology is good, but how is it possible to develop and improve the use of technology for the improvement of the learning and its quality (U.S. Department of Education, 2017).

The use of technology in the field of education was undertaken in two directions, from the rationale to the instrument or from the instrument to the rationale. The intellectual partnership between the learner and the suitable computer software programs can support constructivist learning. The transfer of some of the burden of the intellectual functions from the learner to the instrument (offloading) will free the learner from secondary activities and thus he can focus on high order thinking tasks (Salomon, 2000). According to Voogt and Roblin (2012), 21st century skills include collaboration, communication, digital literacy, citizenship, problem solving, critical thinking, creativity, and productivity.

High order thinking. According to Levy and Murnane (2004, p. 47), the schools of today following the technological revolution began to prepare students for master skills, which require a high order of thinking, such as ability to solve new problems, critical thinking, evaluation of possibilities and alternatives, making decisions, drawing conclusions, and critical assessment of information sources. According to Mioduser, Nachmias, and Forkosh (2006), the ICT environments are rich in information, visual representations, and tasks that enable a high level of thinking.

Communication and collaboration. Collaborative learning is the basis for the promotion and development of social skills and communication ability (Ministry of Education, n.d.-a). According to Levy and Murnane (2004, p. 47), this is one of the master skills that must today be inculcated in students so as to prepare them for the new work world. he ability to build relationships with others to solve problems, the ability to work in a team, awareness of the need for collaboration, the involvement of peers in information and knowledge, partnership in the learning communities, ability to manage a dialogue and conversation, with the exchange of information and the development of ideas (Estervan et al., 2017). Collaborative learning places the learner at the center. It includes experiences that emphasize the teamwork and reciprocal relations and interaction among all the group members for the finding of a solution to the problems of the real world (Horizon Report, 2017).

Literacy in the ICT era. The identification of knowledge, the management of knowledge, ability to connect, and attribution between new knowledge and existing knowledge, search strategies, merging of knowledge, phrasing of questions, recognition of sources and ability to access them, evaluation of sources, drawing conclusions, and the creation and dissemination of new knowledge in order to function effectively in a knowledge society (ICISL, 2018; Lemke, 2003). Herbert Lin (2000) presents the expression "fluency with information technology", which hints not only at the individual's ability to understand information but also at the ability to phrase and reproduce the knowledge and express the self creatively and correctly. Gomez and Gomez (2007) determine the concept of "reading-to-learn". This is the learner's ability both inside and outside of the school to produce the information and meaning from the given text.

In the era of the 21st century, and three decades ago in light of the rapid development in the world and to adjust to future changes in the business world, the Secretary's Commission on Achieving Necessary Skills (SCANS) noted in the report the importance of encompassing the learner in a learning environment that enables him to experience the true living conditions. They emphasized in the report skills of communication (Gomez & Gomez, 2007). In parallel, Stephen M. Rutherford (2014) maintains that the learning based on elements considered essential to the professional learner's future increase in the learner commitment to the learning process.

2.1.5 Adjustment of the Education System to the 21st Century

The rapid technological development and the availability and dissemination of information accelerated the process of economic, cultural, and social change, thus shocking the education system. Uzi Melamed and Olsen Goldstein (2017) maintain that digital technology has penetrated into the social structure rapidly and powerfully, to an extent that human culture has not previously encountered, and its influence is on the person's environment and on the person himself. Therefore, the use of technology has influenced our lifestyle, which has become more attractive, contributed greatly in the field of the learning and work and quality of life. This change dictated that the education system would change accordingly in terms of the goals of the curriculum, as well as the pedagogy and the manner of assessment (Ates & Alsal, 2012; Cheung & Slavin, 2013; Dede, 2009).

The education system of today must cope with the fact that the students live in an information-rich environment, the "Information Age" (Ates & Alsal, 2012), and the information in it is varied, open, and available at all times. Melamed and Goldstein (2017, p. 108) see the role of the school today to be the training of an "autonomous learner" who can build for himself the direction of development suitable to his goals and who will be aware of his desires. In parallel, David Chen (2006) asserts that education today is found "at a level of crisis", which is universal and multidimensional and composed of variables of achievement orientation, contents, organizational structure, supervision, and pedagogy. Genit Weinstein (2006) holds that to create innovation there must be "two ways" dialogue between the educational establishment and the learners' creativity and needs. Educators must constantly supervise the products so that they will suit the standards.

The adjustment of the education system to the 21st century has the goal of the assimilation of information and communication technology (ICT) and the construction of a new pedagogy. The changes proposed by the use of technologies obligate the education system to change (Salomon, 2000). There is a change from the traditional approach where the basic learning techniques were the absorption of knowledge from the teacher, the earning by rote of knowledge, and individual learning, when the dominant instrument of learning is the book and the notebook. In the traditional system of studies, the teacher will find it difficult to turn effectively and equally to the students of the class (Katz & Ophir, 1996). This traditional approach contradicts the constructivist perception, which is the basis of the new learning environments, which are computer-supported collaborative learning environments, when the learning techniques in it are based on meaningful learning through individual experience in which the learner accumulates information through interaction between human and technological partners, through active involvement over time and solving of real problems.

The use of the benefits of technology in mathematics increases the effectiveness of the teaching and learning process (Gunbas, 2015; Mayer, 2005; NCTM, 2011). The cognitive theory of multimedia learning of Richard E. Mayer (2014) describes the benefits of learning with digital tools based on three assumptions. The first assumption is the *dual-channel assumption*, which argues that learners organize information into two cognitive structures: the visual and the auditory channel. The second assumption is that each channel has *limited capacity* for processing information. The third assumption is that learners need to *actively engage* in cognitive processing with the learning content to build a new mental representation based on their experience. Using digital tools that based on visual representations supports and improves the learning skills in terms of the memory and in terms of the understanding (Drigas, Kokkalia, & Lytras, 2015; Mayer, 2009), develops the students' involvement in their learning, and increases the motivation, the encouragement of reading, and personal interest (Shalita, Friedman, & Hartan, 2011). Learning with digital environment builds in-depth processes of learning and processing and greatly influences the transmission of knowledge and learning processes (Chen and Law ,2016), emphasizes the learner's personal pace, through feedback and peer support (Dunleavy, Dexter, & Heinecke, 2007).

The interactive learning environments enable the learner to be active and responsive in his own learning. Furthermore, they enable the learner to manipulate and control aspects of the presented material, such as movement, enlargement, and form of presentation of the material (Moreno & Mayer, 2007). The effective assimilation of technological innovations in the education system necessitates change in the perception of the concept of traditional learning and transition to new and suitable concepts such as collaborative learning and learning in pairs or small groups, through reciprocal increase of productivity. Collaborative learning develops the learner's cognitive constructs and increases the desire and motivation for the performance of tasks (O'Donnell & Kelly, 1994). According to Gabriella Ilam (2003), mutual social relationships and social mediation are two basic elements in development of the high mental functions such as perception, attention, memory, and thinking. Educational technology applications enhance and provide positive effects on mathematics achievement (Cheung & Slavin, 2013; Slavin & Lake, 2008; Steenbergen-Hu & Cooper, 2014). In Israel according to the Ministry of Education (2010b), the main goals of the national program proposed are based on ICT and on the assimilation of the technology in the school, as well as the building of a new pedagogy that will give the students the skills of the 21st century with emphasis on digital literacy.

2.1.6 The Educational Challenges of the 21st Century

The development of technology and the extension of the uses of it in the world in all sorts of fields obligated the global education systems to rebuild themselves and to develop in a way that will enable the preservation of the correct continuum between the learner's life in the school and the transformation into an adult suited to the new era. Toffler (1980, p. 399) maintains that learning in the era of technology will occur outside of the walls of the school. The separation of the learners by age will become more flexible, when there will be a mix between the older ones and the younger ones, and the education will become a part of the work that will last throughout the life. He emphasizes that the children of tomorrow will grow up in a society that is focused less on the child and on his needs. Hence, the concept of the adjustment of the education system to the skills of the 21st century arises. Consequently, in the year 2010 the Ministry of Education implemented the national ICT program, which has the aim of the assimilation of information technology and communication in organizational, pedagogical, and social aspects of the school (Ministry of Education, 2010b).

The education system has become committed to rebuilding the processes of teaching and learning, through the integration of technological digital means in all the schools (Amir & Avidav-Unger, 2018; Cheung & Slavin, 2013). The development of technology and its integration in learning led to many advantages and also to more than a few challenges that need to be dealt with, in order to ensure effective use of new educational technologies (Mayes, Natividad & Spector, 2015). The evolution of educational technologies requires coping with several challenges such as:

1. The Digital Divide, Ethics, Equal Access to Resources, System Security, Privacy, Appropriate Source Verification and Digital Dependency (Mayes, Natividad & Spector, 2015; UNICEF, 2017b).
2. The development of learning environments. The success of an online learning environment is measured according to the degree of consideration of the requirements, goals, and expectations of the learners and the degree of the inculcation of effective strategies for the management of her time (Song, Singleton, Hill, & Koh, 2003). Learning environments based on technology and computerization are considered adaptive learning environments, which can improve the learning experience and broaden the possibility of self-learning. The person's learning experience (optimal flow) changes as he accumulates knowledge and develops skills (Katuk, Kim, & Ryu, 2013). The digital learning environment that uses interactive tools that build on three instructional design features of feedback, pacing, and guided activity can improve learning (Moreno & Mayer, 2007).
3. The correct integration of the technological means. The definition of the term "educational technology" includes a wide range of technological means-based programs or applications helpful for the transition of learning materials and supporting the learning process in order to improve academic learning goals (Cheung & Slavin,

2013). The rapid technological innovations created a challenge to more effectively design the learning tasks, both formal and non-formal. Educational technologies that support educational goals include improving knowledge, skills, problem solving, and critical thinking skills needed to develop responsible citizens and lifelong learners (Mayes, Natividad, & Spector, 2015).

4. The coping of the system with uncertainty. Educational technologies need to be clear and based on expectations and at the same time prepared for unexpected evidence (Mayes, Natividad, & Spector, 2015). People need to improve their capability by learning in order to have new skills to adapt themselves to the information age (Johnson, 2009). Learning is a process that is not limited to school age; it is lifelong learning that depends on diverse situations (Ates & Alsal, 2012).
5. The re-building of the teacher's role. In the future, teachers have to cope with the best way to integrate various educational technology applications into the curricula (Cheung & Slavin, 2013). Educational technology suggests that teachers put forth significant efforts in educational technology integration (ETI) (Mayes, Natividad & Spector, 2015). However, teachers who implement a new method of teaching need to be whole with three things: *congruence*, the practical value of the reform, *instrumentality*, its suitability to the school structure, and *benefits*, whether this is worth the effort (Shirley, Irving, Sanalan, Pape, & Owens, 2011). Therefore, successful integration of educational technology depends on teachers' beliefs on and experiences with the technology that impacted pedagogical change (An & Reigeluth, 2011). Hence, it is important to empower teachers and increase their commitment to technology that improves teaching-learning processes (An & Reigeluth, 2011; Mayes, Natividad, & Spector, 2015). In the digital era the teacher's role is to be the instructor and mediator between the learning content and the learner (Ministry of Education, 2010b). Jonathan Bergmann and Aaron Sams (2012) maintain that the teacher's role has changed over in the era of technology. The teachers` role is that of a facilitator who directs and arranges the process of a productive discussion and generates new experiences (Klarin, 2016). Weinstein (2006) emphasized that the teacher has constructive and creative thinking during the change and her importance is in the re-examination of the needs of the field

so as to perform constant changes for the improvement of the system and the development of appropriate pedagogy.

2.1.7 Effective Assimilation of Technology as a Promotion of Pedagogy

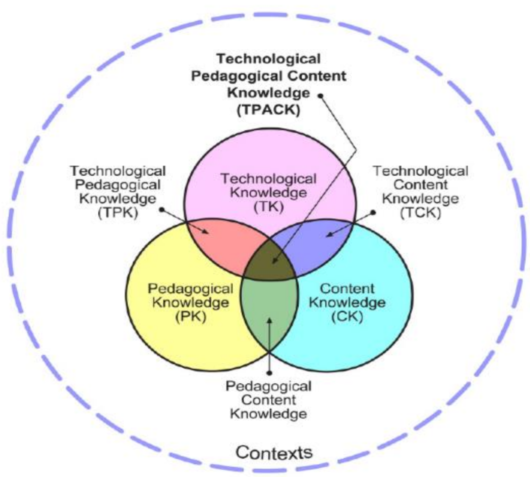
Why do we need this, and why do we need to join and invest efforts and considerable money and resources? If we know how to answer this question, then we will know our goal and way of success. Tiffany A. Koszalka (2001) maintains that the use of ICT in teaching in the schools has the power to promote the process of teaching and learning. Sumedha Chauhan, Neetima Agarwal, and Arpan Kumar Kar (2016) hold that if the technology becomes an integral part of pedagogy it can serve as a powerful tool for increasing learning effectiveness.

The education system of today is committed to adjust itself to the world that is rich and accessible in the learner’s diverse information sources. Therefore, the school has the role of training learners with high independent thinking skills who know to move safely and to move independently to achieve their goals (Melamed & Goldstein, 2017; UNESCO, 2009) since education of today is less to teach and more self-learning. In contrast, Chen (2006, p. 12) holds that there is a gap between the renewal of scientific information in the world and the knowledge proposed in the curricula and in the schools, which influenced the difficulty in the development of new appropriate pedagogy (UNESCO, 2009). Effective assimilation of ICT in the education systems obligates a clear definition of the goals, the supply of software programs and digital contents, the professional development of the teachers, and the flexible approach of both the teachers and the students to the sources of digital information through the development of methods of assessment of the product so as to increase the effectiveness and improve the process of integration.

The content was the essential component of the education system for hundreds of years. From the 1980s and through the 1990s, the dimension of pedagogy and teaching skills developed, and the definition of a good teacher and of the necessary skills was formed. In the 2000s technology joined the two dimensions of content and pedagogy. Today there are a number of new models of teaching through the use of technology, when the leading one is called the TPACK model (Technological Pedagogical and Content

Knowledge; Koehler & Mishra, 2009). According to the TPACK model, the education system is composed of three basic circles: pedagogy, learning content, and the use of technology in teaching. The TPACK model was presented in the work of Shulman (1986, 1987), who presented the PCK, in which he described the teachers' understanding of educational technologies and the importance of the interaction with the PCK as a means of the creation of effective technological teaching. According to Matthew Koehler and Punya Mishra (2009), in this model (see the following figure) there are three main components of teacher knowledge: content, pedagogy, and technology. The most important is the correct interaction in and among the bodies of knowledge represented by PCK, TCK (technological content knowledge), TPK (technological pedagogical knowledge), and TPACK, so as to create effective teaching and assimilation of the content and skills.

Figure Number 6: The TPACK Framework and Its Knowledge Components (Adapted from Koehler & Mishra, 2009, p. 63)



Education systems in the world recommend to teachers to allow the learners to use technological instruments and to emphasize the essentialness of the use of the support of their learning, in other words, that the information and knowledge sources will be relevant, with a goal and personal benefit for the learner. They must ascertain that the learners use technological tools to find, manage, and create content. The perspective is that the technological instruments are instruments that provide flexibility in terms of the ability to learn at all times and from all places (Conole, de Laat, Dillon, & Darby, 2006). The

TPACK model shows that technology integration requires, beside the technical skills of teachers, the emphasis of professional development, which is important in order to create problem-based learning environments that are based on meaningful uses of technology (Ertmer, Lehman, Park, Cramer, & Grove, 2003).

2.1.8 Models in the Assimilation of Technology in the Education System

The education system and the schools have begun to adopt the use of technology in the schools, and this is to close part of the gap that has developed between the lives of the learners outside of the school and the learning system, a gap that was expressed in values, ways of learning, and ways of approaching the information (Sancho, 2009). This gap led to the disconnection between the learners' learning patterns and their lives outside of the school. The use of ICT as a part of learning has increased the students' participation in the lessons as well as the hours of learning invested in the learning and has improved the class dynamics and cooperation (Heggart & Yoo, 2018). The assimilation of educational technologies in the education institutions occurs according to one of two models: islands of innovation and comprehensive innovation (Blau & HaMeiri, 2012).

- In islands of innovation, the assimilation focuses on a certain content field or defined task and exists only in part of the educational organization. According to Juana M. Sancho (2009), despite the use in the schools of technological means such as smartboards, interactive boards, computer laboratories, laptops, and PowerPoint presentations, most of the lesson remains fundamentally a structured lesson, generally the process of assimilation that is conducted according to this model leads to an essential change in the organizational culture.
- The process of technological assimilation according to the model of comprehensive innovation exists in all the layers of educational organization and can create a new organizational culture. David Chen (2006) identified a change from "third order" the goal of which is change of the education system on the level of the community, district, or state.

The integration of technology into teaching is a complex task that depends on the coping of the system with many challenges, so that there is no "one best way" to integrate the technology into the curriculum (Koehler & Mishra, 2009). Therefore, integration efforts should be creatively designed or structured for particular subject matter ideas in specific classroom contexts.

2.1.9 Directions for the Development of New Technology-Based Pedagogies

2.1.9.1 Connected Classroom Learning and Interactive Learning Environments

Today learning should be changed from traditional frontal learning to learning based on the use of technology, learning that enables students to cooperate with themselves and to practice new ways of problem solving. A computerized and technology supported class enables cooperation between the learners and their teachers. The use of technology and digital means enables flexibility in the form of the preparation of the lesson and in the learning methods. The use of digital instruments offers the learner complete and easy access to all the learning materials from every place and at every time and enables immediate, easy, and daily communication between the teachers and their parents. The class is linked with the outside world, when the learner can create a relationship and develop cooperation with their peers in other classes and even in cities and countries around the world.

Today's classroom is a connected classroom. A connected classroom is based on two principles: how is learning performed, and who will perform it? (Center for Digital Education, 2018). The development of technology enabled the improvement in the systems of communication, and the appearance of classroom connectivity technology (CCT), which uses wireless communication that links between the teachers' computers and the students' handhelds (Pape, Irving, Owens, Boscardin, Sanalan, Abrahamson, & Silver, 2013). The advantage of handheld devices are that they are relatively inexpensive, and therefore it is easy to integrate them as a part of the educational process (Allan, Carbonaro, & Buck, 2006; Cheung & Hew, 2009). According to Jason MacLeod, Harrison Hao Yang, Sha Zhu, Yinghui Shi (2017), the connected classroom is a class that encourages, and its importance lies in the increase of the flexibility in terms of the educational processes that occur in the classroom. Online learners have the “ability to be both together and apart and to be connected to a community of learners anytime and anywhere, without being time, place, or situation bound” (Garrison & Kanuka, 2004, p. 96). The connected classroom technology (CCT) is improving the development of two learning opportunities: the representational fluency and the flexibility of students to manipulate and represent to the entire classroom a rich construction (Gunpinar & Paper, 2016). CCT is one of a wide range of interactive assessment devices that enable easy and effective communication in terms of the collection of data about the learner (Shirley, Irving, Sanalan, Pape, & Owens, 2011).

The use of interactive digital tools should provide three crucial elements (Hillmayr, Ziernwald, Reinhold, Hofer, & Reiss, 2020):

- Dialogue that is based on the feedback of the learner`s solution.
- Control of the individual learning pace and the best representation.
- Manipulate the presented information using the learner`s preferred order in the learning environment.

For learners, using features of the interactive learning environments based on cognitive activity are improving deep learning and enabling learners to construct their knowledge (Moreno & Mayer, 2007). The feature of interactivity enables learners to be more active on the construction of new learning content and their learning processes (Hillmayr et al., 2020). For teachers, instructional design using interactive learning environments provides features of feedback, pacing, and guided activity that improve the learning processes (Hillmayr et al., 2020). Also teachers have the ability and awareness for the needs of more students (Garrison & Kanuka, 2004). Effective and intensive use of technology innovations improves the learning experience (Hsu, 2016), influences the students' levels of thinking (Garrison & Kanuka, 2004), and has positive impact on student achievement (Cheung & Slavin, 2013; Shirley et al., 2011). According to Freddy Mampadi, Sherry Y. Chen, Gheorghita Ghinea, Ming-Puu Chenc (2011), interactive environments that are based on a personalized learning system and adapted to the needs of different learners` cognitive style could improve students` performance and achievements.

2.1.9.2 Technology-Supported Discovery Learning

Discovery learning is a type of learning that has gained momentum following the development of digital technology, so that the learning is no longer dependent on a certain time or place and the acquisition of knowledge has become flexible. The learner collects the information from different sources and diverse forms of representation according to the need and the goal (Melamed & Goldstein, 2017). This method enables the effective utilization of the lesson time between the teacher and her students (Bergmann & Sams, 2012). Discovery learning such as problem-based learning (PBL), case-based reasoning (CBR), and inquiry learning (IL) are methods of teaching that are similar, including pedagogy, when the learner is responsible for his learning and for the search of information, the concepts, and the understanding of the content (target information). In discovery learning, the learner receives help such as providing instructions, use of assistive material (lectures, viewing relevant films, summaries). This type of help is considered a scaffolding, to reach the goal and to end the task successfully (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011).

2.1.9.3 Transformation of the Class: The Foundations of the Flipped Classroom

The business world expects to obtain a creative graduate with higher order critical thinking skills (Bristol, 2014). Therefore, the education system needs to challenge itself with pedagogy and new learning methods that will provide a solution to these requirements. One of the approaches of learning that developed with the rapid development of technology around the world was the flipped classroom. The flipped classroom is a model that developed quickly because of the technological development that transformed what was accepted and instead to listen to the teachers' lectures or lessons the students observe at home short video clips, films, or presentations of the learned topic before they come to the classroom (Lafee, 2013; Sherzer-Glazer, 2013). In the class itself, the lessons will be dedicated to more in-depth learning and implementation and this analysis improves the learning and enables the teacher to reach with the learners' high levels of thinking (Bristol, 2014).

The flipped classroom has become one of the approaches that develop high level thinking, and this has increased in recent years following the rapid development of the technology (Lafee, 2013) and the ability to prepare at home films and presentations using applications and almost for free. Two goals will be shaped during the paradigm change and the flipped class's construction: higher order thinking and realism (Bristol, 2014). These are two skills that are important to the learner in his future development. According to Min-Kyung Lee (2018), the flipped class has positive long-term influences: the learning and teaching are more experiential, an improvement in apparent in the learner’s self-confidence, and the flipped class becomes more equal and democratic. Of course, this influences the building of a learner who can adjust to all types of learning. In contrast, Sherzer-Glazer (2013) holds that the difficulty in this model focuses on the requirements and considerable effort that is required of the teachers in the preparation of the materials, the loss of the learner’s relationship with the teacher, the digital gap between the learners in terms of the access to the technology, and the loss of the learner’s free time at home dedicated to the free activity with friends and family.

To flip a class, there is still no clear policy or instructions how to do this (Lafee, 2013). Therefore, the flipping of the teacher’s class needs to think about a number of components: effective use of innovation during the face-to-face learning with the learners, the empowerment and motivation of the learner, the learner in the center, emphasis on high order thinking and not through learning by memorization (Morris, Thomasson, Lindgren-Streicher, Kirch, & Baker, 2012). Minkung Lee (2018) emphasized that the flipped class motivated the learners, the learning was active, and the learners improved their learning achievements.

2.1.10 Barriers that Delay the Assimilation of ICT in Teaching

For more than two decades there has been the use of ICT in the education system, but its influence still is not clear, and it is difficult to measure it (UNESCO, 2009). Therefore, many countries preferred to slow the pace of the technological integration in the system. Peggy A. Ertmer (1999) maintained that technology, like other innovations, brought many advantages and challenges. The barriers to the entry of technology into the education system are connected around two main focuses:

- Management organizational barriers related to the pedagogical and technological adjustments needed to assimilate ICT into the system. Since the definition is vague and not sufficiently clear of "what is considered as success" tension is created and awakens question marks regarding the desired processes of assimilation (Aplalo, 2012; Kaniuka, 2012).
- Barriers related to teachers include perceptions and beliefs regarding the ICT alongside their teaching.

According to Ertmer (1999), there are two main barriers in the assimilation of technology**. First order** barriers are external barriers that include lack of support of teachers, difficulties in access or use of computers, and deficient technical software programs and managers, and in addition time is lacking for the construction of guidance and support for the teachers (Hsu, 2016). According to the TALIS[5] research results, which were published in the year 2018 (Ministry of Education, 2018b), most school principals reported that the set of possible barriers that harm the school's ability to provide quality teaching for its students includes the lack of appropriate physical conditions. In the research, 50% noted the teaching spaces (as opposed to 25% in the OECD countries), about 40% of the principals in Israel noted the lack of digital technology for teaching, or that what exists is not suitable, and lack of physical infrastructures (as opposed to 25% in the OECD countries), and 34% noted limited access to the Internet (as opposed to 19% in the OECD countries).

Second order barriers are internal barriers, related to the teachers' belief in computerization and the ability to the technology to effect the change needed in pedagogical terms (Ertmer, 2005). Aliza Amir and Orit Avidov-Ungar (2018) indicate a

[5] Teaching and Learning International Survey. This is a large-scale international survey that engages in teachers of the seventh to ninth grades, the principals of their schools, and the school environments. The survey is held by the OECD. The TALIS research examined the perceptions of the teaching staffs in the seventh to ninth grades. Teachers and principals from middles schools from 48 member states participated in this iteration of the research. The research results provided comparative and useful information that enables the examination of the educational policy of every state, the identification of the required changes in the system, and the learning how other education systems cope with similar challenges.

“disciplinary barrier”, when the teachers see the technological instruments and the use of them as a need for another interesting interactive instrument that creates interest and diversity in the lesson, and this indicates a lack of understanding of the teachers that the technological instruments were intended to promote a certain area of knowledge. According to Koehler and Mishra (2009), professional development even emphasizes this barrier, when it offers the teacher a “one size fits all approach to technology integration”, when in actuality the teachers deal with the challenges of teaching and learning in different contexts. Pi-Sui Hsu (2016) adds the barrier of competence of learners and teachers to use a computer.

The use of information technology in teaching involves risks and side effects, which are often overlooked or minimized (Spitzer, 2014; Johnson, Jacovina, Russell & Soto, 2016). The technology has negative impact on education on four areas (Alhumaid ,2019):

- Deterioration of students’ competencies in reading, writing, and arithmetic, which are the basic three skills any student is expected to master (Spitzer, 2014).
- Dehumanization of education in many environments and distorting relationships between teachers and students.
- Isolation of students in a digital and virtual world that distances them from any form of social interaction (Chen ,2020).
- Deepening of social inequalities between the wealthy and the poor.

To minimize these impacts, and for effective technology integration we, as teachers, parents, educators and stakeholders, need to work together and to provide continuing education to teachers (Johnson et al., 2016).

2.2 Importance of Teachers in the Assimilation of Technology

Teachers are the foundation stone in the development of the learning community, and they are the ones responsible for the building of the knowledge among the learners (Brody & Davidson, 1998). Therefore, it is very important in the processes of the development of new approaches to learning and teaching to focus on the development of the professional teacher. The main work in the schools begins and ends with the teachers, who are responsible for and constitute the direct contact with the learner.

There is a change in the role of the teacher, the learner, and the processes of teaching and learning (Mioduser, Nachmias, & Forkosh, 2006; Nachmias, Minz, & Ben Zadok, 2006). In environments rich in the use of technology, the learner advanced at a personal pace, is more active and investigative, and the feedback is immediate. The teaching methods are based on cooperative learning. The teacher in the ICT environments chooses the contents important for learning and the appropriate teaching instrument. He serves as an instructor for learners and guides them. Koszalka (2001) finds that the degree of use of ICT in schools is influenced by the teacher's pedagogical perceptions and her belief in the importance of the process and the degree of support. According to Aliza Amir and Orit Avidov-Ungar (2018), when the teachers see technological tools as another instrument that is not inherent in the learning process and when difficulties are encountered, the choice is made not to use this instrument. To improve and increase the use of technology and to leverage the online learning we need to see to the teachers who take part of the burden in the assimilation. According to the Center for Digital Education (2018), technology is a tremendous source of power for the teachers, and therefore they need to be provided with technical support to start and they need to be given constant professional training, when each one advances at her own pace. Ertmer et al. (2012) maintains that the teachers' knowledge and ability to use technology and the teachers' beliefs about the effectiveness of the use of technology are considered problematic barriers to the assimilation of technology in schools. Ertmer suggested placing the emphasis in the professional development of teachers on the reinforcement of attitudes and the teachers' attitude towards the use of technology. According to Koszalka (2001), a positive attitude of the teachers towards the use of ICT is an essential condition in the assimilation of innovation in the schools. He proposed to build support groups in which the teachers will share successes, experiences, and challenges. Aliza Amir and Orit Avidov-Ungar (2018) proposed a focused professional development in which it is emphasized that the integration of ICT is essential in the neutralization of the disciplinary barrier.

2.2.1 The Role of Teachers and Students in the ICT Age

A number of educators brought up different models of the roles of the teacher and the student and the interaction between them in the technological era (Siemens, 2008). The common denominator among all the models is the integration between the concept of educator expertise and the learner's ability to construct knowledge. The teacher's expertise lies in the dynamism of the role, while giving information and assessment on the students' activities, as opposed to the learner's building of knowledge through the cultivation of high levels of student involvement in their manner of learning (National Survey of Student Engagement, 2007). Since the learners use technological instruments for communication, collection of information, cooperation, and playing (New Media Consortium, 2007), these technological means of learning enable the learners to find and synthesize information and integrate across multiple sources of data (Conole, de Laat, Dillon, & Darby, 2006).

The use of digital technology brought the learner from frontal learning, in which the learner is more passive, receiving the information, to active learning, in which he creates knowledge. In the classes and subjects in which there is the use of digital technology, the learning is carried out in a flexible manner (Bergmann & Sams, 2012). The learner advances at his pace, is helped by many interesting and experiential materials. The learner can assemble and use different resources of technology at a time convenient for him, and for as long as he needs. The learning occurs independently or in cooperation with friends of the same age group in online communities (Horizon Report, 2014). The use of digital technology in the fields of teaching and learning increase student engagement; the students become more involved in their learning, they manage the content they use, the pace, and the style of learning, and they choose the means in which they illustrate their knowledge (Millard, 2012).

The increased use of collaborative computerized tools created and increased the learners' autonomy and mastery of the access to the information. The rapid change in the technological world puts the education system under the pressure of carrying out a change of the teacher's role and teaching process (Siemens, 2008). Fezile Ozdamli and Gulsum Asiksoy (2016) emphasize that unlike the traditional approach, in which the teacher is the source of knowledge, in the digital era, with the increased use of technology both in teaching and in learning, we have shifted from the approach of the teacher in the center to the approach of the student in the center. Technology innovation redesigned the teachers` role from the deliverer of content to the role of facilitator in problem-based contexts (Graves, Abbitt, Klett, & Wang, 2009). Technology-based pedagogies produce a new relationship between teachers and students (Spector, 2007). Teachers who use ICT in their classes and direct the learners regarding the effective use of technology develop their students' computer and information literacy (CIL) (Fraillon et al., 2014).

The teacher in the digital era has constructive and creative thinking during the change, and his importance lies in the re-examination of the needs of the field so as to carry out constant changes for the improvement of the system and the development of suitable pedagogy. The teacher adopts approaches of instruction that will suit the learners for the learning purposes (Center for Digital Education, 2018; Horizon Report, 2014). Teachers explain lack of understanding and close content gaps between the learners through the dedication of more time to the learners who have personal difficulties (Bergmann & Sams, 2012; Melamed & Goldstein, 2017; Tomlinson, 1995; Weiss, 2010). Elizabeth Millard (2012) holds that the teachers determine the continuation of the lesson on the basis of the learners' questions and thus create interactive discourse conditions. The technology is a tremendous source of power for the teachers, and therefore the empowerment of the teacher's role has become a main objective in the education system (Day, 2012; Fraillon et al., 2014). The intricacy and intensity of the change in the last two decades and the appearance of the new generation of "screen culture" influenced the teachers` work and lives (Day, 2012).

2.2.2 The Education System in Israel

The right to education is one of the basic social rights that were included among the human rights adopted by the United Nations in the year 1948. The right to education and the possibility of access to education for every child and adolescent were determined in the international convention on the child's rights. In Israel, this right is anchored in the Student Rights Law, 2000 (Nevo Website, n.d.), according to which "every child and youth in the State of Israel is eligible for education according to the provisions of all law". The goal of the law is "to determine principles for the student's rights in the spirit of human dignity and the principles of the United Nations Convention on the Rights of the Child". In Israel, the implementation was through the legislation of the Compulsory Education Law, 1949 (State of Israel, 1949), which assigned to the state and the local government the obligation to operate official institutions of education for children and youths. At the date when the law was legislated, the mandatory period of studies was set to be the first to the eighth grade (then the elementary school period). However, over the years the period was gradually extended, until its present situation, from age three to the end of the twelfth grade.

The present structure of the education system in Israel includes formal education and informal education. Formal education is found under the responsibility of the Ministry of Education and is based on the state budget. The education system in Israel is composed of four divisions: age-based division (stages of education), legal status of the education institution, type of supervision of the education, and population sector.

Division by Age Group

Formal education is divided into four main stages of education according to the students' ages:

1. Pre-elementary education, ages 3-5.
2. Elementary education, ages 6-13.
3. Secondary education, divided into middle school, seventh to ninth grades, and high school, tenth to twelfth grades.
4. Post-secondary and academic education, for ages 18 and above.

This main division is the basis, but in every level of education there is a variety of frameworks with possibilities of the combination of two stages and the division of every stage (Weissblay & Winiger, 2015). Informal education includes actions in society and among youth and in the areas of adult education.

Division by the Legal Status of the Education Institution

In the education system in Israel there are a number of types of education institutions (State Comptroller, 2014).

1. Official education institutions are the state education institutions and state religious education institutions funded completely by the state and the local governments and most of the teaching workers therein are employed by the state.
2. Recognized non-official education institutions are those that are not owned by the state but are supervised by it. These institutions received budget from the state at a rate less than that given to the official education institutions. These institutions have greater freedom in accepting students, in employing teachers, and setting curriculum.
3. Exemption institutions. On the basis of section 5 of the Compulsory Education Law, 1949, the law allows students to be exempted from the provisions of the law under certain conditions. These institutions do not accept the supervision of the state but it budgets them at the rate of 55% of the budget per student in official education.

The difference between the three types of institutions is in the degree to which they are subordinate to, funded by, and answer to the supervision of the state.

Division by Type of Supervision

1. State. Non-religious institutions in the Jewish and non-Jewish sectors. The education is supervised
2. State religious. Zionist religious institutions. This is state education whose institutions are religious in their lifestyle and curriculum implemented in them and the teachers and the supervisors are religious.
3. Ultra-Orthodox.

Division by Sector

- Jewish: Most of the children who learn are Jewish and not Arabs.
- Not Jewish – Arab, Bedouin, Druse, and Circassian.

Israeli society has a diverse and complicated human texture. This complexity is reflected in its education system, which is heterogeneous, expressed "in different levels in the structure of the system and in its budget and in the existence of many types of education institutions adjusted to the needs of the different sectors" (Weissblay & Winiger, 2015).

2.2.3 The Israeli Education System in Connection with Arab Society

The level of education is one of the dimensions for the determination of the level of modernization and development of human society. Therefore, one of the most important challenges facing the Israeli education system is the reduction of the gaps on the one hand between the Arab education system and the Jewish education system and on the other hand between the Arab education system and the needs of the Israeli Arab population.

The inculcation of education for the Arab population improves its status and realizes the human potential of the growing population (Haj-Yahia & Rodnizki, 2018). According to the data of the Central Bureau of Statistics of Israel, updated for the beginning of 2018 (Central Bureau of Statistics of Israel, 2018), the Arab population is about 1.85 million. According to the report of the Center of Research and Information of the Knesset in 2018 (Winiger, 2018), the number of students in Arab education is about 555,000 students, who constitute about 25% of all the population of the students in the State of Israel. 45% of the students learn in elementary schools, 20% of the students learn in the preschools and kindergartens, and 35% of the students are secondary students. The number of middle schools is 213, with 3,193 classrooms and 85,837 middle school students. According to the OECD report, published in September 2018, classrooms in the State of Israel are among the most crowded of the member countries. The mean of students in the study classes in Israel in the elementary schools is 27, when in the advanced states the mean is 21 students in a class. The trend continues in the middle school, so that the mean of the number of students in Israel is 29 as opposed to the mean of 23 in the OECD member states. The rate of dropping out of students aged 14-17 in Arab education is higher by about 4% than the percentage of dropping out in Jewish education (Central Bureau of Statistics of Israel, 2018). The percentage of overt and covert dropping out in the Arab education system is about 20%. The phenomenon of dropping out occurs for the most part at the ages of the transition from the middle school to the high school, and most of the dropouts are boys (Abu Asbah, Fresko, & Abu Nasra, 2013).

We note that in the Arab education system in Israel the language of instruction is Arabic. The Arab education system in Israel is divided into four sectors: Arab, Druse, Circassian, and Bedouin. A report published in the year 2019 by the Taub Center for the Research of Social Policy in Israel (Blass, 2019), relying on the data of the Ministry of Education, determines that the gaps in the budgets between Jewish society and Arab society – both for the student and for the class – are still very large. Although the percentage of increase in the budget of Arab education was higher than in Jewish education, the budget per student in Jewish education is considerably higher than the budget per student in Arab education, when the student learns in the school at the same level of cultivation. Thus, it becomes clear that not only the budget that the Arab student in formal education receives does not cultivate him but also in essence the Arab student is discriminated against. If the calculation includes the budgets that the local authorities and the parents add to the education basket, the budget that the Arab student in the State of Israel receives is lower by 78%-88% than that of the Jewish student (Orlozorov, 2016). The gaps between the Arab education system and the Jewish education system are apparent in the lack of structures, in the study classes, in the laboratories, and in the gymnasiums, and even in the inadequacy of existing buildings and facilities (Abu Asba, Fresko, & Abu Nasra, 2013). There is discrimination in the status of Arab education also in the political aspect of the allocation of resources, the non-inclusion of Arab communities in the regions of national priority although their social-economic situation obligates preferential reference[6]. It is important to note that in ideological terms **the Arabs are not involved in the setting of the learning contents[7]** and the Arab population is ignored as a national minority and as citizens of equal rights (Abu Asbah, 2006, 2007).

The Dovrat Report (2005) recognized the importance of the reduction of the gaps between the two sectors, while promoting and constantly improving the academic

[6] The intention of this reference is what is called the "cultivation basket", or in other words, the investment in weak populations in which the children suffer from an educational lack.

[7] The Jewish shapers of policy are those who determine that learning contents and values according to which it is necessary to educate the Arab students.

achievements of all students, cultivating the school climate, life skills, prevention of violence, and use of drugs and alcohol, cultivating values and heritage, promoting the status and professional development of teaching personnel, improving the perseverance and reducing the covert dropping out. Therefore, one of the important recommendations was to allocate resources in a differential manner, namely, to allocate according to the personal data of the learner according to the number and profile of the students learning in the budgeted institution. Most of the students in the Arab population belong to the weak and moderate weak cultivation group, as opposed to most of the Jewish students who belong to the strong cultivation and moderate strong cultivation group. This gap is reflected in the poor social-economic situation of the Arab population in Israel, and to close this gap the Ministry of Education is supposed to budget differential hours, subordinate to the cultivation indices. According to the survey of the DeMarker newspaper from the beginning of 2016 (Orlozorov, 2016), which was prepared following the adoption of decision 922 (Prime Minister's Office Website, n.d.), the Ministry of Education was supposed to budget differential hours in the Arab education system, for the sum of 11.6 billion shekels a year for the years 2016-2020. The decision was not implemented, and the Ministry of Education refused to allocate the budget although it was its decision. According to the Mossawa Center, the Advocacy Center for Arab Citizens in Israel (2019)[8], the budget of the Ministry of Education of the year 2019 does not include a section of differential budgeting for the Arab education system, although the book of the main tenets of the budget emphasized the reduction of gaps and the allocation of additional learning hours for students from a weak socio-economic background. The unequal allocation of the Ministry of Education of the reinforcement classes, which were supposed to be budgeted primarily in the Arab communities following their weak social-economic background and their moderate-weak position on the cultivation indices, was also apparent. However, from a budget of 2 billion of programs, the Arab communities received only 18.5% of all the allotted reinforcement classes.

[8] Mossawa Center, the Advocacy Center for Arab Citizens in Israel, was established in the year 1997 and registered as a nonprofit association in the year 2000. The center acts to promote the political, social, economic, and cultural rights of the Arab citizens in Israel. The budget of the Ministry of Education for the year 2019 and Arab society: http://www.mossawa.org/heb/?mod=articles&ID=800

The State Education Law does not take into consideration the needs of Arab education, blurs the national-ethnic identity of the Arabs, and educates Arab children for inferiority and idleness with the Jewish majority. This is expressed in the curriculum that does not express the cultural and national uniqueness of the Arab minority (Abu Asbah, 2006). The Ministry of Education adopts the integration approach towards the Arab minority and does not give autonomy to Arab education (Haj-Yahia & Rodnizki, 2018). These factors influenced the level of achievements in the Arab education system, which is far lower than that in the Jewish education system (and this will be discussed in the section in the continuation).

The Arab education system in Israel has seen accelerated development from the establishment of the state until today. Nevertheless, socio-economic gaps between Arab children and youth and Jewish children and youth still exist and it is clear that the improvement is not fulfilling the growing needs of Arab society in the field of education. The Ministry of Education is aware of the gaps between the two sectors, and along with the government in Israel attempts to act with all methods to close these gaps. Thus, the policy of the Ministry of Education in Israel aspires to instill knowledge and skills, to broaden the circle of learners at all levels of education, to reduce dropping out among students, and to increase the level of education in the population at large (Haj-Yahia & Rodnizi, 2018). Khaled Abu-Asbah (2007) sees this to be a very important step but not enough since the Ministry of Education still has the responsibility to act to improve the quality and manner of teaching, in which affirmative action is required. It is necessary to re-build the trust between the community and the education system and operators of the system and in parallel to act to change the dominance of the outdated teaching methods, through the professional development of teaching workers (Abu Asbah, 2006).

2.2.4 The Curriculum in Mathematics, The Ministry of Education of Israel

Assumptions of the Mathematical Education Program in Israel (Ministry of Education, 2013c)

The mathematical curriculum in Israel is built on a number of main assumptions, as follows:

- Spiral Nature

The curriculum is "spiral", and the learner is therefore exposed to the same topic or main mathematical idea a number of times during the three years, in all the areas. The repetition on the topic is through the deepening and establishing of the knowledge and the adding of layers. This gradual development intends to empower the knowledge in the learner and cause the effective assimilation of the mathematical knowledge through the connection between the rest of the topics learned in mathematics.

- Increased Depth of Mathematical Knowledge

Intelligent integration of the three mathematical areas (numerical, algebraic, and geometric) in the curriculum is recommended, through the diversification of the instructional instruments (use of technological instruments and different visual illustrations, such as drawings, paper folding), for the purpose of the strengthening and furthering of the mathematical knowledge among students with different learning styles.

- Skills of Calculation and Thinking

Integrated learning of calculation skills, skills addressing the knowledge of the performance of arithmetic and algebraic procedures, with emphasis of the mathematical understanding, will improve the skills that will support the development of the understanding, and the development of the understanding will support the learning and reinforcement of the skills. The learning topics are learned through the development of different levels of thinking: knowledge and identification, algorithmic thinking, process-based thinking (implementation in familiar contexts), and open search.

- Mathematical Literacy

The curriculum emphasizes the need for integrate problems from the learner's real world and to focus on the solving that will bring up conclusions in the context of the problem. From the solution it is possible to understand whether the learner has the ability to determine the reasonableness of the mathematical solution and its logic in relation to and in the context of the problem, while providing a suitable explanation.

- Understanding of Concepts

The learning is based on understanding that includes the understanding of concepts, the finding of relationships between different concepts, and solving problems (mathematical and from other knowledge fields). The work methods and conversation methods characteristic of mathematics are the development of an idea from experiences and generalizations.

The curriculum in mathematics emphasizes the ways of thinking and mathematical discourse that include the use of correct mathematical language, including the understanding of concepts and the building of relationships between the different concepts, the presentation of a problem or an idea through the use of the different representations.

- Definition, Argument, and Proof

The recognition of the definition and its implementation, the use of mathematical language for the representation of problems, for solving problems, and for critiquing the solution, the development of explanations and arguments and giving the explanations and critical examination of other explanations, follow up after processes of proof and their understanding and the ability to build simple proofs, use of representations and different tools, and development of strategies for solving problems.

Structure of the Middle School Curriculum in Mathematics

The curriculum in the middle school includes three content areas:

- The numerical field (including statistics and probability)
- The algebraic field
- The geometric field

The three areas are based on the contents learned in the elementary school (first to sixth grades), while extending them. They are learned in the intelligent combination between them, when the program of study is *spiral* in nature and serves as a basis for the contents that will be learned in high school.

Specifications of the Seventh Grade Curriculum

The seventh grade mathematics curriculum is intended for 150 hours of study (at least) and is built to further and deepen the mathematical knowledge learned in the elementary school by the learner through the inculcation of new contents, when the recommended distribution of the Ministry of Education of the hours is 68 hours for the field of algebra, 30 hours for the field of numbers, and 52 hours for the field of geometry. The curriculum of the seventh grade includes three rounds (Ministry of Education, n.d.-i):

The first round includes three areas:

- Algebra: Variables, algebraic expressions, and generalization of numerical phenomena allocated 15 hours.
- Numbers: Arithmetic actions and their laws, powers and quadratic roots, allocated about 10 hours.
- Geometry: Rectangle, box, perpendicularity and parallelism, allocated about 15 hours.

The second round includes three areas:

- Algebra: Solving equations and word problems, allocated 15 hours.
- Numbers: Negative numbers, positive numbers, and zero, allocated about 20 hours.
- Geometry: The topic of areas, allocated about 12 hours, and angles, allocated about 15 hours.

The third round includes three areas:

- Algebra: Functions, allocated about 18 hours, and equations and word problems, allocated 20 hours.
- Numbers: Negative numbers, positive numbers, and zero, allocated about 20 hours.
- Geometry: Triangle and triangular prism, allocated about 10 hours.

Description of the Topics of Learning of the Mathematics Curriculum according to the Three Rounds (Ministry of Education, n.d.-f)

First Round:

Geometry: Rectangle, box, perpendicularity, and parallelism (15 hours)	Numbers: Arithmetic actions and laws, powers and quadratic roots (10 hours)	Algebra: Variables, algebraic expressions, and generalization of numerical phenomena (15 hours)
Topics of study: - Rectangle - Perpendicularity - Parallel lines - Congruent shapes - Traits of the rectangle - Square - Perimeter and area of a rectangle - Box (interior area of a box/volume of a box/slicing of a box)	Topics of study: - Rules of arithmetic actions - Laws of commutativity and associativity of addition - Laws of commutativity and associativity of multiplication - Impossible to divide by zero - Neutral terms - Opposite numbers - Law of division - Subtraction of a sum: $a - (b + c) = a – b – c$ - Subtraction of a difference: $a – (b – c) = a – b + c$ - Multiplying the divider: $a : (b \cdot c) = (a : b) : c$ - Dividing the divider: $a : (b : c) = (a : b) \cdot c$ - Powers with a natural exponent - Quadratic root	Topics of study: - Variables and algebraic expressions - Insertion of numbers in algebraic expression and calculation of the numerical value of the arithmetic expressions obtained - Equality between algebraic expressions - Collection of similar terms

Second Round

Geometry: Areas (12 hours) Angles (15 hours)	Numbers: Negative numbers, positive numbers, and zero (20 hours)	Algebra: Solving equations and word problems (15 hours)
Topics of study: - Area of polygons, triangles, parallelograms, trapezoids, and general polygons. - Perimeter and area of a circle. - Angles: Equal angles and comparison of angles / addition and subtraction of angles / measurement of angles / adjacent angles / vertical angles / bisecting angles / alternating and corresponding angles / alternating angles between parallelograms / corresponding angles between parallelograms	Topics of study: - Presentation of negative numbers on the number axis, order on the number axis, inverse number. - Four arithmetic operations - Integration of the algebraic in the study of integers - Powers with a natural exponent and base of the exponent is an integer - The system of axes – marking points and reading points	Topics of study: - Equations and solutions. - Solving equations of the first order with one variable. - Word problems that can be solved using first order equations with one variable.

Third Round

Geometry Triangle and triangular prism (10 hours)	**Algebra** Introduction to functions (18 hours), solving equations and word problems (20 hours)
Topics of study: - Triangle - Knowing the triangle - Angles in the triangle / angles in the quadrilateral / angles in polygons - Sides of the triangle - Triangular prism (knowing the shape, calculating interior area, volume, surface area)	Topics of study: - Useful graphs – reading and drawing - Introduction to functions: different representations of functions / change of function / Increase and decrease of function / change of function at a uniform pace and at a non-uniform pace - Solving linear equations - Word problems with linear equations

2.2.5 National and International Mathematical Tests

In the 1980s the neo-liberal wave grew in the education systems in Israel and around the world (Volansky, 2014). This growth was following the low level of achievements in the education systems in the world. The reforms in education appeared so as to improve the students' level of achievements. The policy that was adopted was based on a uniform curriculum that examines achievements through standardized tests, with the publication of these achievements (Sahlberg, 2015; Volansky, 2014). J. Dewey (1857-1952), who formed the progressive perception of education, sought to make the learning relevant for students by connecting it to the experience the students bring, to their lives, to their topics of interest, and to their social needs, through the adoption of pedagogical tools for cultivation (Dewey, 1969). In the 20th century, Dewey's doctrine, which encouraged the need for difference, began to weaken, and a new era in education commenced, which was based on the adoption of the principles of the job market and competitiveness, similar to that of economics, who encourages freedom of educational initiatives and competiveness (Volansky, 2014).

The neo-liberal policy spread around the world (Kliger, 2013), and alongside it criticism developed. Pasi Sahlberg (2015) from Finland maintains that the difference in human society and in student society does not allow the use of a uniform scale of measurement (standardized tests). He believes that the curricula, teaching methods, and method of evaluation need to adjust themselves to the diversity in the students' abilities and tendencies. Sahlberg further emphasizes the importance of the humanistic progressive thinking based on principles that respect the students and the teachers and maintain transparency, collaboration, and culture of trust among all those involved in the educational process.

The standardized tests have become a measure according to which the effectiveness of the curricula and teaching methods are assessed, and they enable the comparison between the students' achievements in one country and those in other countries. According to the Taub Center for Social Policy Studies in Israel (Blass, 2016), since the year 1960 dozens of international tests have been carried out by education systems in the world.

According to the Ministry of Education (n.d.-j), Israel participates in a number of international tests in order to enable comparison between the students' achievements in central areas of knowledge, to examine the relationship between achievements and different factors that have influence on them. In parallel, the tests examine the social, economic, and cultural influences of the state in which the research is carried out on the achievements. These tests enable comparison between sectors and groups in the population of the state and between states. The tests associated with mathematics and the sciences are:

1. TIMSS (Trends in International Mathematics and Science Study)

The research study belongs to the research of the International Association for the Evaluation of Educational Achievement (IEA). It is the oldest international study aimed at assessing mathematics and science knowledge. The TIMSS examines the level of mastery of eighth grade students of the fields of mathematics and sciences, while referring to the existing curriculum that is deployed and achieved in each one of the participating states. The first TIMSS exams were administered in the 1960s (Blass, 2016). The research study is held once every four years, when it was most recently administered in 2019.

The TIMSS in Israel was carried out in the months of April-May 2015 in a representative sample of 5,513 eighth grade students from 200 schools (not including Ultra-Orthodox and special education institutions). The test evaluated the knowledge and skills of the eighth-grade students in mathematics and science and the educational context of teaching these subjects in the different countries. The research results, both in mathematics and in the sciences, indicated that the mean of the achievements of the students of Israel is higher than the international mean of the countries that participated in the research. Israeli is situated in the upper half of the ranking of the countries according to the mean of the students' achievements (rank 16 in mathematics and 19 in sciences, of 39 countries). The percentage of excelling students in Israel is among the highest of the participating countries. In the ranking of the countries according to the percentage of excellent students, Israel is found in the eighth place in mathematics and the ninth place in science. Conversely, the percentage of students who have difficulties is also high, similar to the values of the international median (rank 20 in the ranking of the countries according to the percentage of those with difficulties). In both areas of knowledge, the dispersion of the scores in Israel is among the largest of the participating countries).

From an internal, domestic perspective, analysis of the achievements of the students of Israel indicate apparent gaps are seen, in both areas of knowledge, in the achievements of students from different populations in Israel. First, the achievements of the Hebrew speakers are higher than those of the Arab speakers. The research found large gaps, of about 70 points, in the means of the achievements in mathematics and sciences in favor of the Hebrew speakers. Second, as the socio-economic background is higher, the achievements are higher. There are very large gaps of about 110 points (among the Hebrew speakers) and about 130 points (among the Arabic speakers) in the means of the achievements between students from a high socio-economic background and those from a low socioeconomic background. When students from the same socio-economic group are compared, the gaps in the means of the achievements between Hebrew speakers and Arab speakers are considerably reduced.

2. PISA (Programme for International Student Assessment)

This test evaluates knowledge in mathematics, reading comprehension and science among 15-year-olds (National Authority for Measurement and Assessment in Education (RAMA), n.d.-a). The research study is administered by the OECD – the Organisation for Economic Co-operation and Development (2018b). Many countries around the world participate in it. The questions included in the research examine the acquisition of general thinking capabilities that enable good and effective coping of the students who are close to the end of compulsory education with their environment. The questions in the research examine practical knowledge, life skills, and ability to solve complex problems that necessitate the integration between different fields, with emphasis on skills. The research is carried out once every three years and examines three areas of literacy, but each time special emphasis is placed on one of the three areas (reading, mathematics, and science). For instance, in the year 2003 and the year 2012 the emphasis was placed on the subject of mathematics. The PISA is completely computerized; both the tests and the questionnaires are done in a computerized manner. Israel will participate in the 2021 PISA (OECD, 2018b).

In the last PISA, conducted in the year 2018 (National Authority for Measurement and Assessment (RAMA), 2019), the mean of the scores in Israel was lower than the OECD mean in each one of the three areas of literacy. In the field of literacy of mathematics, for example, the mean score of Israel is 463 points, while the OECD mean is 489. Israel is ranked in the 41st place of 78 countries that participated in the research. The percentage of excellent students in all three areas of literacy together is identical to the OECD mean (3%), while the percentage of students with difficulties in all three areas of literacy together is higher than the OECD mean, 22% in Israel versus 13% in the OECD. In all areas of literacy, the dispersion of the scores in Israel is the greatest in the world. In all areas of literacy considerable gaps were noted in the achievements between Hebrew speaking students and Arabic speaking students, in favor of the Hebrew speaking students, 144 points in reading literacy, 11 points in mathematics literacy, and 116 points in science literacy.

Israel has adopted the international tests and made them standards that influence its educational policy and curricula (Kliger, 2013). In parallel to the international tests, internal assessment tests such as the MEYZAV (a Hebrew acronym meaning positioning and standing for measurements of effectiveness and school growth; National Authority for Measurement and Assessment (RAMA), n.d.-b) were built, with the goal of the evaluation of the intra-school achievements. The standards in these instruments, which include detailed rubrics and guidelines, enable the teacher and the school staff to collect data throughout the year on an individual, class, and grade level. The goal of the analysis of the data is to monitor the process of learning in the school through the formation of pedagogical insights and the promotion of learning through the construction of an intervention program on an individual/class level.

These tools complement the evaluation tools customarily and routinely used in the school. These instruments are used to follow up after the progress of the learning process and to identify content gaps that were accumulated among some of the students. These internal instruments in the middle school include:

- Internal MEYZAV test. The analysis of the results can identify the abilities of the learners in the classrooms and in the schools and their meeting of the expected requirements and the standards defined in the curriculum in the different areas.
- Performance tasks. The goal is the collection of information on a wide range of complex learning skills, such as planning processes, critical thinking, learning processes, and drawing conclusions about the ways of teaching and manner of learning.

The education system in Israel believes and hopes that the analysis of the data and results of the achievement tests, both national and international, will help the education system in general and the school system in particular build a personal individualized work plan that is commensurate with the difficulties and content gaps in every school, when the aim is to allow each and every boy and girl in Israel to realize the best of their ability and to bring about the reduction of learning gaps in Israeli society (National Authority for Measurement and Assessment (RAMA), 2017).

2.2.6 Mathematical Competence of Teachers in the Digital Era

Despite the development of the world in terms of technology and industry, the schools have not changed both in structure and in the manner of carrying out the learning process. The teachers remain the source of information, and they are perceived as the one with authority. National surveys emphasize the importance of the change in the instruments and methods of traditional formal education, with emphasis of its relevance to the nature of developing society (Canadian Council on Learning, 2006) and the cultivation of high levels of student involvement in the manner of their learning (National Survey of Student Engagement, 2007). Since the learners use technological instruments for communication, collection of information, cooperation, and play (New Media Consortium, 2007), the technological learning means enable the learners the "finding and synthesizing information and integrating across multiple sources of data" (Conole, de Laat, Dillon, & Darby, 2006, p. 5).

The rapid growth of science and the dissemination of science through the use of technology and Internet networks have greatly influenced society, so that the instruments for the production of information have become the province of people at large. In society, in this stage, the teacher's role has changed and has become multifaceted when the requirements from the profession's requirements differ from what was customary. This development brought up the question: who is the expert and who has knowledge? This led to the re-consideration of the manner of representation of the knowledge and the process of learning and form of pedagogy, when the focus is on the subjects of mathematics, physics, and chemistry (Scharnhorst, 2003).

The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes that "technology is essential in teaching and learning mathematics; it influences what is taught and enhances students' learning" (p. 24). Therefore, Taylor Martin, Ayiesha Lukong, & Raven Reaves (2007) maintain that the mathematics teachers in the technology-rich environment must have expertise and ability to guide students in a number of directions: planning and implementation of the experience of mathematics that develops the potential in the learner and the desire to learn mathematics, development of a mathematical discourse that advances mathematical ideas, and encouragement of learners to use technological tools and aids that help in the increase of the depth of the mathematical understanding. Yael Yondler et al. (2018) maintain that teachers in the digital era fill different roles in terms of the processes of teaching-learning and assessment, this produces four prototypes of the teacher: the teacher as guide, the teachers as inviting, learning, the teacher as a partner, and the teacher as leading the learning. The common denominator of all the prototypes is the perception of their role as innovative teachers who are willing to try new ways of teaching in a technology-rich environment.

The emphasis today is on teachers who develop ways of teaching learner-focused mathematics more than on the teacher who teaches techniques. The aim is to create active learners who engage in solving problems, who can provide explanations, and who can build relationships between concepts through the use of technology to solve problems related to real life. Robert Powers and William Blubaugh (2005) indicate that one of the most important skills of mathematics teachers in the digital era is to display expertise in mathematical topics and applications. Consequently, the preparation of the teachers of the future who can integrate and use technology has become the paramount objective in the programs of teacher training. The use and integration of technology in a correct manner by the mathematics teachers can influence different aspects in mathematical education, how mathematics is taught and learned, and how mathematics is assessed (National Council of Teachers of Mathematics [NCTM], 2000).

2.2.7 Professional Development of Mathematics Teachers in the Digital Era

The improvement of the education system does not lie in the worsening of the conditions of supervision and the firing of teachers or the adoption of models of management from the corporate business world in the education system (Sahlberg, 2015). Rather, a large part of the success of the education system depends on teachers. In his book, *Finnish Lessons: What Can the World Learn from the Educational Change in Finland*?, Sahlberg describes the principles that Finland adopted and that has led it to be at the apex of achievement for many years. These principles are a high degree of professional freedom for teachers in the implementation of the curriculum, the use of assessment as a diagnostic tool of the teacher for the purpose of the advancement of the student, and not for measurement and competition, the cultivation of cooperation between the teachers, the professional development of teachers, and primarily the establishment of a culture of trust among teachers, principals, parents, local governments, and the political leadership. He shows how all these have contributed to the reinforcement of the teachers' public and professional standing. Andy Hargreaves and Dennis Shirley (2009) show in their book *The Fourth Way* that through training, professionalism, and shared responsibility it is possible to re-shape the education system and make the changes necessary in it. Thus, the closing of the gaps was made through the emphasis of the importance of the professional empowerment of the teachers in two directions:

- The first way is the emphasis of the change, beginning in the teacher's role and the teaching and learning processes (Nachmias, Mintz, & Ben-Zadok, 2006; Nachmias, Mioduser, & Forkosh-Baruch, 2006). The process of professional development will help teachers cope with the challenges of teaching in their classes and with heterogeneity in the levels of learning in the classes (Hsu, 2016). The teacher is the pillar of differential teaching; he looks at the learner, assesses the learning needs, and provides an answer personally to every learner (Keshet & Meor, 2017; Tomlinson & Strickland, 2005). The challenge of tailoring the teaching to the individual and making it personal teaching obligates the educational staff to work on the adjustment of the learning program in a personal manner for every learner according to the learning needs, learning pace, areas of interest, and strengths (Corno & Snow, 1986; Keshet-Meor, 2017). In the TALIS research held in the year 2018, the results of the research in Israel show that the school principals in Israel attribute considerable importance to the guidance activities[9] as a part of the professional training. This action will contribute to the improvement of the teachers' skills and to the extension of their knowledge and professional identity, to the support of beginning teachers, to the promotion of the sharing among teachers, and to the improvement of the students' achievements.

- Second, the topic of the use of technology out of the belief that in technology-rich environments the learner advances at a personal pace (Collins & Halverson, 2009), is more active and investigative, and the feedback is immediate, a criterion that is very important in differential teaching. Professional development empowers the teachers in terms of professional knowledge and improves their abilities as leaders of change. Allan Collins and Richard Halverson (2009) maintain that the use of technology is burdensome to teachers, who are considered "digital immigrants" (Prensky, 2002, p. 1). Thus, to improve and increase the use of technology and to leverage online learning, it is necessary to develop them professionally.

When in the subject of mathematics, the teachers' professional development is a basic condition of the extension of the didactic mathematical knowledge that assures the intelligent integration of the technological innovations and means, this improves the quality of the teaching, the pedagogical skills, and in the areas of knowledge advances and raises the students' achievements. The emphasis of the use of technology comes from the recommendation of The Association of Mathematics Teachers (AMTE, 2006) regarding

[9] Guidance addresses the constellation of activities in the school, during which an experienced staff member (teacher, subject coordinator, school instructor) guides a less experienced teacher. The guidance actions include guidance in the regular staff meetings, guidance following observation of a lesson, personal conversations, and so on.

the training programs of mathematics teachers, which “provide opportunities [for teachers] to acquire the knowledge and experiences needed to incorporate technology in the context of teaching and learning mathematics” (p. 1). In the changing reality of today, the Ministry of Education in Israel sees importance in the development of the teaching strategies through the professional training of the teachers (Center for Digital Education, 2018). The innovations in the changing world are the challenge that the teachers are required to adopt and to be an active part in the development and creation, out of the belief and commitment to create dynamic teachers and to give them the opportunity to attempt current tools. This makes the teachers into having a high level of professional expertise (Beijaard, Verloop, & Vermunt, 2000).

2.3 Methods of Teaching-Learning Mathematics in Israel in the Digital Era

2.3.1 Importance of ICT and Use of Technology as a Part of the Mathematics Curriculum in Heterogenous Classrooms

The shapers of policy in the world, following the heterogeneity and steadily increasing differences among learners and the inability of traditional teaching to provide a solution for coping with the differences between the learners, began to recognize the need to adjust the education system in its transition from a “one size fits all model” to personally tailored teaching (Becker, Rodriguez, Estrada, & Davis, 2016; Tomlinson, 1995). The learning among the learner is carried out effectively only if the teachers attribute importance to “goodness of fit” or in other words, teaching that is tailored to the learner’s manner and preference of learning, teaching that is fundamentally differential (Tomlinson, 2005). Differential teaching is designed according to every learner’s learning preference, the manner of teaching and method are suited to the style and characteristics of the learner’s learning, when the goal is the identity of the knowledge and final content for all the learners (Bray & McClaskey, 2013). Differential teaching is a learning environment that provides an answer to the learners’ different needs, in terms of the skills, abilities, and learning styles. It enables the learners to choose the manner of the learning, the nature and means of activity, the level of the task and the pace of progress; in other words, the teacher determines according to the learner's needs the way of suiting the five elements of content, process, product, learner tendency, and learning environment to the learners (Tomlinson & Eidson, 2003; Tomlinson & Strickland, 2005).

Online differential teaching, which makes use of technology and the database of interactive digital contents, can provide a differential solution to the learners' range of abilities and needs and enables the teacher to support the learners in diverse situations of learning and to provide feedback in real-time on the students' performances, tools of continuous assessment, which enable tailored planning of the continuation of the learning processes, ensure the challenge of the learner, and increase the learner's motivation and ensures the effectiveness of the teaching (Weiss, 2010). The policy of the Ministry of Education emphasizes the importance in the integration of online tasks in teaching mathematics in the heterogeneous classes in terms of the learners' abilities, so as to reach as many students with difficulties as possible and to advance each one and his ability, taking into account his learning style. Differential teaching today, which uses technology, is more flexible, enables coping in better way with large and heterogeneous classes of students regarding their different learning ability (Sota, Clarke, Nelson, Doabler, & Fien, 2014). Dobby Weiss (2010) maintains that the combination between a smart teacher and smart technology produces successful classes. The technology improves the teachers' teaching and the learners' learning. On the one hand, it offers the teachers different tools that help the students reduce the gaps in the different areas of knowledge, and on the other hand this motivates the learners, increases the motivation, and enables academic growth while using in a variety of mediums (Stanford, Crowe, & Flice, 2010).

The differentiated instruction with technology is an effective tool for teachers and students because of the ability of teachers to modify student progress (Cobb, 2010), to support the learners' needs (Logan, 2011), and to monitor the students' learning processes, thus leading to success (Bicard, Casey, Nichols, Plank, & Finley, 2009). The use of technology in differential teaching enables the teacher to become "smarter" and more effective and to work less hard, since the technological environment enables the teacher to create differential content in less time and to create active engagement that leads to on-task students (Stanford, Crowe, & Flice, 2010). Thus, online differential teaching can be the panacea that the teachers are looking for (Logan, 2011). Aries Cobb (2010) holds that, on the basis of the research he carried out, teachers who used online differential teaching based on the learner's needs showed an improvement in their students' learning achievements. One of the results of differential teaching is the promotion of students from a low socioeconomic background who were defined as having difficulties in the traditional education system and the reduction of gaps (Steiner, Baird, Hamilton, & Pane, 2015; Hecht, 2018). The strong and effective influence of the use of technology appears in the subject of mathematics, science, and English (UNESCO, 2009).

2.3.2 Teaching-Learning Mathematics with Interactive Digital Tools

Information and communications technology (ICT) has affected our modern life and is embedded in societies and in everyday life, particularly at school (Cheung & Slavin, 2013; Fraillon et al., 2014; Misir, 2018). Educational technologies that developed and advanced rather rapidly in the past fifty years (Kllogjeri, 2010; Ouyang & Stanley, 2014) accelerated the development of a new era in teaching and the appearance of new challenges, such as the manner of integration of technology in the teaching-learning (Lytras, 2007). Today educational technology is a set of electronic tools and applications used to transfer learning materials and promote the learning process in K-12 classrooms (Cheung & Slavin, 2012). The portable technologies become a part of classrooms and the educational sector and enable flexible learning anytime and anywhere. Those technologies increase the students' ability to be involved in their learning and in learning activities from every place and at all times and at a pace suited to the learner, to hold interactions with one another in a variety of forms and situations (Howard & Scott, 2017). Information and communication technologies are an important tool with which the learners create and master the learning environment in an open and supporting manner (Levin, 1995).

The applications of educational technology produced a positive effect (Cheung & Slavin, 2012). Innovation in teaching increased the students' motivation and positive attitude towards learning and the learners' involvement in their process of learning and in their academic achievement (Rosen, 2011). The large impact of using educational technology appeared on a secondary level (Cheung & Slavin, 2012). The reason is that knowledge is found everywhere, and the learners of the "new generation" have critical thinking that requires the relevance of the learned knowledge to real life (Lytras, 2007). These reasons emphasized the importance of digital learning. Digital learning is a constellation of teaching-learning methods that combine ICT in an online learning environment and empower the learning (Rotem, 2013). The main characteristics of this learning is personal adjustment and flexibility in terms of the knowledge management, accessibility, and learning contents; management of the teaching and the teacher's responsibility, whose role is learning support for the learner that enables personal autonomous and cooperative learning; collaboration among the learners; and up-to-date digital learning materials personally tailored to the learner's needs. Digital learning environments enable the decentralization of the learning because of the flexibility given to the contents, time, place, and forms of information representation (Melamed & Goldstein, 2017). They advance the learner by setting goals for the learning, managing the learning content and process, and communication with others during the learning (Harmelen, 2006). Computerized learning environments facilitate the interaction between the teachers and the students and cooperation in the learning process (Kllogjeri, 2010).

Educational technology enables full access to the educational content and includes collaborative learning activities (Mayes et al., 2015). New technology facilitates the presentation of information from a number of sources and the illustration of lessons. Accordingly, the use of learning using digital tools challenges the students according to their level and gives them appropriate learning (Zucker & King, 2009). This can be explained using two theories. The first theory is cognitive flexibility theory, according to which the learning occurs in a complex and ill-structured manner. Therefore, the teaching process must provide the learners with a variety of learning possibilities in order to build their knowledge. The theory proposes in the context of educational technology collaborative digital learning environments that will facilitate the accumulation of knowledge and cognitive development (Ouyang & Stanley, 2014). The second theory is the dual coding theory, according to which the brain receives, processes, and organizes information through two channels, visual and auditory channels, when each channel has a capacity limit and therefore from the perspective of educational technology the use of digital learning environments enables the interaction between the two channels and therefore increases the efficiency of the learning process (Ouyang & Stanley, 2014). Roxana Moreno and Richard Mayer (2007) emphasized the importance of interactive multimodal learning environments as a factor that helps the learner build his knowledge in a dynamic and in-depth manner. They indicated five main characteristics of these environments, according to which the teaching-learning model is shaped: “dialoguing, controlling, manipulating, searching, and navigating” (p. 311). Effective learning environments combine verbal and non-verbal representations of the knowledge (Moreno & Mayer, 2007) by using interactive features such as dynamic presentation, data visualization, and multisensory media (Baldwin & Ching, 2017). In other words, interactivity led the learner to understand the relationship between verbal and dynamic visual information (Bus, Takacs, & Kegel, 2015). Therefore, Pellumb Kllogjeri (2010) maintained that learning through digital tools is very important in the field of the teaching of mathematics. A technology-rich environment facilitates the development of mathematical abilities and even reduces the anxiety of the subject of mathematics and can increase the students’ learning motivation (Mevarech, 1993).

In Israel, mathematics is considered a central subject in the education system, since it is the basis for logical-quantitative thinking, for learning, and for the development of topics related to science and technology (State Comptroller, 2014). The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes that “technology is essential in teaching and learning mathematics; it influences the mathematics that taught and enhance students` learning” (p. 2). Additionally, digital tools have the potential to enhance and support competencies important for the mathematical content, such as real world problem solving (Greefrath, Hertleif, & Siller, 2018). In recent years, the use of the application of the digital tools in mathematics has become important (Greefrath et al., 2018). They have a positive influence on the inculcation of digital skills and the improvement of specific cognitive processes of interaction and spatial orientation (Mascia, Perrone, Bardi, Agus, Penna, & Lucangeli, 2018).

One of variety of the most adopted interactive mathematical digital learning environments is the GeoGebra software that enables dual structuring of the geometric and algebraic representation simultaneously of the mathematical object (Kllogjeri, 2010). A research study published by Boo Jia Yi & Leong Kwan Eu (2016) examined the degree of influence of the use of GeoGebra on the understanding of the concept of the angle in schools in Malaysia. During the research, the teachers used six GeoGebra applications that included the creation of angles and the measurement of angles in triangles and in regular polygons. The applications were prepared ahead of time and tailored to the curriculum. The findings showed that teaching using the dynamic instrument that enables use of a graphic presentation facilitated the understanding of the concept of angles and enabled an experiential and effective learning process. The students wanted to continue to learn geometry in this dynamic method also at the end of the research. Pavethira Seloraji & Leong Kwan-Eu (2017) studied the influence of learning through GeoGebra on the level of understanding of the concept of geometrical reflection. The research findings showed the importance of the integration of GeoGebra in teaching and learning as an environment that advances in a meaningful manner the learning of geometric transformations among the students and especially advanced the understanding of the concept of reflection.

The integration of learning tools that include an interactive and dynamic virtual learning environment is effective in the development of the visualization ability in space in a meaningful manner and reduces gaps in the learners' spatial ability (Widder et al., 2014). Spatial perception is very important to the students in the learning of mathematics. Therefore, computerized illustrations provide concrete objects that facilitate the ability to build connections and strengthen different representations of mathematical ideas that will help the building of knowledge (Clements, 1999). One of the tools for coping with the difficulty in spatial visualization in this mathematical field is the use of interactive concrete aids. The virtual representations, enable virtual manipulatives of a visual object through the control of the physical actions on these objects, in a dynamic way that enables the learner to discover and build contexts and mathematical rules and consequently to build mathematical knowledge (Moyer, Bolyard, & Spikell, 2002). A virtual manipulative is defined as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer et al., 2002, p. 373). Thus, virtual manipulatives is a mathematical approach that is fundamentally cognitive (Zbiek, Heid, Blume, & Dick, 2007). Its main characteristic as a cognitive tool is the ability and flexibility to provide different representations concurrently and to provide visual feedback. This increases the motivation to learn and promote meaningful learning. From a pedagogical perspective, this is teaching that encourages inquiry-based learning and problem solving (Moyer et al., 2008; Moyer et al., 2002).

The digital constructivist learning environment that facilitates the inquiry learning processes (Solomon, 2000) and helps with the representativeness of data in a concrete and interactive dynamic way that directs some of the learner's resources and encourages the conceptual learning (Shaughnessy, Garfield, & Greer, 1996). Patricia S. Moyer-Packenham, Gwenanne Salkind, and Johnna J. Bolyard (2008) noted that virtual manipulatives can help build scaffolding in the transition between concrete activities and abstract concepts in the learning of mathematical concepts. Hence, dynamic geometry software (DGS) is software that enables the creation and inquiry of geometric constructs. In these programs, the visual product can undergo transformations while maintaining the features defined during the building.

The explanation regarding the effectiveness of the dynamic environment is through the cognitive load theory (CLT) of John Sweller (2010), according to which the model explains the cognitive ability of the working memory to succeed in the learning process. The working memory is limited in capacity and especially in the learned material is complicated and has many information components. This load leads to cognitive overload, which is expressed in the burden on the working memory. Therefore, successful learning is learning that occurs in the boundaries of the memory. When the learning conditions are suited to the structure and boundaries of the human cognition and address the load imposed on the working memory during the learning, thinking, problem solving, or visual illustration, the learning itself will be more effective, because "students can focus on decision making, reflection, reasoning, and problem solving" (Saha, Ayub, & Tarmizi, 2010, p. 687).

The digital learning environments have the potential to provide feedback as a part of the improvement of the learning process (Lytras, 2007). Providing immediate and individualized feedback to the learner (Cheung & Slavin, 2012), especially to learning disabled students, enables the learners to work on mathematical problems and to advance at their own pace (Mascia et al., 2018; Ouyang & Stanley, 2014), through the internalization of the form of the planning of the learning process in the continuation (Hattie & Timperley, 2007; Shute, 2008). “Checking and controlling the solutions obtained is another important mathematical activity that can be supported by digital tools, for example by means of graphical representations when performing numerical calculations, solving equations, rearranging terms, or when working with discrete or functional models” (Greefrath et al., 2018, p. 234). Another potential of digital learning environments is that they enable personalized learning that can offer students of varying interests, attention spans, and diverse needs a chance to be in control of their learning (West, 2012).

2.3.3 Online Collaborative Learning in Mathematics

The Ministry of Education in Israel (n.d.-b) supports the significant advancement of the model of online collaborative learning. Collaborative learning is a pedagogical approach that puts the learner in the center and includes methods of teaching and diverse models when the common denominator among them is the cooperation between the learners for the achievements of a shared goal, such as the performance of a project, the solving of an assignment or a problem, when the basis for the achievement of the goal is reciprocal dependence between the learners that leverages the process and advances it (Brody & Davidson, 1998; Horizon Report, 2017). The collaborative learning developed in the digital technology era following the requirement to adjust the education system to the skills of the 21st century, such as critical thinking, solving complex problems, collaboration, and adjustment to change (U.S. Department of Education, 2010), through the assimilation of information and communication (ICT) technologies, and new pedagogical structuring. The importance of the required combination between collaborative learning and use of online learning environments as a part of the learning process was expressed in a number of directions:

- The increase of the students’ participation in the lessons and the hours of learning invested in the learning improved the class dynamics and cooperation (Heggart & Yoo, 2018) and improved the learning experience (Hsu, 2016), when the involvement and communication do not depend on the time or place (Horizon Report, 2017).

- The quality of the learning, the teaching methods, and the learners' achievements were improved (UNESCO, 2009).
- The collaborative constructivist nature in the online learning environments increases in the learner his active participation in the learning process, enables the learners to build knowledge in a team (Rutherford, 2014; Salomon, 2000).

Collaborative learning develops the learner's cognitive constructs and increases the desire and motivation for the performance of tasks (O'Donnell & Kelly, 1994). Learning in an online collaborative environment builds in-depth processes of learning and processing and greatly influences the transmission of knowledge and learning processes, and emphasizes the learner's personal pace, through feedback and peer support (Dunleavy, Dexter, & Heinecke, 2007; Chen & Law, 2016),). The importance of the use of online learning environments and collaborative learning in mathematics is emphasized among students with difficulties. Students with learning difficulties are students with low motivation to study mathematics, but computerized activity has the potential to cause students to be active when it allows trial and research without the fear of failure. Therefore, researchers offer to create an "optimal experience" in mathematical education, when one of its components in the learner's growth is internal motivation, which he experiences in an online collaborative learning environment (Shalita, Friedman, & Hartan, 2011; Skadberg & Kimmel, 2004).

The social reciprocal relationship and the social mediation are two basic elements in the development of the high mental functions, such as perception, attention, memory, and cognition (Ilam, 2003). The use of the qualities of technology in mathematics increases the effectiveness of the learning process (Mayer, 2005). ICT is used as a collaborative learning environment that supports and improves the learning strategies and the coping with problems of memory (Drigas, Kokkalia, & Lytras, 2015). The technology offers to the students with difficulties the reduction of the burden of the mental load. Difficulties in active memory and long-term memory influence the students' ability to solve problems and to examine thinking processes (Geary, 2004: Solomon, 2000). The means of illustration are essential in teaching mathematics since they bridge between the abstract idea and its representation and the tangible concrete level (Linchevsky & Tuval, 1993; Mayer, 2009). The use of technology will increase the interest in learning because of the reduction in the mental load in which technology takes part and in this it increases the visual means that are important to the acquisition of knowledge and the development of intellective skills (Solomon, 1981). The technology offers the learners control of the learning processes and a maximum of knowledge transmission (Chen & Law, 2016).

Chapter 3: Development of the Learning Ability of Adolescents with Mathematical Difficulties in the Digital Era

3.1 The Structure of the Middle School and Its Importance for Adolescents

The concept of middle school in the countries of the world is a new concept (Ellerbrock, Main, Falbe, & Franz, 2018). In the State of Israel, following the Rimlet Committee in the year 1968, it was recommended to embark on an extensive reform of the education system in which the structure of the division of the elementary and secondary education institutions changed[10]. It was decided to change the structure of the schools and to transition from a bipartite structure (elementary school from first to eighth grade and high school from ninth to twelfth grade) to a tripartite structure. Today, the structure of elementary education and secondary education in Israel includes elementary education for six years (first to sixth grade) and secondary education for six years that consists of two divisions, the middle school (seventh to ninth grade) and the high school (tenth to twelfth grade). In Israel, the middle school is generally operated as an independent framework, and sometimes the middle school and the high school operate in one complex, called the six-year school (Vargan, 2010). Today, most of the cities and communities in Israel use the tripartite structure (Rash, 2008).

The importance of the middle school as an independent entity was described by G. Stanley Hall (1846-1924) (in Dahl & Hariri, 2005) emphasizes that the stage of adolescence is a *unique* period of development and that this period can be represented as a *new birth*. Therefore, it is important to create a special framework for adolescents. The creation of the middle schools emphasizes the difference between these students and the elementary school students and between them and the secondary school students, from the

[10] The Rimlet Committee in the year 1968 was composed of Knesset members and recommended an extensive reform following which the education system moved from a structure of a division into two learning institutions over the years in the system (eight years in the elementary school and four years in the high school) to a structure of three institutions: six years in the elementary school, three years in the middle school, and three years in the high school. The Committee recommended also extending the Compulsory Education Law to include the tenth grade and to establish integrated middle schools.

understanding of the education system of the young learner's unique pedagogical needs associated with adolescence (Ellerbrock et al., 2018; Rash, 2008). The requirements and uniqueness of the middle school students led the education system in Israel to build special academic training programs suitable for the teachers. These programs instruct the teachers specifically to deal with the academic and emotional requirements of the population of adolescent students (Vargan, 2010).

One of the challenges of the middle schools is the system's coping with the transition of students from the elementary school, a transition that occurs in three dimensions: social, emotional, and academic. The stage of the transition in the education institutions that was expressed in the creation of the middle schools is an important step in terms of the development of the learners in psychological and social terms, and its aim is the transition from the "greenhouse" of the elementary school to the academic demandingness of the high school (Rash, 2008). Of course, the transition between the educational frameworks or to a new school raises concerns, curiosity, and tension[11] and creates many changes, changes in the learner's social environment and changes in the academic environment and academic requirements. The existence of a separate framework of studies – the middle school – for students who are found in early adolescence prevents frictions between them and older students who can constitute an undesired personal example and enables flexibility in educational-academic terms (Vargan, 2010).

The transitions from elementary school to the middle school and from the middle school to the high school influence the development of the young adolescents (Eccles & Roeser, 2009). The goal of the middle school is to create a learning environment that encourages the development of academic responsibility of the young adolescents

[11] The Dovrat Committee Report: "The National Task Force for the Advancement of Education in Israel", more commonly known as the Dovrat Committee, was established at the initiative of the government of Israel in September 2003. The establishment of the committee was in response to the extensive criticism leveled at the education system in Israel, following the failure of the Israeli students in the international tests (PISA [Programme for International Student Assessment] and TIMSS [Trends in International Mathematics and Science Study]). In January 2005 the plan was submitted to the Minister of Education and to the government of Israel that included: description of the situation of the education system in Israel, reference to the crisis in the education system and to reforms around the world, presentation of the principles of the plan for the advancement of the education system, and six chapters of recommendations, and themes for implementation.

(Ellerbrock et al., 2018). According to the Rimlet Committee on the issue of the structure of education, the aim of the middle school is to follow up after the adjustment of the students in the academic, behavioral, and emotional aspects.

The middle school is responsible to direct the students according to their preferences and ability to continue their secondary school studies (Vargan, 2010). In recent years, there has been increasing awareness of the importance of supporting adolescents by building socioemotional skills and providing a positive environment in schools and between the adolescents and their families (World Health Organization, 2021). These programs empower the adolescent and reduce the tension that is created in this period. Still, the challenges that adolescent students face are many, and they appear along with the learner's physical, mental, and cognitive development.

3.2 Unique Challenges of the Adolescent Learner

Adolescence is a Latin word that means growth or growth towards adulthood. Adolescence is a unique period of the transition from childhood into adulthood (from ages 10 to 19 years) (Jaworska & MacQueen, 2015; Nimri & Phillip, 2010; UNICEF, 2017a; World Health Organization, 2021). Adolescence is a most rapid period and one of dramatic change that occurs in the individual development experience (Crosnoe & Johnson, 2011; Eccles & Harold, 1993). The rapid biological, cognitive, and psychosocial changes that occur in the second decade of adolescents' lives have critical effects on their future (World Health Organization, 2014; World Health Organization, 2021). Many factors can impair both the physical and mental health of adolescents such as "violence, poverty, stigma, exclusion, and living in humanitarian and fragile settings" (World Health Organization, 2021). As a result of these factors, adolescence is defined as a period of tension and mental stress, which is accompanied by rapid changes in physical, emotional, social, and behavioral terms (Dahl & Hariri, 2005; Nimri & Phillip, 2010; UNICEF, 2017a). This period is characterized by behavioral changes that occur in parallel to the physical development that is influenced by the child's environmental factors and internal factors. Children in this period spend more time with their peers than with the parents, and they display more independence and less dependence on their parents (Jaworska & MacQueen, 2015).

Adolescence is characterized by internal pressures and mental tensions (Carmel College Website, 2014). Nevertheless, it is a main and critical period in which there is active structuring of the adolescent's identity (Gogol-Ostrovsky & Lazer, 2010) and gives an opportunity for the adolescent's self-development, progress, and success (Steinberg, 2014). The period of adolescence is associated with the adolescent's self-perception that undergoes physiological and cognitive changes. This is a period in which dramatic cognitive development occurs through a range of cultural events. These changes occur in the context of social factors out of the adolescent's experience that derives from the social interaction and from the events in the cultural context that influence the building of the identity (Gogol-Ostrovsky & Lazer, 2010). Adolescence is universal, but it is also dependent and influenced largely by the individual's characteristics, the adolescent's environment, and the socio-cultural environment (World Health Organization, 2020).

In the period of adolescence, there occurs a dual transition, a transition from elementary school to the middle school and in parallel a transition from childhood to the beginning of adolescence (Rash, 2008). This process is accompanied by concerns, anxieties, and deliberations. The implications beyond this are on the behavioral, cognitive, and emotional levels. The changes that occur are not only external but also internal. During adolescence, the adolescent accumulates and develops knowledge and skills, such as abstract thinking and generalization of events, development of discretion, ability to ask questions, logical thinking, and development of judgment. These responsibilities are a part of the cognitive abilities that develop in the period of adolescence (Gogol-Ostrovsky & Lazer, 2010; World Health Organization, 2020).

3.2.1 Influences of the Transition from the Elementary School to the Secondary School on Adolescents in the Subject of Mathematics

'Transition' expresses a process of change. This change is described as a process of the children's moves from one school system into another, or the transition in the same school between different years like the transition from primary to secondary schools (Evangelou, Taggart, Sylva, Melhuish, Sammons, & Siraj-Blatchford, 2008; Gueudet, Bosch, Disessa, Kwon, & Verschaffel 2016). This unique intermediate framework is accepted for early adolescence between elementary school and the later stage of secondary school education in the developed countries (Rash, 2008). If the experience of the transition is difficult, this can detrimentally influence the learners' self-esteem and achievements (West, Sweeting, & Young, 2008). A primary difficulty of the young learner is the transition to a larger framework, which is more competitive and demanding in terms of the academic requirements (West et al., 2008), and in parallel there is the developmental transition (adolescence), in which there are physiological, emotional, social, and cognitive changes (Eccles & Roeser, 2009), and adjustment to the new form of learning focused on specific and diverse areas of knowledge (Addi-Raccah, Biran, & Friedman-Goldberg, 2011).

In the period of transition, the young adolescent experienced physical, emotional, and cognitive changes. The expectations and requirements of the young adolescent increase, and the adolescent is required to be more independent and more responsible academically (Niesen & Sachs Wise, 2004). Therefore, to understand the process of the development of adolescents, it is necessary to perform an integration in terms of the influence of the biological, behavioral, social, and environmental factors and the integration between them (UNICEF, 2017a). The influence of the transition to the middle school on the adolescents appears in the decline of the students' academic achievements. Organization difficulties, lack of motivation, and decline in the achievements of the studies are linked to the students' lack of interest in academic activities and the investment of more efforts in the social realm and in their being more focused on themselves (Eccles & Roeser, 2009; McGee, Ward, Gibbons, & Harlow, 2003). One of the important indicators of the success of the transition is curriculum interest and continuity (Evangelou et al., 2008).

In middle school, the academic demands of the study subjects increase, as does the level of thinking. The students are required to develop complex thinking processes, such as building a concept, inquiry, making decisions, and problem solving. The carrying out of the thinking processes is through the inculcation of thinking strategies. Thinking strategies are cognitive actions focused on asking questions, formulating hypotheses, making comparisons, and drawing conclusions. The development of the students' thinking is revealed considerably in the subject of mathematics in the middle school (Ministry of Education, 2009). The adolescent learner's thinking ability develops and changes from concrete thinking to abstract thinking that enables the solving of high order problems (The Advising Council, Trump Fund, 2020). Therefore, the aim of learning mathematics in the middle school is to develop the students' ability to deal with situations of uncertainty and with tasks for which the solution does not have a predetermined procedure (The Advising Council, Trump Fund, 2020). The requirements of the subject increase and focus not only on doing a rapid calculation but on assimilating "mathematical thinking" in the learner, such as the ability to ask questions, to map out pathways, to provide an explanation to a solution of a complex question, and to build different models (Boaler, Williams, & Confer, 2015).

The great challenge in teaching mathematics is to create interest in mathematics and to make the learning relevant and real (Manitoba Education and Advanced Learning, 2015). Relevance for the learner increases the motivation to learn mathematics since it creates a relationship between the world of the learning content and the learner's learning goals (Frymier & Shulman, 1995). The relevance helps the students develop mathematical thinking through which the learner can connect between mathematics as a subject of study and as a relevant subject that he uses to solve everyday problems (Iyer & Pitts, 2017). During the period of the middle school, the students learn to be independent, they are partners in the definition of the learning goals and in the building of the criteria for assessment and participate in the assessment.

The role of the teachers is to provide an opportunity for students to advance in the learning processes (Iyer & Pitts, 2017). The teachers are responsible for the development of an advanced mathematical discourse that emphasizes different forms of thinking (Hufferd-Ackles et al., 2004; National Council of Teachers of Mathematics – NCTM, 2000). The teachers focus on providing mathematical activities that instill mathematical facts not through learning by rote by through the creation of logic and the building of a number sense (Boaler et al., 2015). The teachers follow up on them by providing suitable and immediate formative feedback (Manitoba Education and Advanced Learning, 2015). The use of technological means in middle schools emphasizes this perception in mathematics and encourages the students to become more independent and to master their learning method (Baya'a & Daher, 2010).

3.2.1.1 The Social-Emotional Effect of Transition on the Seventh Grade Student

Transitions during adolescence are undertaken on two levels: personal and social. On the personal level, the adolescent undergoes physiological development in the neural structure of the brain and sexual maturation (Lupien, McEwen, Gunnar, & Heim, 2009). The adolescent transforms from a child to an adult who can take responsibility for his actions. On the social level, according to McGee et al., (2003) the adolescent develops requirements and desire to belong, to be accepted, and to be esteemed by his peers and in parallel the expectations of him as a responsible adult increase. In the period of adolescence, the learner searches for and develops self-identity in the sense of the self-discovery, but the transition disrupts this process (Akos et al., 2005; Eccles & Harold, 1993). Of course, what influences and shapes the adolescent's functioning is the personal experience and the environment (Eccles & Harold, 1993; UNICEF, 2017a). The influence of the experience of transition from elementary school to middle school on the young adolescent appears in two dimensions – future learning and wellbeing (West et al., 2008).

Development during adolescence requires time and energy and therefore causes stress and lack of comfort that is expressed in changes in the mood, changes in their opinions, lack of self-acceptance, and sometimes bullying (Akos et al., 2005). The assumption is that the constant pressure and the chronic production of stress hormones because of the changes during adolescence influence three regions in the brain responsibility for cognitive deficits – learning, memory, and emotional regulation (Lupien et al., 2009; UNICEF, 2017a). Every adolescent experience the changes during adolescence in a different way since this depends on many external factors, experiences, stimuli, and influences of the environment, and therefore the challenge of every school is to cope with the difference between the adolescents (Akos et al., 2005). The difficulty in coping with adolescence and the period of transition becomes harder for students from a low socioeconomic situation. These are a mentally vulnerable group that develops more difficulties in the process of transition (Akos et al., 2005; West et al., 2008). An ineffective transition from elementary school to middle school influences the learner's ability to cope socially-emotionally, or in other words, the adolescent will be more mentally vulnerable and may develop health problems (Akos et al., 2005).

The students spend most of their time either in school or at home with the family (Eccles & Harold, 1993; Eccles & Roeser, 2009). The schools play an important role in the adolescent learner's mental development and grant the learner mental confidence (Eccles & Harold, 1993). The support of the family and the school staff in the sensitive period of transition has importance in mental terms. The involvement of the parents and the teachers in the process of the transition parallel to the period of adolescence is important, and it has a positive impact on the adolescent's emotional and mental health. Children who live life in families that support and encourage autonomy display better adjustment ability in the process of the transition, and there is positive influence on the academic achievements (Asmussen, Corlyon, Hauari, & La Placa, 2007; Eccles & Harold, 1993; Jeynes, 2007). In contrast, the teachers in the schools are a very important factor in the adolescent's life. The young adolescents need a framework that is not of the family, such as the teachers, in order to develop their identity and independence (Eccles & Harold, 1993).

A successful transition depends also on the development of the learner's social skills, which are expressed in the learner's ability to be accepted and a part of the group of peers. The group of peers can be supportive and encouraging through conversation, and in the group, there can develop conflicts and provocations that have an important part in the adolescent's development (Asmussen et al., 2007; Niesen & Sachs Wise, 2004). Goal theory emphasizes the importance of the social field out of the understanding that the social field significantly influences the academic success and motivation of the students in the school (Covington, 2000).

3.2.1.2 The Educational Effect of Transition on the Seventh Grade Student (age 11-12 years)

The transition from elementary school to middle school has been considered a very important topic during the past three decades (Akos et al., 2005). The transition influences the students' wellbeing and achievement (West et al., 2008). Since the period of transition of the students from the elementary school to the middle school is parallel to the period of adolescence, in this period between age ten and fourteen there are the greatest and most essential changes of the adolescent's life (Akos et al., 2005). Middle school is a problematic period in educational terms; the heterogeneous composition of the learners is expressed in the gap in the learning achievements that are intensified by the transition. The middle school is a relatively large framework, heterogeneous in terms of the learners' characteristics both socially and economically and culturally (Byrnes & Ruby, 2007).

The adolescent students' successful adjustment to the transition positively influences their learning (McGee et al., 2003). Difficulties in the transition between elementary school and middle school detrimentally influence the students' learning achievements in the future and sometimes this leads to dropping out from the school (Akos et al., 2005). The student's learning achievements in middle school have precedence in the priorities of the teachers, principals, and educational staff. The emphasis of the learning achievements and the placement of the achievements at the top of the school priorities put the student under tremendous pressures, but with fewer resources and more difficult teaching and learning conditions because of adolescence. Therefore, the realization of the learning ability of the students in the middle school will be more limited (Addi-Raccah, Biran, & Friedman-Goldberg, 2011). In addition, research studies prove that students with a background of a low socioeconomic background find it difficult in the transition between the elementary school and the middle school (Akos et al., 2005; Evangelou et al., 2008; West et al., 2008). In contrast, other research studies indicate positive implications of the transition to middle school. Those who support the transition indicate a challenging and interesting opportunity that gives them a "fresh start", to change and develop in terms of learning and in terms of personal and social ability (Barber & Olsen, 2004; McGee et al., 2003).

The transition from elementary school to middle school entails a significant change in the learning environment. The learning environment in the middle school is different from that in the elementary school, relatively more complex than in the elementary school. In the middle school there is a significant jump in the academic requirements and in the academic level. The students develop abstract thinking ability and develop formal thought. This is the most dramatic change in cognition that occurs in the person's entire life (Akos et al., 2005; West et al., 2008). The students must develop the ability to learn independently and to meet the mandatory standards of tests and assignments. Since the manner of learning in the middle school is different and requires more independent learning, sometimes in a group setting and project based, in certain cases there is a need to help the adolescent develop learning skills. The importance of providing challenging tasks encourages the use of diverse cognitive operations that increase the investment of effort, the development of interest, and of course achievement (Eccles & Roeser, 2009; Jeynes, 2007).

Adolescents have motivation and ambition to achieve if they are challenged by assignments that require critical and complex thinking (McGee et al., 2003). In addition, it is recommended to teach the adolescent to organize their schedule and to deal with the load of information. To advance adolescent students in terms of their learning and achievement in the period of the transition to the middle school, it is necessary to focus on two things (Eccles & Harold, 1993; Eccles & Roeser, 2009; McGee et al., 2003):

1. The content of the curriculum and the manner of the teaching need to be clear, challenging, interesting, and meaningful and relevant to the learner with high expectations about their learning achievements.
2. The time of the adjustment is reduced through personal support of the students who have difficulties with the involvement of the parents and the teachers.

The parents' involvement in the period of the transition has a great positive influence on the students' achievements. The parents' involvement, which focuses on the parents' expectations and the development of aspirations among the children, learning support of the learners (such as enforcement of the times of doing homework, help with homework and adjustment of a comfortable work environment including the supply of materials required for the learning), increases the students' expectations of themselves and this leads to an increase in their academic achievements (Efraim, 2014). The period of adolescence is a period of lack of organization and confusion for children, but this is a necessary period. Youths in this period of trial and error develop into responsible adults who can correctly retrieve important information and know to plan activities and make correct decisions that are important to their future, and of course they have control of their emotions (Amussen et al., 2007).

3.2.1.3 Coping with the Emotional and Educational Effect of the Transition

To deal with the difficulties of the transition from the elementary school to the middle school in Israel, the Ministry of Education works to empower the learner in the social dimension and to provide academic solutions for all the students, each student according to his ability and achievements. Children, parents, and schools measure the success of the transition of children from the elementary school to the middle school according to the level of social integration and the reinforcement of the self-image, the belonging to the school, the lack of worries on the part of the parents of their children, the adjustment to the learning routine and the new school organizational structure, and of course the continuity of the curriculum (Evangelou et al., 2008; Niesen & Sachs Wise, 2004).

The preparation of the learners through the cooperation between the elementary school and the middle school will allow the learners to develop adjustment abilities and will enable a smoother transition (Ministry of Education, Culture, and Sport, 2004). Today many schools are aware of the importance of providing an atmosphere of encouragement and support for the students and providing information relevant to the students regarding the change in the manner of the teaching and the learning (McGee et al., 2003). A well-planned plan of transition is one that involves all the interested parties: students, the school staff, and the parents. These plans are the ground upon which the beginning of the positive experience for the learner moving to the middle school (Addi-Raccah, Biran, & Friedman-Goldberg, 2011) is constructed. The preparation programs of students that emphasize the form of the learning and the coordination of the expectations of the students and the new school system facilitate the adjustment of the children to the transition (Evangelou et al., 2008). A successful transition between elementary school to middle school depends on the two vital concepts, communication and planning. The communication is between all the relevant factors, such as the school staff, the teachers, the parents, and the students, and appropriate planning of transition plans, based on the social aspect (Akos, Lineberry, & Queen, 2005).

The aim of preliminary social programs is to lessen the stress created from the transition and to increase the self-esteem and belonging among the students. Social programs can include the preliminary visit to the school, which enable the orientation with important focuses, recognition of their peers in the classroom, the accompaniment of mentors of the new students, and the students' involvement in school programs and activities such as sports and art (McGee et al., 2003). The involvement and support of the family environment in the period of the transition also encourages and advances the learner academically and in terms of achievement (Niesen & Sachs Wise, 2004).

From an academic perspective, in mathematics in the middle school there is a lowering of the performances, and this is related to the reduction in the hours of weekly learning, increase in the heterogeneity of the students in the classrooms, lack of appropriate training for the teachers to deal with students, lack of adequate mastery of the students of the main social-emotional skills – self-organization, self-assessment, determination, ability to cope independently, and difficulty in learning abilities that include ability to understand logic in mathematical language, mastery of arithmetic competencies, identification of the need for proofs, familiarity with traits of functions, ability to model, and basic geometric ability (The Advising Council, Trump Fund, 2020). To reduce the academic pressure that accumulates in mathematics, the middle schools in Israel adopt the division of the students into groups and the diversification of the teaching methods. The aim of the division into grouping is to provide a suitable solution for students with different abilities. Thus, they will learn together in the homeroom in most of the subjects and in main subjects like mathematics and English they will be separated according to their achievements on tests into homogeneous classes. The division into groupings in mathematics is customary in the seventh, eighth, and ninth grades (Ayalon, Blass, Peniger, & Shavit, 2019). As indicated by some researchers the separation of the students into different classes according to their ability contributes to the effectiveness of the teaching and enables the teachers to customize the teaching according to the students' abilities and thus to realize their abilities (Harkabi & Mandel-Levy, 2014), however, even though teaching in a classroom with students with diverse abilities is difficult, a well-thought-out strategy avoids stigmatization of students with lower potential while at the same time allows those with high potential to develop[12] .

3.2.2 Aspects of the Mathematics Curriculum in the Middle School in the Digital Era in Israel

The requirements of the 21st century have reshaped the figure of the adolescent and the skills the adolescent requires in order to succeed in life. The employment requirements have changed, and the performance of the worker's role shows an increase in the requirement of complex thinking, the ability of problem solving, creativity, and collaboration (Brandes & Strauss, 2013). Skill is considered an acquired ability influenced by the manner of learning. The goal of the inculcation of the skill is to help the learner carry out complex tasks through effective use of knowledge and cognitive components, both intrapersonal and interpersonal, in the learning context and in everyday life. Cognitive and metacognitive skills include linguistic literacy, mathematical literacy, scientific literacy, digital literacy, information literacy, critical thinking, and creative thinking. Intrapersonal skills include self-awareness and self-direction. Intrapersonal skills include social awareness, social conduct, collaboration, global literacy (Ministry of Education, n.d.-c), and practical and physical skills associated with the effective use of new digital means (Organisation for Economic, Co-operation and Development – OECD, 2018a).

The importance of the mathematical field increased with the technological development in the 21st century. The structure of the curriculum in mathematics must inculcate in the learner the skills of the 21st century, which will include: the ability to solve complex problems, analysis and synthesis of information through systemic and creative thinking, interpersonal communication and teamwork, ability to exchange information,

[12] H. Krauze - Sikorska Individualization in education - a challenge for modern school in: Krauze - Sikorska H., Kuszak K. (eds.) Selected problems of psychosocial functioning of children and youth with developmental disabilities. Prevention - diagnosis - therapy. New perspectives - new challenges (2nd edition, expanded). WSPIA Scientific Publishers, Poznań, pp. 55-73

transformation of knowledge and use of knowledge to cope with different issues and diverse problems, abilities of critiquing and evaluation and their use for the examination of alternative solutions, self-discipline, management of self-regulated learning, which includes the ability to set goals, the ability to determine strategies for the achievement of the goals, and the ability of time management (Anaoniadou & Claro, 2009; Brandes & Strauss, 2013; Laskey & Hetzel, 2010; Zimmerman, 2002). In addition, it is important to instill interpersonal skills, the ability to work collaboratively, self-regulated learning ability, flexibility, and adjustment, and personal and social responsibility (Vidislavsky, Peled, & Pavasner, 2011).

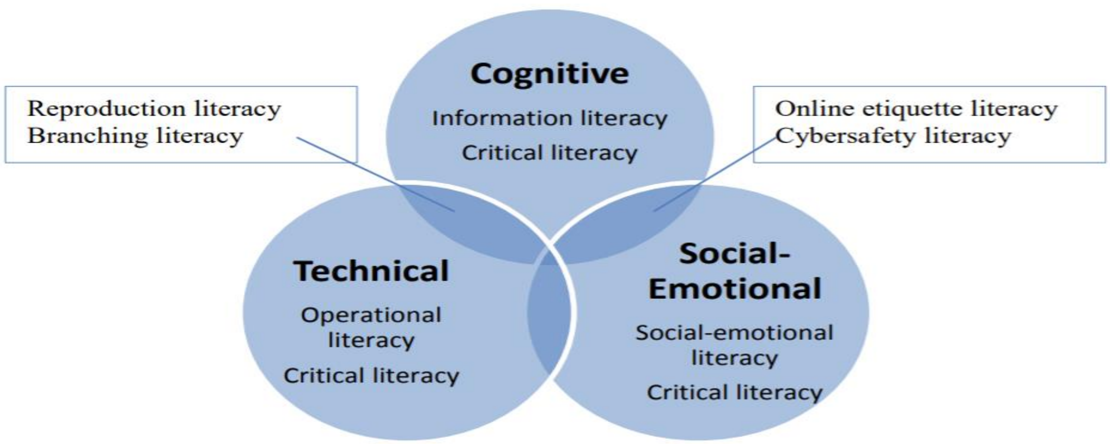
The mathematical curriculum of the new era needs to offer the students a rich mathematical program that will give the learners mastery in different aspects of the knowledge field. The program must offer the students access to technology while providing tools that help broaden and deepen their mathematical understanding. The program will also encourage students to love mathematics through aspiration for better achievements that will contribute in the future to specialization in fields that require high mathematical ability (National Council of Teachers of Mathematics – NCTM, 2000). According to the Organisation for Economic, Co-operation and Development – OECD (2018a), a well-planned curriculum is essential to the cultivation of skills suitable for the 21st century and ensures:

- The motivation of students, when the topics will be challenging and will enable in-depth thinking.
- The learning contents are focused, thus ensuring in-depth understanding of the learning for all the students.
- Coherence between the different topics, a continuum that helps to build relationships between concepts according to the level and the age.
- Alignment between the curriculum, the instruction, and the assessment.
- Transferability, the ability to transfer the knowledge, skill, or position learned in a certain context to another context.
- Choice, support of the learners so they can choose from a wide range of topics the topics of the project in which they are interested.

In Israel, one of the components upon which great emphasis was placed in the new curriculum in mathematics is the topic of digital literacy. In an era of digital transformation, it is most essential to inculcate in the learner skills of digital literacy and data literacy (Organisation for Economic, Co-operation and Development – OECD, 2018a). Digital literacy is defined as the individual's ability to use ICT to find, to evaluate, to research, and to create effective communication in the school and in the learner's environment (Fraillon & Ainley, 2010). The use of digital means and environments obligates the inculcation of digital literacy. Digital literacy is composed of three dimensions (see Figure Number 7) (Ng, 2012):

1. Technological – abilities to use technological tools.
2. Cognitive – abilities of the search for and evaluation of information in a critical manner, through the examination of the trustworthiness of the information.
3. Social – abilities of the creation of communication and socialization in a responsible manner.

Figure Number 7: Digital Literacy (Adopted from Ng, 2012, p.1067)



The new curriculum in mathematics and the teaching approaches in Israel were adjusted to the requirements of the 21st century. The curricula in mathematics, with the transition to the inculcation of knowledge and skills in the learned mathematical topics, emphasize the inculcation of skills of thinking, research, and criticism and encourage independent thinking through the creation of linkages between mathematics and life and the ability to implement skills and mathematical knowledge in different areas, use of mathematical language to represent problems and to solve them, development of explanations and arguments, and ability to work in groups (Lival et al., 2018). The development of mathematical ideas through communication with the peers in the class encourages students and helps mathematical understanding (National Council of Teachers of Mathematics – NCTM, 2000). The students will research the problem, will identify ways to solve it, and will present the solution before their classmates – this will encourage students and will lead to meaningful learning (Hufferd-Ackles, Fuson, & Sherin, 2004).

3.2.3 Mathematics Education among Adolescents in the Middle School in Israel

Mathematics includes in its many domains: arithmetic, algebra, calculus, geometry, trigonometry, and so on (Peterson, 2001, in Savion & Seri, 2016; Stevens, 2011). Arithmetic that is learned in elementary school is basic and vital for the continuation of mathematical studies. The understanding of mathematics is based on the understanding of arithmetic actions (such as addition, subtraction, multiplication, and division), square roots, and mastery of the order of operations between them (Peterson, 2001, in Savion & Seri, 2016; Weintraub, 2004).

The curriculum of the studies of mathematics in Israel, in all the stages of the learning, is spiral (Ministry of Education, 2013b). The idea of spirality is based on the exposure of students to the same subject or main mathematical idea a number of times during the three years in the middle school, in all areas, when each time layers, levels of detail and/or depth are added according to the knowledge, experience, and mathematical sophistication accumulated over time. This meaningful repetition and extension of the topic is for the purpose of the establishment and formation of knowledge, for the gradual development of a broad mathematical perspective on the entirety of the learned topics. There is a differentiation between what is learned in elementary school (arithmetic) and what is learned in middle school (mathematics). The Ministry of Education (2006b) details the curriculum in mathematics for elementary schools, in which two subtopics are included: numerical insight and geometric insight. The numerical insight includes mastery of mathematical skills and recognition of basic calculation facts, learning of mathematical concepts, and building of connections between them in writing and orally, and the understanding of mathematical representations through illustrations and ability to change between them through the correct use of the mathematical language. The students in elementary school learn arithmetic as a basis and foundation for the subject of mathematics that is learned in the middle school. The relationship between arithmetic and mathematics in comparison to language is described as the relationship between spelling and writing (Weintraub, 2004).

In middle school, the program continues in a spiral manner that emphasizes repetition of the arithmetic laws that were learned in the elementary school, and in this the extension is done and the students begin to learn algebraic expressions. From the start of the seventh grade, there is integration of the three mathematical areas: numerical, algebraic, and geometric, when the learner is asked to provide explanations, proofs, and solving of problems through the implementation of tools or approaches of one of the areas in another field (Ministry of Education, 2013b). The integration of the three areas is for the purpose of the strengthening, deepening, and enrichment of the mathematical knowledge, the building of connections between the topics through emphasis of the different forms of teaching/learning as a response to students with different mathematical tendencies/ preferences.

In terms of the studies of mathematics, in Israel the division of the students according to levels and the students' learning abilities is accepted. The division into levels is specific to the subject of mathematics (Lival et al., 2018). In the middle schools in Israel the Program for the Realization and Excellence in mathematics is implemented. The aim of the Program "Realization and Excellence in Mathematics" is to promote two populations at the extreme edges in the subject of mathematics: the first population is realization – for the students with difficulties, while the second population is excellence, for the students who are most talented in the study of mathematics. In addition, a program of the scientific technological reserves track is deployed, a six-year program from the seventh grade to the twelfth grade. The scientific technological reserves track is a program for excellence that has the aim to increase the number of students who receive the high school matriculation certificate in 5 units of study in mathematics and in two higher levels scientific/technological subjects. In the year 2015, the Program for “Give Five”, the National Program for the Advancement of Mathematics, was launched, with the cooperation with the “5 times 2” initiative. The goal of the program is to double in four years the number of students studying 5 units of study in mathematics.

In the studies of mathematics in the middle school in Israel, the method of the division into levels of learning, according to the students’ abilities as determined by mapping tests, is customary. The subject of mathematics is a mandatory subject required for the student to be eligible to receive the high school matriculation certificate. The process of the division of the students in the middle school, which lasts for three years, constitutes a criterion for the division of the students in the continuation in the high school (Lival, Lerman, Atzmon Hecht, & Sholga-Raz, 2018). In the high school, it is possible to choose between three levels of the studies of mathematics, according to the level of the high school matriculation tests at which the students intend to be tested: 3 units of study, 4 units of study, and 5 units of study (Lival et al., 2018).

3.3 Growing Up with Mathematical Difficulties

Difficulty in learning is an outcome of every phenomenon that harms the person’s ability to acquire knowledge or new skills (American Psychiatric Association – APA, 2013). The difficulty is expressed in the reduction of the student’s learning curve and achievements. Comprehensive definition of learning disability, “Learning disability refers to a retardation, disorder, or delayed development in any one or more of the processes of speech, language, reading, spelling, writing, or arithmetic. These problems are due to disorder or deficiency in any one or more of the basic psychological processes involved in understanding or in use of spoken or written language” (Rachamalla & Rafi, 2016, p. 111).

A learning disability is genetic and/or a neurological disorder (Szklut, Cermak, & Henderson, 1995; Rachamalla & Rafi, 2016; Radhika, 2018). The symptoms that characterized learning disabled students are different and diverse. Not all the symptoms appear among all the students; the symptoms and their degree of severity differ from one student to another (Szklut, Cermak, & Henderson, 1995). The disabilities are accompanied by difficulties with organization, difficulties with perception, and difficulties with social interaction. Learning disabilities are life-long (Radhika, 2018). There are seven types of learning disability: (1) dyslexia, (2) dysgraphia, (3) dyscalculia, (4) auditory processing disorder, (5) language processing disorder, (6) nonverbal learning disabilities, and (7) visual perceptual/visual motor deficit (Muktamath, Hegde, & Chand, 2021). It is possible to see that there is a wide range of concepts used by researchers in order to describe students with mathematical learning difficulties.

Students are defined as having difficulties in the acquisition of mathematical skills if they have one of the indicators persevering at least six months despite interventions: "Difficulties mastering number sense, number facts, or calculation (e.g., has poor understanding of numbers, their magnitude, and relationships; counts on fingers to add single-digit numbers instead of recalling the math fact as peers do; gets lost in the midst of arithmetic computation and may switch procedures). And difficulties with mathematical reasoning (e.g., has severe difficulty applying mathematical concepts, facts, or procedures to solve quantitative problems)." (DSM-5, 2013, p. 66). In addition, the phonological processing is important for learners to transition thinking processes into mathematical concepts and representations, difficulties in developing language and reading skills develop difficulties among students in mathematics (Montis, 2000). Fluency of reading is important to produce knowledge and is influenced by the learner's ability to decode and vocabulary and of course metacognitive skills, psychological factors (motivation, emotion), and environmental components (Meltzer, Katzir, Miller, Reddy, & Roditi, 2004).

Students with difficulties are a very broad population of mathematics learners who cope with a variety of difficulties that arise in the learning in the regular class. Students with difficulties in mathematics develop difficulty and underachievement in mathematics regardless of the source of the disability or difficulty. The learning difficulties can derive from a certain learning disability, today there is awareness that the difficulty in learning can derive also from environmental factors (Schmidt, 2016; Westwood, 2016). Many students come to middle school when they have not yet acquired and assimilated the basic skills in literacy. Therefore, there is the need to work on the basic skills in a personal manner according to the learner's level and in parallel to the work on high order skills (such as summary, control, and need to deploy previous knowledge) (Brasseuer, Hock, & Deshler, 2005).

3.3.1 Origin of Math Learning Difficulties

The difficulty in learning is an outcome of every phenomenon that harms the person's ability to acquire new knowledge or skills. The difficulty is expressed in the lowering of expectations and motivation to learn and to achieve. Students with difficulties are a very broad population of learners of mathematics who cope with a range of difficulties that arise in the learning in the regular class.

From a socio-cultural perspective, students with difficulties in mathematics are students whose academic achievements are low (Denvir, Stolz, & Brown, 1982) and who experience a feeling of frustration, failure, and lack of self-confidence to participate in mathematical activities (Haylock, 1991). Professional literature differentiates between students with difficulties in mathematics and students with learning disabilities. There is a differentiation between two different characteristics of students with difficulties, which require different types of coping: environmental characteristics and learner characteristics:

- *Environmental Characteristics*. Students with learning difficulties are a larger group of students who do not respond to the school curriculum because of environmental factors, such as social factors, factors associated with the curricula, or lack of support of the learning (Elkins, 2002; Westwood, 2016). They are defined as "students with difficulties in math", and receiving tailored help is supposed to help them overcome the difficulties. Ben Yehuda and Licht (2013) maintained that students with math difficulties are students who have not acquired basic learning skills. Therefore, the difficulty derives primarily from environmental factors and from learning that is not suited to their unique needs. These are related to the nature of the teacher's instruction, the curriculum, the teacher, and the teacher's style of teaching (Kidron, 1985). The misleading use of mathematical language and means of illustration, and rigid requirements of the learner, such as requirement to memorize with the internalization of the meaning, rapid pace of learning, and lack of reference to the cognitive differences among the learners cause the accumulation of gaps in mathematics among students and makes it harder for the learners in the continuation (Broza & Ben-David Koliknet, 2009; Kidron, 1985).

- *Learner Characteristics* Students with learning disorders make it difficult for the learner to acquire basic learning skills (reading, writing, and arithmetic) and the learner cannot cope with the difficulties. A student with difficulties does not meet the learning requirements expected of him according to his age, and this is reflected in the achievements commensurate with his abilities. The Diagnostic and Statistical Manual (DSM, 2013) defines learning disabilities as a specific learning disorder, or in other words, a change was made in the definition, with a transition from the term 'learning disability' to 'learning disorder', with emphasis on the obligation of learning intervention as a condition of the definition of the disorder in the learning. Consequently, they frequently make mistakes, experience repetitive failures, and develop a sense of lack of inner responsibility for learning and lack of motivation (Ben Yehuda & Licht, 2013; Geary 2005, 2013; Goldman, 1989).

3.3.2 Characteristics of Students with Difficulties in Mathematics

Students with learning difficulties in mathematics have difficulties with process thinking. Success is an outcome of flexible perceptual thinking, which enables the learner to move between the mathematical concept and the process of solving and the reverse, smoothly and fluently (Tall & Razali, 1993). The difficulty is more profound and derives from a "learning disorder", one that has long-term and multidimensional influence.

Dyscalculia is not a marginal disorder; this is a phenomenon that necessitates the investment of efforts in order to provide a solution for the students who suffer from it (Przybyla, 2021). The percentage of students who experience dyscalculia at different levels of severity is 5-7% of the population (Bugden, Peters, Nosworthy, Archibald, & Ansari, 2021). In contrast, some maintain that this percentage is 5%-8% of the school population (Geary, 2004). These disorders decrease the students' performances and influence the learning of mathematics.

The characteristics of students who have difficulties in mathematics include the following:

1. The slow or inaccurate retrieval of basic arithmetic facts: the main difficulty among students in mathematics is the understanding of number, number operations, and relationship, number estimation, counting knowledge, and arithmetic (Geary, Hoard, Byrd-Craven, Nugent, & Numtee 2007; Jordan & Levine, 2009).
2. The problem of the creation and preservation of mental images of mathematical concepts (the number line as a visual means for the presentation of subtraction as a process of change) (Geary, 2004) or representations of basic concepts of the decimal structure (Geary, 2005).

Most of the students with difficulties in mathematics have weak ability to produce numerical meaning from symbols (Jordan & Levine, 2009).

1. Difficulty to keep information in their working memory (Baroody, 2006; Schuchardt, Maehler, & Hasselhorn, 2008). Difficulties in active memory such as: understanding the concept of the number and understanding basic arithmetic facts and solving problems that require for solution a number of steps (Baddeley, 1986; Bull & Johnston, 1997).
2. Difficulties in long term memory that expressed in two types of contents: (a) memory of facts – basic facts, laws, and formula and (b) memory of procedures / memory of a continuum of processes (Baddeley, 1986; Geary, 2004), lack of number sense, difficulties in processing numerical information, and lacks in metacognitive skills.
3. Difficulties that develop in the learning-disabled students in mathematics or in students who developed learning disability and difficulties on the background of environmental reasons intensify over time. One of the means for examining the degree of difficulty or disorder is the identification of the type of difficulty and the examination of its causes – whether the origin is environmental or neurological. Difficulties that depend on the environment can be resolved through the cultivation and empowerment of the teachers through professional development and through development of learner-tailored teaching methods. In contrast, regarding difficulties that derive from the learning disability, it is necessary to work with students in a special manner and to examine the cause of the disability. There are three main difficulties for the development of learning disability in mathematics:

(1) difficulties in working memory, (2) attention difficulties, and (3) dyscalculia, and they are described in the continuation.

3.3.2.1 Memory Difficulties – Working Memory

Learning is defined as a long-term change that occurs in the mental representations as a result of experiences (Ormrod, 2017). Learning leads to the acquisition of information and skills, and saving this information is called memory (Gupta, Singh, & Sisodia, 2018). Memories are not stored in a special region of the brain but are an outcome of many processes and a number of systems in the brain (Westwood, 2016). The concept of memory in psychology is composed of three 'databases' that are different in purpose and in the quantities of information that they can include: sensory memory or short-term memory, working memory, and long-term memory (Gupta et al., 2018; Tripathy & Öğmen, 2018). Working memory (WM) is compounded from mental processes that hold limited information in a temporarily accessible state in service of cognition (Cowan, Elliott, Saults, Morey, Mattox, Hismjatullina, & Conway, 2005; Geary et al., 2007). "Working memory refers to a mental workspace, involved in controlling, regulating, and actively maintaining relevant information to accomplish complex cognitive tasks (e.g., mathematical processing)" (Raghubar, Barnes, & Hecht, 2010, p.110). WM is a cognitive system that has a central role in learning and thinking, it helps in understanding individual differences in academic performance (Cowan et al., 2005; Wilson & Swanson, 2001). In other words, to store and to remember information it is necessary first to process this information in the WM (Cowan et al., 2005). A student with learning difficulties cannot cope with the requirements of advanced information processing, because of the difficulty with controlled attentional processing that include, monitoring activities with limited resources, suppressing conflicting information, and updating information (Wilson & Swanson, 2001).

Students with learning difficulties in mathematics develop difficulties in the WM that are expressed in the understanding of the concept of the number and understanding basic arithmetic facts or solving problems that require for solution a number of steps (Passolunghi & Siegel, 2004; Wilson & Swanson, 2001; Bull & Johnston, 1997). In addition, there is difficulty with the retrieval of relevant information in complex tasks that include verbal and numerical information (Passolunghi & Siegel, 2004). Deficiencies in the WM make it difficult to save information that is relevant to the task because of distraction or to differentiate between relevant information and information that is not important or necessary for the purpose of the solving of the task (Wilson & Swanson, 2001). The WM and especially the central executive system strongly predict the learner's mathematical abilities, such as the ability to solve mathematical problems and problem-solving strategies (Passolunghi, Vercelloni, & Schadee, 2007; Wilson & Swanson, 2001). The success of the learning depends on the processes that occur in the WM and that are necessary for the achievement of automation in information in the learner (Cowan et al., 2005; Dehn, 2008).

The inculcation of information processing strategy and effective use of WM enable the learner's focus on the task, filtering of irrelevant information, and integration of information from a number of sources including the retrieval of information from the long-term memory (LTM). LTM includes the memory of facts – basic facts, laws, and formulae (Geary, 2004). Mathematical tasks become more complex with the advancement from elementary school to middle school. The students need to remember and to follow up after the considerable information, while carrying out complex processes of solving that include mathematical calculations and automatic retrieval of information and mathematical facts and rules. Difficulties in the WM ability in the learner and lack of attention harm the learner's abilities in mathematical terms.

3.3.2.2 Attention Difficulties

Attention disorder is a common phenomenon, estimated to appear among about 5%-10% of the population of school children. Attention disorder is a phenomenon that does not disappear in adulthood; it accompanies most of the people who have it during adolescence and adulthood (Ministry of Education, 2017). "Inattention manifests behaviorally in ADHD as wandering off task, lacking persistence, having difficulty sustaining focus, and being disorganized and is not due to defiance or lack of comprehension" (DSM-5, 2013, p. 61). In other words, attention disorder is defined as lack of attention, lack of differentiation in information or ignoring the information, high distractibility from stimuli that disrupt the individual's functioning (Ministry of Education, 2017).

Children with attention-deficit/hyperactivity disorder (ADHD) develop an academic and scholastic gap that is expressed in underachievement in mathematical tests (Hart, Petrill, Willcutt, Thompson, Schatschneider, Deater-Deckard, & Cutting, 2010). There is a significant empirical relationship between WM and children with ADHD (Kasper, Alderson, & Hudec, 2012). Attention and memory are essential for learning; and lack of attention and memory are one of the main reasons for learning failure (Westwood, 2016). In addition, the ability to pay attention is influenced by difficulties in the students' fine motor skills.

Difficulties in the students' motor abilities inhibit their development in cognitive, emotional, and social terms. Motor difficulties and delays are characteristic of students with developmental disorders, since the difficulties in cognitive, social, and perceptual abilities are intensified because of the motor disabilities (Libertus & Hauf, 2017). Difficulties in motor skills predict mathematical abilities (Pitchford, Papini, Outhwaite, & Gulliford, 2016), since the difficulties in motor skills require considerable attention from the learner that could be directed towards the understanding of mathematical principles and more complicated operations (Ziegler & Stoeger, 2010). In the mathematical context, fine motor development significantly predicts the development of mathematical cognitive abilities during the elementary school (Cichy, Kaczmarczyk, Wawrzyniak, Kruszwicka, Przybyla, Klichowski, & Rokita, 2020; Libertus & Hauf, 2017), "for math, fine motor skills may underlie the acquisition of quantitative and spatial concepts" (Pitchford, Papini, Outhwaite, & Gulliford, 2016, P.2). Becker, Miao, Duncan, McClelland (2014) argued that the good fine motor skills lead to automation in writing letters and numbers, this promotes cognitive resources toward conceptual processes. It was found that the development of motor skills positively influences academic achievements (Harrowell, Hollén, Lingam, & Emond, 2018). The development of motor physical abilities among students positively influences the acquisition of mathematical abilities and the acquisition of mathematical knowledge (Cichy et al., 2020).

3.3.2.3 Dyscalculia

Many students find mathematics difficult as a result of a phenomenon called dyscalculia. Dyscalculia is a psychological medical concept that addresses the considerable difficulty in the individual to acquire mathematical concepts and harm to the person's ability to perform accurate and correct mathematical calculations (Montis, 2000; APA, 2013). The source of dyscalculia is psychological, neurological, environmental, emotional, and genetic factors associated with neurodevelopmental abnormalities (Geary, 2004; Hannell, 2005; Pandey & Agarwal, 2014; Muktamath, Hegde & Chand, 2021). This phenomenon is not accorded considerable attention in the education system although the percentage of students who experience memory difficulties or cognitive deficit that disrupt their ability to learn concepts or carry out mathematical operations is not marginal (Geary, 2004; Geary et al., 2007).

Students who are diagnosed as students with dyscalculia are students with normal intelligence (Hannell, 2005; Pandey & Agarwal, 2014). Students who suffer from dyscalculia have a specific learning disability in mathematics, a disorder that influences their development of basic abilities ranging from the ability to understand numbers and to acquire basic arithmetic skills, and the ability to manipulate numbers and to learn mathematical facts (Mark-Zigdon, 2011; Pandey & Agarwal, 2014). Students with dyscalculia find it difficult to master the basics of mathematical thinking and as a result find it difficult to acquire mathematical skills necessary for carrying out mathematical tasks (Hannell, 2005). They are characterized by slowness and difficulties with automation, difficulties in mental calculations, use of strategies of counting with the fingers in order to carry out calculations, difficulties in carrying out tasks with a large number of stages, difficulty in remembering mathematical facts, and difficulties in spatial organization (Hannell, 2005; Mark-Zigdon, 2011; Ansari & Karmiloff-Smith, 2002). "Students with low visual-spatial intelligence have difficulty on three metacognition aspects, namely knowledge on strategy, cognitive tasks and self-knowledge" (Rimbatmojo, Kusmayadi, & Riyadi, 2017, p. 7). Furthermore, students who are diagnosed as suffering from dyscalculia will have difficulties in the fields of math reasoning, calculation ability, math memory, and ability of mathematical expression, both in writing and in speech, and in addition in visual-spatial orientation (Pandey & Agarwal, 2014).

Mathematical learning disability is related to general cognitive abilities, some of which were mentioned previously, which include intelligence, working memory, and processing speed and of course attention in the classroom (Geary et al., 2007). The cognitive abilities influence the mathematical cognition. Mathematical cognition entails complex mental activities such as the identification and coding of quantities for internal representations, mental comparisons, and calculations (LeFevre, DeStefano, Coleman, & Shanahan, 2005). Therefore, students who are mathematically learning disabled need structured tasks that emphasize cognitive flexibility, problem solving, speed of processing, spatial orientation, and visuo-spatial abilities (Mascia, Perrone, Bardi, Agus, Mirian; Penna, Lucangeli, 2018).

Difficulties in mathematics obligate the education system in general and the teachers in particular to instill in the students' strategies for coping with the difficulties and with this specific disorder. The next section discusses these strategies.

3.3.3 Adolescents with Learning Difficulties in Mathematics in Israel

Mathematical understanding is a process and goal that focuses on the performance of tasks, while providing explanations through the use of concepts and ideas, giving examples, and ability to project information that is acquired in other contexts and to generalize it on new contents through the ability to explain and justify it (House & Tirosh, 2015). The success in the acquisition of mathematical content is an outcome of flexible *proceptual*[13] thinking. The flexible thinker can automatically switch from the process to the concept and back fluently (Tall & Razali, 1993).

The goal of mathematical education is to bring students to success in the subject, but while some of the students succeed there is a large group that is struggling and having

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[13] *Procept* is an amalgam of three components: a **process** which produces a mathematical **object** and a **symbol** which is used to represent either process or object. The term is a construct used frequently in mathematics education research.

difficulties understanding – and they need help (Tall & Razali, 1993). In the education system, teachers tend to accuse students with difficulties of being "lazy or unmotivated learners" (Westwood, 2007). Students with difficulties do not have the experience of success and they generally are defined as "low achievers" and "slow learners" (Westwood, 2016). The difficulties of students with mathematics appear as a result of a number of reasons (Tall & Razali, 1993).

1. The learner does not perform manipulation of the concept to adjust to the process.
2. The learner does not have the ability to manipulate concepts in a creative manner.
3. The learner uses routinized processes without the use of meaningful ideas.

These difficulties influence the learner, who experiences failures in an accumulating manner. Cumulative and continuous failure contributes to the development of learning difficulties in the school (Westwood, 2007). Students with difficulties in mathematics need help and treatment. The lack of diagnosis of students with difficulties leads to difficulties in in post-secondary mathematics programs (Tall & Razali, 1993).

Today the task to diagnose and identify students with difficulties is assigned to the education system. The main role of the teachers is to combine between their knowledge about pedagogy, topics, and learning contents and technological means, in order to create a suitable learning environment that enriches the learner (Prensky, 2010). The teachers are assigned the role of the development of teaching and learning methods that will help the learner with difficulties accumulate positive and successful learning experiences (Westwood, 2007). Ways of thinking and manner of understanding of students are the foundation stones in the teaching of the subject of mathematics. Teachers who look at the ways of thinking based on ways of solving of their students evaluate correctly their way of knowledge development and develop strategies and plan ways of teaching effectively (Weizman, in House & Tirosh, 2015).

Pedagogically there is still no learning method that is defined as effective for all the children who have difficulties, since the difficulty of every student stem from a different cause (Westwood, 2007). In Israel, a number of pedagogical methods have been adopted to assist students with difficulties especially in the subject of mathematics, which requires the systemic work of the learning educational school staff. One program is the Program of Realization and Excellence that is implemented in Israel in the middle schools in the subject of mathematics. The program of Realization and Excellence provides an answer to the difference in the level of learning among the student. Differences in the level of learning are carried out, taking into account the learner, his abilities, and his learning needs, assuming that it may contribute to the student's development and motivation for achievement (Brandes & Strauss, 2013). Students with difficulties in mathematics are placed for the purpose of the mathematics lessons into groups called realization classes, where they learn for three years, from the seventh grade to the ninth grade, in a special program (Linchevsky, 2013). Second, there is the development of an intervention program for the treatment of students with learning disabilities based on the Response to Intervention (RTI) model. During the 2014 school year, the steering committee of the Ministry of Education recommended the implementation of a comprehensive program that emphasizes the continuous process of work with students with learning and attention disabilities. The committee emphasized the work in two dimensions. The first is the teacher who teaches in the classroom and here the need is emphasized to inculcate in the teacher the subject knowledge, tools, and skills that will help her adjust the teaching, assistance to the students with difficulties, and implementation of an appropriate intervention program. The second is the construction of an infrastructure for supervision and the creation of a professional standard regarding the assessments and the professional factor that diagnoses and that confirms eligibility for accommodations in the testing methods (Ministry of Education, 2018a).

The RTI model is a systemic approach with the goal of the discovery and early preventative intervention that provides support to students with difficulties in a manner tailored to their needs, before they will experience failures in the future (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). According to the model, the support needs to be continuous and the follow up methodical. In addition, the support is examined and evaluated frequently during the intervention that is multi-stage (Ministry of Education, 2018a). The first stage in the RTI model is the identification of the students who have difficulties, a process that is based on the collection of information from different sources (such as learning history, previous interventions, and so on).

In addition, in the education system there are mapping tests and tools for the evaluation of achievements, such as the MEYZAV[14] test held in the fifth and eighth grades, the AMIT[15] test in the ELA[16] program that is held at the start of the seventh grade and tests in specific areas such as mathematics and English. The identification of the students is carried out through reference to academic aspects and/or behavioral emotional aspects. The second stage is the data analysis, and according to the results achieved the students who need focused intervention are selected and directed to suitable intervention programs. The RTI model addresses two levels of identification: (1) universal identification of all the students and (2) individualized identification focused on specific skills, with the aim of differential diagnosis and choice of intervention methods tailored to the needs of the student (Ministry of Education, 2017). In terms of the intervention on the mathematical level, Gersten et al. (2009) proposes eight recommendations of interventions in mathematics according to the RTI model, as follows:

1. Screening all students in order to identify those at risk for potential mathematics difficulties and provide interventions for those at risk.
2. The instructional materials for students in risk will be based on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8.
3. Intervention should be explicit and systematic and include providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.
4. Interventions should include instruction on solving word problems that is based on common underlying structures.

[14] MEYZAV is an acronym for Measures of Effectiveness and School Growth. It is a battery of tests held in the elementary and middle schools in Israel in the subjects of science and technology, native language (Hebrew or Arabic), mathematics, and English. They are held annually in one third of the schools of Israel, so every school participates every three years. The tests are administered by the National Authority for Measurement and Evaluation in Education.

[15] AMIT is an acronym for Kit for Mapping Students and is intended for the identification of students who have difficulties in the native language and for the mapping of all the students to the performance levels.

[16] ELA is an acronym for identification, learning, assessment. The ELA Program is a program operated by the Ministry of Education. Its objective is to advance students with learning ability through the enrichment of the teaching methods and the learning of learning strategies and literacy skills in word-intensive subjects.

5. Interventionists should be proficient in the use of visual representations to enable providing intervention materials that include opportunities for students to work with visual representations of mathematical ideas.
6. Interventions at all grade levels should devote about ten minutes in each session to building fluent retrieval of basic arithmetic facts.
7. The progress of students receiving the intervention should be monitored.
8. Interventions should base on motivational strategies.
9. The recommendations focus on the practice of basic arithmetic facts, intensive in-depth treatment of whole numbers, progress monitoring, and motivational strategies.

This approach emphasizes that methodical support, follow up, control, and feedback provided early and tailored to the unique needs of the learner with difficulties will prevent the future experience of failure. This model emphasizes that the idea of intervention becomes more intensive as the student's needs are more complex; this ensures that the student receives learning opportunities that will enable progress and achievements. One of the main challenges in the teaching of mathematics for students with difficulties is to involve them actively in the building of mathematical knowledge from understanding and to avoid the learning by rote of procedures (Haylock, 1991). Therefore, to cope with the learners' difficulties, it requires intensive intervention on the part of the professionals and the building of a suitable intervention program. Teachers have the responsibility for the manner of the inculcation of knowledge among the students who have difficulties and have the ability to cope with the different problems in the field, and the inculcation of strategies for overcoming the difficulties (Elkins, 2002).

3.4 Learning Strategies of Adolescents/Students with difficulties during puberty

The teaching of students with learning difficulties requires many interventions on the part of the professionals and the construction of an appropriate intervention program (Ministry of Education, 2014). Therefore, some of the responsibility for the treatment of students with difficulties is assigned to the education system in general and the school in particular. The intervention program begins with a preliminary diagnostic evaluation, which is important for the identification of the learner's abilities and difficulties and will serve as a basis for the construction of a multidimensional profile of the learner with difficulties in mathematics, through the formation of the rationale for intervention and the design of a program for tailored teaching (Ben Yehuda & Licht, 2013; Westwood, 2007).

3.4.1 Teaching Strategies for Students with Learning Difficulties

The challenge of the Ministry of Education in general and the teachers in particular is to reshape the teaching and learning strategies of the subject of mathematics among the students with learning difficulties and with difficulties. Students with learning difficulties need special teaching methods through which it is possible to identify and implement didactic ways that enable the correction and diminishment of gaps (Bashara, 2020). Special teaching methods have the aim of lessening the gaps in the achievements of students with difficulties in mathematics relative to the rest of the students and enable the focus on the curriculum of the Ministry of Education (Ministry of Education, 2014).

The scaffolding strategy. Students who are underachieving and who have difficulties in mathematics need help and support to build mathematical concepts. According to the learning theory of Vygotsky (1980), learning and cognitive development is a social process that occurs in the interaction between people. In other words, appropriate support of an adult, during learning, enables the children to advance and to lessen learning gaps in what he defined as the zone of proximal development (ZPD). Learning support and help for the learner so the learner can achieve the performance of the task independently is called *scaffolding* (Wood, Bruner, & Ross, 1976; Lepper, Drake, & O'Donnell-Johnson, 1997). The concept of scaffolding developed over time and its definition was extended. The concept became a basis of many sociocultural ideas. According to Roy D. Pea (2004, p. 423), "this is surely too much complexity to take on at once".

Today digital learning environments are considered to be scaffolding. The use of technological representations, such as visual representations, graphs, and diagrams, improve the student's learning process (Mayer & Moreno, 2005). The load of multidimensional stimuli may create cognitive load that harms the learning processes

(Sweller, 2011). Therefore, the contribution of multimedia to learning is the help to the cognitive system in order to overcome disability (Mayer, 2014). The digital environment serves for the learners as an intellectual partner that helps advance thinking, learning, and understanding (Ashburn & Floden, 2006). Technology serves as a support of the learning, helping develop students' cognitive capacities during thinking, learning, and problem solving (Crompton, Grant, & Shraim, 2018). In other words, different learners can derive benefit from the learning materials in diverse situations of learning. The use of extensive illustrative means and tailored ways of instruction may make it easier for the students with difficulty and simplify the learning material. Digital media enables the creation of a diverse lesson that includes practice, immediate control of the products, and promoting feedback (Ministry of Education, 2020). In geometry, the use of dynamic geometry software provides the information on which the proof was based, and in problem solving sessions dynamic geometry software fosters the verification process of students (Özçakir, 2019). The virtual manipulations enable the learners to make their thinking visible and to visualize the shapes as geometric entities (Zbiek, Heid, Blume, & Dick, 2007). This leads the students to be partners in the learning process. They supervise and control learning, while controlling the thought processes, motivation, and behavior, thus improving their learning (Chen & Law, 2016; Zimmerman, 2001).

In the teaching of students with difficulties, it is important to also develop the learner's ability in terms of metacognition and organization, to base on pedagogy of **self-regulated learning** (SRL). Metacognition is the knowledge of the learner about one's own thinking (Zimmerman, 2002). Ernesto Panadero (2017) maintained that the self-regulated learning (SRL) strategy is a process in which the learner builds the learning, through the use of metacognitive, behavioral, and motivational characteristics. These characteristics increase the effectiveness of the learning, since the learner directs his thoughts to learning from the awareness of cognitive strategies, self-efficacy and desire that he uses in the learning to achieve his learning objectives. Students need to cope with their difficulties out of self-awareness of the learning limitations. Learning difficulties among students are attributed to lack of metacognitive awareness of their personal limitations and an inability to compensate (Zimmerman, 2002). Self-regulated learning (SRL) skills are not acquired spontaneously, but it is possible to develop them through the development of learning environments that support and provide opportunities for active experience in them (Kramarski & Michalsky, 2009). The role of the teachers is to strengthen the students and to give them self-awareness about their limitations, to allow them to develop a capability to self-regulate (Zimmerman, 2002). In mathematical terms, self-direction strengthens the intuitive view of mathematical structures and the construction of contexts with arithmetic actions, flexibility in the development and understanding different solution strategies, through the use of previous knowledge and experience (Bashara, 2016). This encourages the development of learning based on high order thinking and ability to solve more complex problems (Kramarski & Michalsky, 2009).

Mathematics is considered one of the most important and difficult subjects in the school, and as a result many students have difficulties with the studies of mathematics. The development of the ability of calculation, numerical insight, and calculation fluency is one of the important strategies for the advancement of students in mathematics. The difficulty with the study of mathematics is the ability to combine convergent-algorithmic thinking and divergent-creative thinking (Berg, 2001). In addition, the subject requires mastery of previous material as a condition of the mastery of new material (Geary, 2004). The curriculum in mathematics (Ministry of Education, 2006b) emphasizes not only the products but also the ways of thinking and necessitates mastery of basic facts in written and oral arithmetic. One of the strategies that can promote students who have difficulties is *numerical insight and computational fluency*.

Numerical insight is expressed in this program in an intuitive view of the numbers, mathematical structures, and arithmetic actions that develop the learner's ability to cope with numerical problems through the development of different and creative strategies of solution. If there is numerical insight, the ability to calculate improves and the speed of the calculation increases. Calculation is a complex cognitive action, and many students have difficulties with it (Almog, Ben Yehuda, & Sharoni, 2014). The development of thinking and mathematical abilities addresses three stages that characterize the progress of children until the mastery of the foundational facts: counting strategy, thinking strategy, and mastery through the effective and accurate production of the answers (Baroody, 2006). Computational fluency is characterized as mastery of a rapid computational process that develops with numerical insight. The learner can solve an exercise in a quick and accurate manner on the basis of numerical insight, which is an essential component of the development of mathematical strength (Baroody, 2006; Boaler et al., 2015; Fuson, 2003). Children need practice accompanied by support, to let them work with numbers in different ways and not only to mechanically memorize what is learned (Boaler et al., 2015; Fuson, 2003).

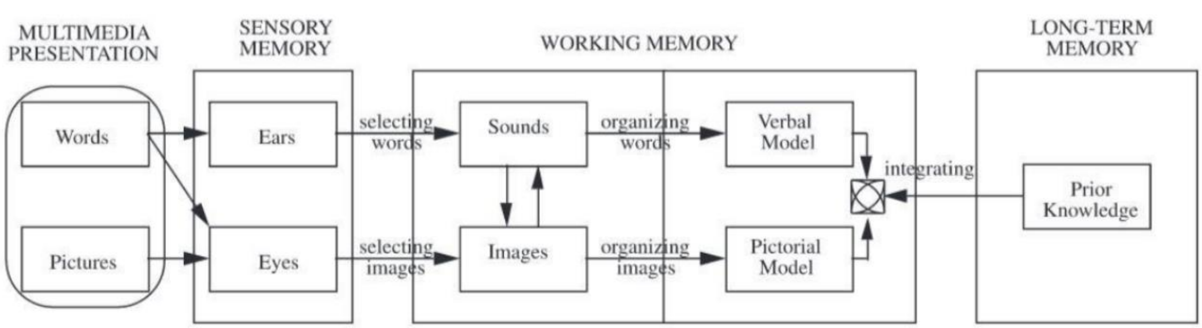
Calculation ability, numerical insight, and computational fluency are expressed in that the student must have the ability to compute and display expertise in the basic facts so that he can focus on the meaning of the more complicated calculation actions he is required to carry out (Almog, Ben Yehuda, & Sharoni, 2014). In other words, it is necessary to teach the students to think and not only to calculate. To achieve numerical insight and automation in the calculation ability, students with difficulties need considerable illustration. The concretizations help the learner who has difficulties and are considered a learning strategy that influences the process of understanding and the ability to build mental models in an easier, more organized, and better understood way.

3.4.2 Using ICT as a Strategy to Help Students Cope with Mathematical Learning Difficulties

The Cognitive Theory of Multimedia Learning (Mayer, 2009) and the Cognitive Load Theory (Sweller, 2011) are cognitive theories that derived from the integration between the Working Memory Model (Baddeley & Logie, 1999) and The Dual Coding Theory (Clark & Pavio, 1991) that holds that the person’s ability to process data is undertaken through two separate channels: the one auditory and the other visual. The two channels are limited in terms of the capacity to absorb stimuli from the environment, and they are responsible for the processing of information that is important to the learning process (Paas & Ayres, 2014). The two channels can work together or separately (Clark & Paivio, 1991). The brain can use the two channels, auditory and visual, in order to organize, process, and store information in order to retrieve it for the purpose of its use (Sternberg, 2003). The working memory is responsible for the data processing and can process only two or three items together (Kirschner, Sweller, & Clark, 2006). New information that is not practiced or handled is lost in 15-30 seconds (Paas & Ayres, 2014). The learning process occurs when we store new information in the LTM, but the limited ability of the working memory to absorb stimuli and information from the environment can harm the acquisition of knowledge and the learning process (de Jong, 2010; Sweller, 2004). In other words, it is important to increase the effectiveness of the learning process through the reduction of the cognitive load imposed on the working memory.

The Cognitive Load Theory differentiates between three characteristics that contribute to load: intrinsic cognitive load that addresses the material and the learning content, the extraneous cognitive load caused as a result of the teaching materials that are used to present the content, and the germane cognitive load that addresses the load caused by the learning process (de Jong, 2010; Wong, Leahy, Marcus, & Sweller, 2012). Today the important challenge is to build teaching methods and environment that reduce the cognitive load (Wong et al., 2012). The adoption of an appropriate learning environment will make the learning process more effective. The working memory model, the dual coding theory, and the cognitive load theory are three theories that overlapped and include interdependencies relationship (Doolittle, McNeill, Terry, & Scheer, 2011). The three theories contributed to Mayer's development of the cognitive theory of multimedia learning (Mayer, 2001). Multimedia learning is based on learning from words and pictures (Mayer, 2001; Mayer & Moreno, 2003).

Figure Number 8: Multimedia Learning (Adopted from Mayer, 2001, p. 44)



Mayer's cognitive theory of multimedia learning makes three assumptions about how people process information. First, the dual-channel assumption is that learners possess separate information processing channels for verbal and visual material. Second the limited capacity assumption is that there is a limited processing capacity available in the verbal and visual channels. Third, the active processing assumption is that active processing learning requires depth cognitive processing in the verbal and visual channels in order to gain new information (Mayer, 2001; Mayer, 2014; Mayer & Moreno, 2003). The Cognitive Theory of Multimedia Learning provides a guideline that may help the designers to design multimedia instruction and to implement effective cognitive strategies to help learners learn efficiently (Sorden, 2012). The implementation of these three assumptions is possible because the digital environment provides interactive learning in which the learners can influence actively their learning processes (Mayer & Moreno, 2007). The interactive text (Ministry of Education, 2013b) enables the use of literacy skills, thinking skills and technological abilities that encourage active learning in which the learners can influence actively their learning processes (Mayer & Moreno, 2007). The interactive text enables the deployment of literacy capabilities, thinking abilities, and technological abilities that encourage active learning. The learners can update and improve the information easily, and they have the ability to carry out virtual manipulatives on objects, which enables the creation of interactive teaching-learning (Moyer-Packenham, Salkind, & Bolyard, 2008).

The feature of a virtual manipulative is defined as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p. 373). The use of virtual manipulatives in teaching mathematics enables the construction of representations of objects in different forms. This helps the learner link between the representations of the objects such as the symbolic notations and the abstract concepts. The learners' physical interaction in the use of computerized environments improves the ability of mathematical thinking and promotes their academic development (Moyer-Packenham & Westenskow, 2013). The manipulations in the digital environment are a part of the interactivity (Mayer & Moreno, 2007); they are important in mathematics in order to achieve an in-depth understanding of the topic since they serve as cognitive technological tools (Moyer-Packenham & Suh, 2011; Zbiek et al., 2007). The use of most of the dynamic representation supports the making of decisions and other skills required to solve problems, such as assessment, choice of representation, and linkage between representations (Yerushalmy, 2006).

Interactivity of the digital environments is provided also by dialoguing and controlling. Dialoguing addresses the additional information from feedback that the learner obtains as a response to the solution done. Controlling occurs when the learner selects the individual learning pace or the preferred order of presentation (Mayer & Moreno, 2007). The interactive digital environment is rich in visual representations. The visual representation (Ministry of Education, 2013a) enables the presentation of nonverbal information in a manner that creates interest and enjoyment through the use of concretizations in the images and simulations at different levels. This helps the learner who has difficulties and who has not yet acquired reading and writing abilities and/or is found in different stages of their acquisition. The manipulations that are done visually on the screen influence the learner's understanding and increase the meaningful mathematical learning in the learner (Moyer-Packenham et al., 2008). In the teaching of mathematics, the solving of problems through the use of learning environments that encourage meaningful learning, in algebra for example, emphasize a process of solving that the learner builds associations between bodily action and analytic symbols (Yerushalmy, 2006) and the creation of connections between the different representations in mathematics (Moyer-Packenham et al., 2008).

In the multimedia digital learning environment, the representation of information is undertaken in a tangible manner (picture, sound, movement, writing, and icons), thus enabling the different learners, and especially those with difficulties, to absorb information in different ways, thus enabling the creation of successes (Ministry of Education, 2013a). "ICT in school education having the potential to influence teaching and learning processes by enabling wider access to a range of resources, allowing greater power to analyze and transform information, and providing enhanced capacities to present information in different forms" (Fraillon, Ainley, Schulz, Friedman, & Gebhardt, 2014, p.41).

Virtual manipulatives are a main element in the interactivity of mathematical learning environments. Virtual manipulatives provide five main traits in the teaching of the learning of mathematics that help the students understand mathematics (Moyer-Packenham & Westenskow, 2013):

1. The learners are more focused and concentrated on the mathematical problem and processes.
2. It encourages creativity and multiplicity of ways of solving among the learners.
3. It increases the learners' ability in the connection between the different concepts and representations in mathematics.
4. The dynamic representations enable precision and effectiveness in the learning process.
5. It encourages the students and motivates them to persevere in the carrying out of mathematical tasks.

It is important to integrate the use of digital learning environments at the start of the learning of algebra. The integration of environments that enable symbolic manipulation helps the students who have difficulties to be fluent in algebra (Yerushalmy, 2006). In geometry, the use of technology such as dynamic geometry environments provide technical and conceptual activities that constitute the basis for the understanding of mathematical concepts. The technology empowers the learners' knowledge and understanding in geometry because of five characteristics: visualization, manipulation, cognitive tools, discourse promoters, and new ways of thinking (Crompton et al., 2018).

To conclude, "Technologies can improve the teaching/learning process by reforming conventional delivery systems, enhancing the quality of learning achievements, facilitating state-of-the-art skills formation, sustaining lifelong learning and improving institutional management" (UNESCO, 2009, p. 11). The interactive digital environment constitutes an answer for the learners who are found at different levels of learning and have special learning styles (Ministry of Education, 2013a). The use of digital environments gives a mathematical experience to the learners and encourages them to research new ideas, thus positively influencing the learners' achievements (Moyer-Packenham & Westenskow, 2013).

Chapter 4: Research Methodology

The issue that this study addresses is the impact of online collaborative discourse as formative feedback for the improvement of the achievements of students with difficulties in learning mathematics. Very few research studies have examined specifically the educational influences of the use of mathematical discourse that occurs through the use of technological means on students who have difficulties in mathematics. According to the National Council of Teachers of Mathematics (NCTM, 1991), the mathematical discourse is used for the development of the mathematical understanding and is considered one of the six main standards in the teaching of mathematics. The combination between mathematical discourse, collaboration, and the use of digital means points out not only the importance of mathematical competences in the process of lifelong education of a person, but also the marking of these competences for the development of children and adolescents with learning disability. The analysis of the mathematical discourse occurring in a collaborative manner through the use of digital means is very important to the development of mathematical education regarding students with difficulties. The analysis promotes the understanding regarding the influence of the integration between the mathematical discourse and the use of information and communication technologies in the teaching of mathematics and their contribution in the promotion of thinking and learning abilities of students who have difficulties. The research presented in the empirical part is based on the result of a quasi-experimental research project. To address this problem, the key objectives of this study were exploring both the direct effect of the online collaborative discourse of math learning on the meta-cognitive thinking ability of students with difficulties in the middle schools in state schools in terms of the building of strategies of processing, analysis, and data control and the effect on academic/intellectual and social/emotional outcomes of students with math difficulties in the Arab sector. This type of research can have important implications for pedagogy because it allows us to promote the learner's mathematical skills in academic, cognitive, emotional, and social terms through the development of a personal intervention plan customized to the learner who has difficulties in mathematics.

4.1 Research Objectives

The research study has two main research objectives, when each main objective has specific objectives.

1. To investigate the impact of the process of learning-teaching by using methods of mathematical discourse based on collaborative digital learning environments on the learning strategies and academic achievements of students who have difficulties in mathematics in the seventh grade.
   1.1 To investigate the degree to which the process of online collaborative mathematical discourse influences the development of learning strategies among students who have difficulties in mathematics.
   1.2 To investigate the degree of influence of the online collaborative mathematical discourse on the achievements of students with difficulties in mathematics.

2. To investigate the influence of the process of the development of the online collaborative mathematical discourse as a means for identification and assessment of the learning abilities (weaknesses and strengths) of students with difficulties and emotional-social learning.
   2.1 To investigate the degree of effectiveness of the process of the online collaborative mathematical discourse as a means for identification and assessment of the learning abilities (weaknesses and strengths) of students with difficulties with mathematics.
   2.2 To investigate the degree of influence of online collaborative mathematical discourse on the advancement of social-emotional learning among students with difficulties in mathematics.

4.2 Research Questions and Hypotheses

The research main questions and sub-questions, and hypotheses are as follows:

Research Question 1

How does the online collaborative mathematical discourse influence the building of effective learning strategies and the promotion of the academic achievements of students with difficulties in mathematics in the seventh grade?

1.1 To what degree will the development of the online collaborative mathematical discourse contribute to the promotion of learning strategies among students with difficulties in mathematics?

Research Hypotheses

- Hypothesis 1.1A: Students with difficulties who learn using online collaborative mathematical discourse develop higher order thinking abilities than do students who learn in the traditional method.
- Hypothesis 1.1B: Students with difficulties who learn using collaborative discourse based on the use of technology are quicker to remember, process, and retrieve facts than are students who learn using the traditional method.
- Hypothesis 1.1C: Students with difficulties who learn using collaborative discourse based on the use of technology have greater control in the development of the stages of solution process.

1.2 In what way does the online collaborative mathematical discourse influence the increase of the achievements of the students who have difficulties in mathematics?

Research Hypotheses

- Hypothesis 1.2A: There is a contribution of using online collaborative environments to the improvement of achievements of the learners with difficulties in mathematics.

- Hypothesis 1.2B: Students with difficulties who learned mathematics using collaborative digital learning environments may succeed more than students who learned in the traditional method.

<u>Research Question 2</u>

How the online collaborative mathematical discourse influences the process of identification and assessment of the learning abilities (weaknesses / strengths) of the students with difficulties and the promotion of emotional and social learning?

2.1 To what degree is the online collaborative mathematical discourse considered by the teacher to be an effective means for the identification and assessment of the learning abilities (weaknesses / strengths) of the students with difficulties in mathematics?

<u>Research Hypotheses</u>

- Hypothesis 2.1A: **A** teacher who uses collaborative online mathematical discourse is more successful at locating the difficulties of specific students than a teacher who teaches in the traditional method.
- Hypothesis 2.1B: A teacher who incorporates in the teaching the use of collaborative digital environments can broaden his knowledge on the reason of difficulty of the learner with difficulties.
- Hypothesis 2.1C: A teacher who incorporates in the teaching the use of collaborative digital environments can follow up dynamically after the development of the learner’s abilities in solving mathematical problems.

2.2 To what extent does online collaborative mathematical discourse help the development of the emotional-social learning ability among students who have difficulties in mathematics?

<u>Research Hypotheses</u>

- Hypothesis 2.2A: Math discourse that develops in online collaborative learning influences the motivation of the student with difficulties to solve math problems and participate in the tasks.

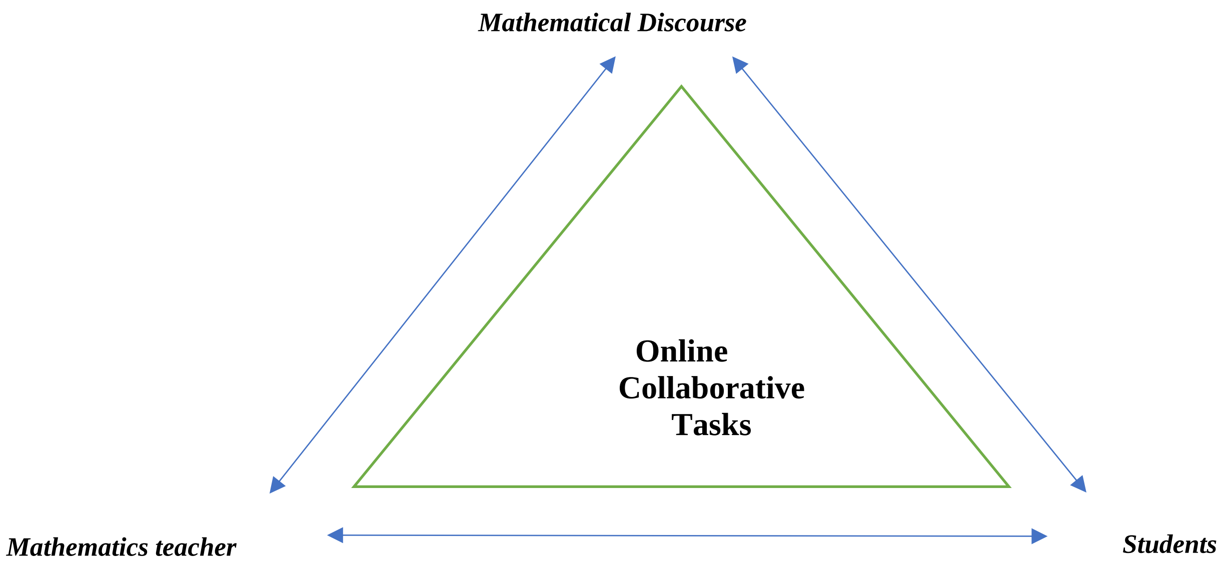
- Hypothesis 2.2B: Students with difficulties who learn with an online environment tend to have more positive learning experience than students who learn in the traditional method.
- Hypothesis 2.2C: Learners with difficulties who learn by online collaborative learning have better self-efficacy to cope with the accumulating math information.
- Hypothesis 2.2D: There is a positive relationship between the use of collaborative social discourse based on digital environments and the promotion in the emotional aspect, or in other words, a positive learning experience and motivation to perform the task on the part of the learner with difficulties.

4.3 Variables and Indicators

Independent Variable

The online mathematical discourse based on student collaboration.

Figure Number 9: The Proposed Model of the Independent Variable (Source: own work)



Mediating Variables

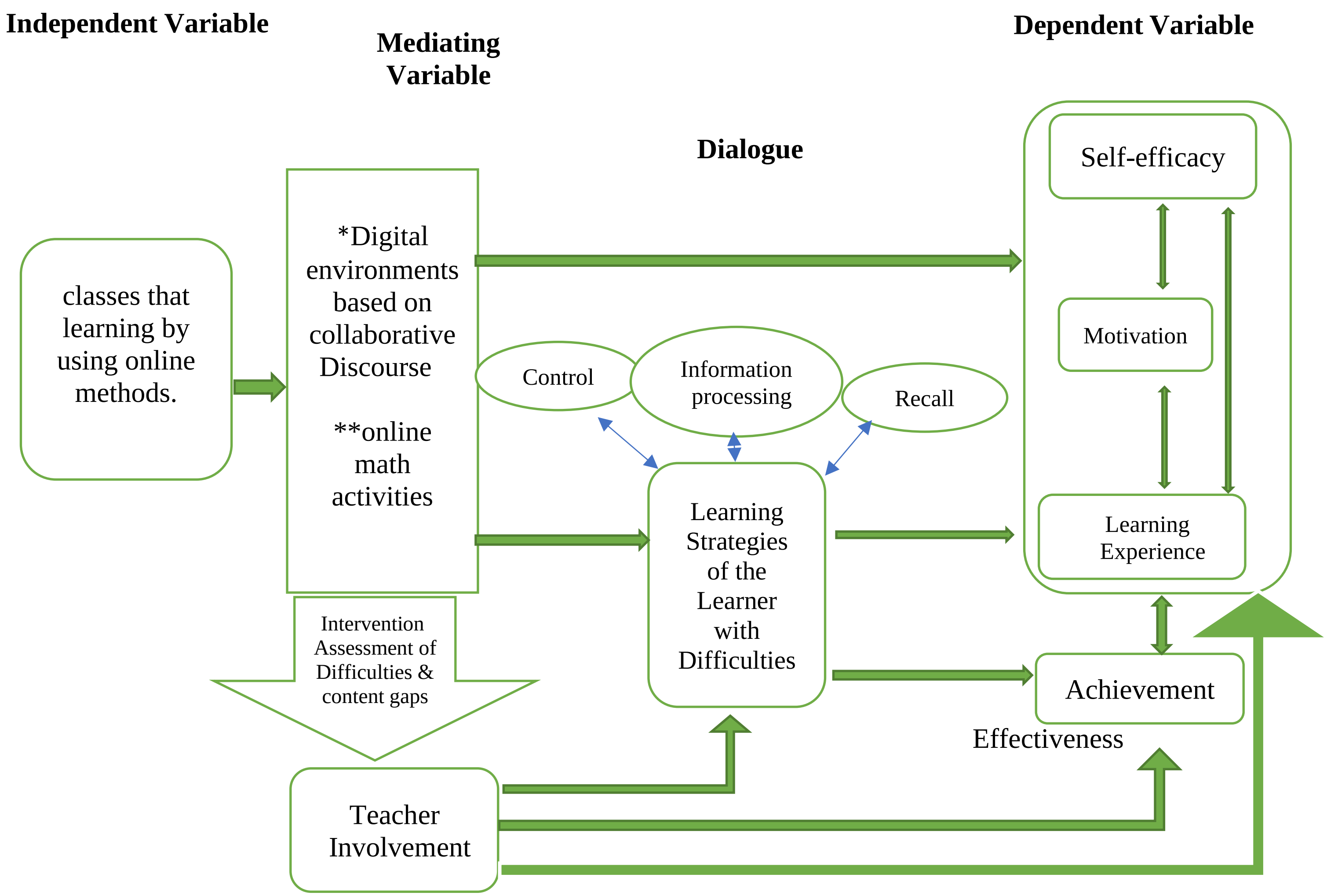
- The digital math environments that encourage developing collaborative discourse.
- The mathematical tasks are based on areas of content.

Dependent Variables

The research dependent variables in this pedagogical experiment:

- **The progress of the learner** with difficulties in terms of academic achievements.
- **The improvement in learning strategies** in the learner in solving in the solving of mathematics problems (memorization, control, and elaboration strategies).
- **The learning experience** of the learner in his participation to learn with his peers in online tasks.
- **The motivation** of the students to participate in the solving of tasks.
- **The learner's self-efficacy**, his participation in solving online tasks with friends.
- **The teacher's ability to identify** content gaps, difficulties in the solution process.
- **The effectiveness** of the teacher's identification the students' difficulties\strengths.

Figure Number 10: The Proposed Research Model (Source: own work)



4.4 Research Method, Research Strategy, Techniques, and Research Tools

4.4.1Method and Research Strategy

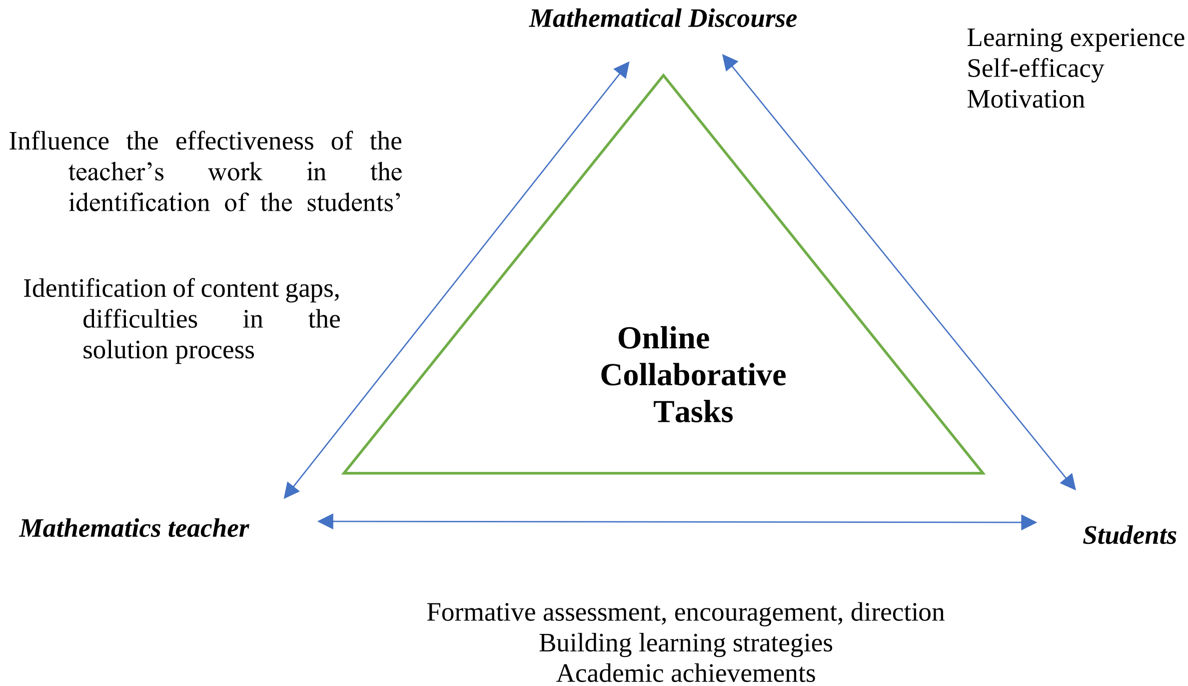
The present research study is controlled experimental research, in a quasi-experimental design, which examines the effectiveness of the integration of the program of online collaborative mathematical discourse in order to advance the achievements of students with difficulties in the mathematics in the seventh grade. In the research the independent variable is manipulated, and the participants are not randomly assigned to terms or orders of terms (Cook & Campbell, 1979). The quasi-experiment of the current research is a time-series experiment. During the academic year there is a set of measurements taken at intervals over the year that are interrupted by using online collaborative mathematical discourse.

In response to the research question, the research was conducted through the mix of two research approaches, the qualitative research approach, and the quantitative research approach. Aliaga and Gunderson (2002) describe quantitative research methods as explaining phenomenon by gathering data using numerical patterns and analyzing with using mathematical methods, in particular statistics. Therefore, according to Creswell (2003) and Williams (2007), quantitative research must base on inquiry techniques such as experiments and surveys and collect data on predetermined aids and tools on purpose to produce statistical data in order to confirm or refute alternative hypothesis. In contrast, qualitative research focuses on the understanding of the phenomenon as they appear in the real world, so that there is no need to research hypotheses that derived from the theory. The planning of the research is organized and structured: the presentation of the problem, the raising of the question, the collection of the data, the analysis of the data, and the giving of answers to the questions that were brought up. However, the researcher's progress in the performance of the research is flexible, since in the research stages there are relationships when the change in one of them influences the others (Sabar Ben Yehoshua, 2001; Shkedi, 2003).

The present research study integrates between instruments that can be statistically analyzed such as questionnaires through which information is collected from the teachers and mapping tests that constitute the basis for the quasi-experiment that is performed in the research framework. These instruments include observation of the learners and analysis of records, transcripts, and interviews to document the qualitative constructivist part of the research (Huberman & Miles, 1994). These instruments emphasize the teachers' course of action, manner and nature of the learning occurrence among students with difficulties, and manner of advancement of the discourse that occurs between them and the teachers and its influence on the student with difficulties.

According to Johnson, Onwuegbuzie, and Turner (2007), mixed methods research is today considered the third approach in the field of research. For the specific research carried out in the school in the seventh grade, the mix of the methods emphasized many important elements that enabled a better analysis of the data and highlighted essential points about the research results. Gelo, Braakman, and Benetka (2008) maintained that research built on mixed methods research can be more effective and can enable the researcher to overcome the limitations of just the quantitative or qualitative approaches. According to Creamer (2018), the importance of the mixed methods approach is its improvement of the reliability of the results, through the triangulation of the multiple sources.

Figure Number 11: The Full Proposed Model of the Research Study (Source: own work)



4.4.2 Research Process

The quasi-experiment was carried out in the following four stages. In the **first stage**, the tests of the supervisor of mathematics coordination were held at the start of the quasi-experiment for the seventh grade (at the level of the end of the sixth grade) for all the students, without differentiation between the students with difficulties and the other students. These results were saved for the purpose of the comparison with the end of the process of the assimilation of the program. The student's scores were divided according to the following table of scores. The purpose of this division is to identify the students who have difficulty, and to identify the degree and type of difficulty of each student.

Table Number 1: Table of Scores (Source: own work)

Grade	0-20	21-40	41-60	61-80	81-100	Learner Difficulty
Student 1						
Student number						

In this stage, after the mapping test, the students who participated in the quasi-experiment were chosen. There were six groups (30 students) that participated in the experiment, and each group included five students and, 28 students that were chosen as a control group. The students of the quasi-experiment were chosen randomly, five students from the two categories of grades (41-60) and (21-40). In middle school, there is a division of students who study in accordance with the groups (A, B and C). In Group B, we select students from these two categories based on the assumption that these categories include students who have academic difficulties but yet maintain an academic foundation (i.e., a good academic base) and motivation to learn.

After the choice of the potential students who will participate in the research group, the interviews were carried out by the subject teacher (see second stage). The aim of the interview is to conduct an initial examination of the form of thinking of the learner who has difficulties in terms of the process until the solution is achieved, the identification of the difficulties (difficulty in understanding the facts and the memorization of the facts or difficulty with the processes of the solution, difficulty with reading comprehension, or difficulty with the translation of the questionnaire from verbal language to mathematical language), with emphasis on the strong and weak points (See Appendix Number 8).

The second stage, the training of the mathematics teachers in the researched middle school (See Appendix Number 10), was carried out with the implementation of the skills of online collaborative mathematical discourse during the 2020-2021 school year, through workshops and exposure to diverse and authentic tasks tailored to the students with difficulties in mathematics. In the process of the training and the implementation, the teachers were asked to participate in the choice and construction of the tasks tailored to the students with difficulties and to provide formative feedback for the students with difficulties and the use of alternative assessment (Birenboim, 1997).

In the **third stage**, the carrying out of the quasi-experiment, the students who will be chosen will receive every week authentic online collaborative tasks in which they will seek to work together with the teacher and the group to solve them (Appendix Number 6 presents an example of an online task). The group worked according to the directive of the mathematics teacher. The students solved together the tasks while using online discourse. The goals of this discourse:

1. The students hold an online discourse according to their understanding. They are the first guides of the process of solving according to insights and interpretations of the frontal lesson held and according to the basic knowledge they had from the frontal lessons.
2. The teacher through the discourse identifies the students with difficulties and their lack of understanding of the learning contents, the reasons for difficulties in the process of the solution, and the learning styles suitable for the participating students. According to this, the teacher builds an intervention program that has a dynamic nature, according to the difficulties of the learners and their learning styles.

In the **fourth stage**, the end of the process, all the seventh-grade students were examined again (in the supervisor of mathematics coordination tests). The results of the summative test of the students who were defined as having difficulties were compared with the results at the start of the year, to determine the scope of success of the Quasi-experiment. Students with difficulties in the schools in the control group also took this test. In addition, the mathematics teachers (in the experimental and control group) of the students with difficulties were asked to fill out the closed-ended and open-ended questionnaire, anonymously (see Appendix Number 1 and Appendix Number 2). Before the tests of the supervisor of mathematics coordination, the students (in the experimental and control group) were asked to anonymously fill out the closed-ended questionnaire (see Appendix Number 4), to determine the depth of the assimilation of the project by the students with difficulties and their teachers.

4.4.3 Research Instruments

The collection of the data using a variety of instruments and different sources influences the findings and the final conclusions (Shkedi, 2003; Yin, 1981). According to Johnson, Onwuegbuzie, and Turner (2007), it allows the consideration of multiple outlooks.

4.4.3.1 Questionnaires

In response to the objective of the research study, which seeks to examine the contribution of the online collaborative discourse to the improvement of the learning strategies and the increase of the motivation and self-efficacy of students with difficulties in the subject of mathematics and the influence on the academic achievements, a closed-ended and an open-ended questionnaire was built for the mathematics teachers and a closed-ended questionnaire was built for the students with difficulties, in the experimental group and the control group.

1. Closed-Ended Questionnaire for Students with Difficulties in Mathematics

This questionnaire consists of three parts, and each part examines a different mediating variable. The questionnaire was distributed to thirty students who participated in the research at the end of the experiment (see Appendix Number 4). The questionnaire is anonymous, and therefore it was distributed by a teacher who is not on the respondent's staff of the mathematics teachers.

The first category of the questionnaire for the student examined the learner's self-efficacy. Hence, use was made of the General Self-Efficacy Scale of Chen and Gully (1997). The questionnaire was translated into Hebrew by Grant-Flumin (1998) and consists of fourteen items that reflect the respondent's belief regarding his abilities. The respondent notes his consent with each one of the statements. The questionnaire is based on a Likert scale and the statements range on a scale of five levels, from "do not agree" to "agree very greatly". The questionnaire authors reported a level of reliability of Cronbach's alpha, α=0.92 (Chen & Gully, 1997). (The items on the questionnaire are 1-14.) These items also were logically validated to adjust them to the research questions.

The second category of the questionnaire for the student is learning strategies. The questions were taken from the Self-Regulated Learning (SRL) Questionnaire, which is included in the PISA questionnaire conducted in Israel and in other countries around the world in the years 2000-2002 (Kramersky & Mevarech, 2004; OECD, 2003). The questionnaire includes thirteen statements about learning strategies that include: (1) memorization strategies the student deploys to remember the learned material, (2) control strategies of the learning process, which are actions of examination and clarification that the student performs and that address the contents and skills the student must learn, and (3) elaboration strategies, which are actions that the student performs to examine the relevance of the information for him and its suitability to his previous knowledge, with the examination of the relationship between the new material and the material learned in other subjects.

Table Number 2: Learning Strategies Items in the Questionnaire (Source: own work).

Strategy	Statement Number	Example of a Statement in the Questionnaire
Memorization strategies	15, 19, 24, 27	When I learn, I try to memorize the material so that I will know to answer each question asked about it.
Control Strategies	16, 18, 22, 23, 25	When I learn, I try to examine myself and to check whether all the concepts are clear to me.
Elaboration Strategies	17, 20, 21, 26	When I learn, I attempt to find relationships between what I have already learned and the learned material so as to understand better.

The students were asked to note the learning strategies that they use following the use of online collaborative learning. The questionnaire is based on a Likert scale, and the scores range from 1 – do not agree to 5 – agree very greatly. (In the original questionnaire, the scores range from 1 – never to 4 – always.) Therefore, as the values are higher, the frequency of use of the learning strategies is higher, and the reverse is also true. Kramersky and Mevarech (2004) reported that the values of Cronbach's alpha were as follows: the general reliability of the questionnaire – α=0.77, memorization strategies – α=0.74, control strategies – α=0.71, and elaboration strategies – α=0.84. (The statements in the questionnaires of the students are 15-27.)

The third category of the questionnaire examines intrinsic motivation (Intrinsic Motivation Inventory). This questionnaire is based on the short intrinsic motivation questionnaire of Ryan, Koestener, and Deci (1991) and consists of 22 items. The authors reported a reliability coefficient of Cronbach's alpha, α=0.86. The intrinsic motivation questionnaire is on a Likert scale from 1 – "do not agree" to 5 – "agree very greatly". (In the original questionnaire the scores ranged from 1 – "do not agree at all" to 4 – "agree greatly"). In statements 38, 40, 41, 45, 46, and 48 the scale was reversed. The level of intrinsic motivation of every student was calculated according to the mean of the student's responses to the questionnaire. As the score is higher, the intrinsic motivation is stronger. The questionnaire examines indices of learning experience and enjoyment, sense of efficacy, and sense of calm (the statements are 28-49).

Table Number 3: Motivation Indices in the Questionnaire (Source: own work).

Motivation Index	Statement Number	Example of a Statement in the Questionnaire
Sense of enjoyment & learning experience	28, 32, 35, 37, 41,42, 44, 47	When I performed the online collaborative task, I thought how much I am enjoying it.
Sense of efficacy	30, 31, 34, 38, 39, 43, 49	I think that I succeeded rather well in this activity in comparison to the members in the group.
Sense of calm	29 ,33 ,36 ,40 ,45, 46, 48	I was not agitated during the performance of the online collaborative tasks.

In contrast, a control group was chosen that learns in the class in the traditional way. The group, which consists of 28 students with difficulties and with the same characteristics as the students participating in the experiment, was given a questionnaire similar in terms of the content of the questions. The questionnaire examined the same variables but in reference to frontal class assignments (See Appendix Number 5).

2. Questionnaire for the Teachers (Appendix Number 2)

The questionnaire examined the attitudes of the mathematics teachers regarding the effectiveness of the use of the online environment as a means that increases the effectiveness of their work and the indices of learning of the learner who has difficulties. In this stage of the research study, the researcher used the logical / face validity of the statements and open-ended questions, which are based on the professional literature and the cumulative experience in the quantification of concepts such as online collaborative discourse, motivation, self-efficacy, effectiveness, and so on (Birenboim, 1993; Friedman, 2005; Sabar Ben Yehoshua, 2001). In the stage of internal validity, the statements of the closed-ended questionnaires of the teachers and the students will be validated by item / total analysis. The item score that will be found to have the highest correlation with the score of the entire scale is the one that will be included in the final questionnaire intended both for the teachers and the students who are participating in the research. The population that will be sampled in the determination of the internal validity of the assessment instrument will serve for the researcher the calculation of the level of reliability of the closed-ended questionnaires according to Cronbach's alpha through the comparison of the mean and standard deviation of each and every statement (Friedman, 2005). The improvement of the level of reliability of the different statements (if necessary) depended on the values obtained (Friedman, 2005). The calculations of internal validity and reliability of the closed-ended questionnaires were performed using the SPSS software.

The questionnaire that the teachers in the experimental group were asked to fill out (see Appendix Number 2) is based on the following table of indices.

Table Number 4: Indices in the Questionnaire (Source: own work).

Index	Statement Number	Example of a Statement in the Questionnaire
Effectiveness of the process	1, 2, 3, 4, 5	The online collaborative discourse gave me information about the student's process of solving in the tasks.
Learning strategies	6, 7, 8, 9, 10, 11	The online collaborative mathematical discourse requires the ability to reason and explain, and this strengthens the student's thinking.
Motivation	12, 13, 14, 15, 16	The online mathematical collaborative discourse causes the students with difficulties to act more because of the encouragement they receive from their classmates.
Learning experience	17, 18, 19, 20, 21	The student with difficulties expresses enjoyment participating in the online tasks.
Self-efficacy	22, 23, 24, 25, 26	The collaborative dialogue in the online discourse develops the thinking and increases the ability to ask questions among the students who have difficulties.
Achievement orientation	27	The achievements in the subject of mathematics will increase if we persevere in this type of learning in our classes.

In contrast to the group of teachers participating in the quasi-experiment, a control group of teachers was selected, who teach in the class in the traditional frontal manner. The group consists of six teachers. A questionnaire similar to the questionnaire distributed to the teachers in the quasi-experimental group in terms of the content of the questions was distributed to the control group. The questionnaire examined the same variables but in reference to the frontal class assignments (See Appendix Number 3).

4.4.3.2 Interviews

In-depth Interview with the Teachers Who Participated in the Implementation of the Experiment (Appendix Number 1)

At the end of the research, after the involvement of the teachers in the process of the implementation of the research and the filling of the questionnaires, the teachers who operated the quasi-experiment were asked to answer the questions prepared ahead of time regarding the quasi-experiment, which was, from their perspective, something that helped the researcher complete the full picture (Friedman, 2005; Shkedi, 2003). In terms of the research results and in relation to the research questionnaire, the questionnaire examined the degree of effectiveness of the use of the online collaborative mathematical discourse with the learners as a means of the identification and assessment of specific difficulties of learners with difficulties in mathematics and in parallel the effectiveness of the use of the source of information, as a means for the structuring of the dynamic intervention program for their advancement.

The interviews will focus on the identification of additional information that may strengthen the quantitative findings. The interviewees were asked to provide examples to clarify the logic behind their responses (Fontana & Frey, 1994).

The interview was carried out with six mathematics teachers who teach in the middle school. All the interviews were held in the school, every interview lasted about an hour on average. The interview included thirteen open-ended questions (see Appendix Number 1). The questions examined the following characteristics: the effectiveness of the use of the online collaborative mathematical discourse as formative feedback to students who have difficulties (questions 1-5 and questions 8-9), the teacher’s attitudes about the use of the online collaborative discourse and its impact on the structuring of the knowledge and the learning strategies (questions 6, 11), the influence of the self-efficacy (question 13), the influence on the learning experience (question 7), the influence on the students’ motivation (question 10), and the influence on the achievements (question 12).

Open Ended Interview with the Learner after the Mapping Test (Appendix Number 8)

The interview is carried out by the mathematics teachers, (Teachers as mentioned above received full training on how to conduct the interview with the students), the goal is, on the one hand, the more in-depth understanding and collection of information regarding the source of the learner's difficulty and on the other hand, the manner and form of thinking of the learner who has difficulties. According to Shkedi (2003), an in-depth interview is an interaction between the speaker and the listener, the goal of which is not only the collection of information but also the creation of meaning from the information together. The type of questions chosen are comprehensive descriptive questions and questions of completion, when, according to Shkedi (2003), the aim is to obtain a greater depth of information about the process of solving the questions, the manner of thinking, and direction. The interviewer does not direct the interviewee in any special manner, but the questions are more descriptive – 'what', 'how', and 'why'.

The interview was carried out at the start of the research process after the preliminary mapping test was held.

4.4.3.3 Mapping/ Summative Test in Mathematics for the Seventh-Grade Student

This is an achievement test, at the level of the end of the sixth grade (see Appendix Number 7), which has the purpose of the identification of students who have difficulties in mathematics. The test was built by the researcher according to criteria determined by the supervisor of mathematics coordination in the Ministry of Education and is tailored to the curriculum at the State of Israel (Ministry of Education, 2018c).

The mapping test was built and carried out at the start of the research study, and through it, students with learning difficulties in mathematics were identified.

The structure of the test and the guidelines in the field of knowledge of mathematics according to the guidelines of the Ministry of Education are presented in the following table[17].

Table Number 5: Structure of the Test and Guidelines-Mapping Test, (Ministry of Education, Israel, 2012).

Use of a calculator	**Not recommended**
Length of the test	90 minutes
Number of items	About 20 items
Exercises in whole numbers and fractions	8
Word situations	3
Completions (for example equation or completing the sign)	2
Comparison of numbers and order between numbers	2
Estimate	1
Algebraic expressions	1 (paragraph in question according to what they have learned)
Geometry	3

Questions in the collection:

- Account operations and order of account operations (including separation between integers/ fractions/ decimal fractions, including separation between exercises with only addition/ subtraction operations or only multiplication/ division operations).
- Understanding the meaning of the number and percentages.
- Geometric features of shapes and measurements of area and circumference.

Note: An estimate and verbal questions are part of each topic.

[17] The test was built from the Ministry of Education's recommended database, a collection of recommended questions. Ministry of Education (2012). Collection of Questions for Examination, September 19, 2012 – September 21, 2012. https://docs.google.com/viewer?a=v&pid=sites&srcid=dWxwYW5pdC5vcnQub3JnLmlsfG1hdGh8Z3g6MWI5MmRmYjBhZDBlYTM1Mg

Summative Test (MAFMAR[18] Test), Supervisor Coordinating Mathematics, after the Experiment (Appendix Number 9)

The test was built at the end of the research by the mathematics teachers, according to two criteria:

1. The guidance of the school instructor accredited by the Supervision of Mathematics in the Ministry of Education and according to the criteria of the test of the supervisor of mathematics coordination proposed by the Ministry of Education for seventh grade students.
2. The completion of the learning materials until the end of the year in the schools.

The goal of the summative test is to compare the achievements between the means of the students between the start of the year and after the end of the process at the end of the year. The test was conveyed to all the seventh-grade students and the students in the control group.

The structure of the test and the guidelines in the field of knowledge of mathematics according to the guidelines of the Ministry of Education are presented in the following table:

Table Number 6: Structure of the Test and Guidelines -Summative Test, (Ministry of Education, Israel, 2015).

Use of a calculator	**Not recommended**
Length of the test	90 minutes
Number of items	About 19 items
Questions in the numerical field	8
Questions in the algebraic field	8
Questions in the geometrical field	3

[18] The word is an acronym meaning Supervisor Coordinating the Subject of Mathematics.

4.4.3.4 Type of Collaborative Tasks Used in the Research

At the end of the 1980s, the world of education began to prefer the open-ended test over the closed-ended test, since it offers complex tasks that are built on the phrasing of answers, justification, adoption of a position, and drawing of conclusions (Ministry of Education, 1996).

The tasks that were given to the students were built based on algorithmic thinking (a series of steps). The tasks include the level of identification, analysis, and implementation of possible solutions to carry out generalization and implementation of the solution process for a wide range of problems. The tasks are based on experiential and multi-literacy learning and teaching that incorporates mathematical discourse and play. The aims of the tasks are:

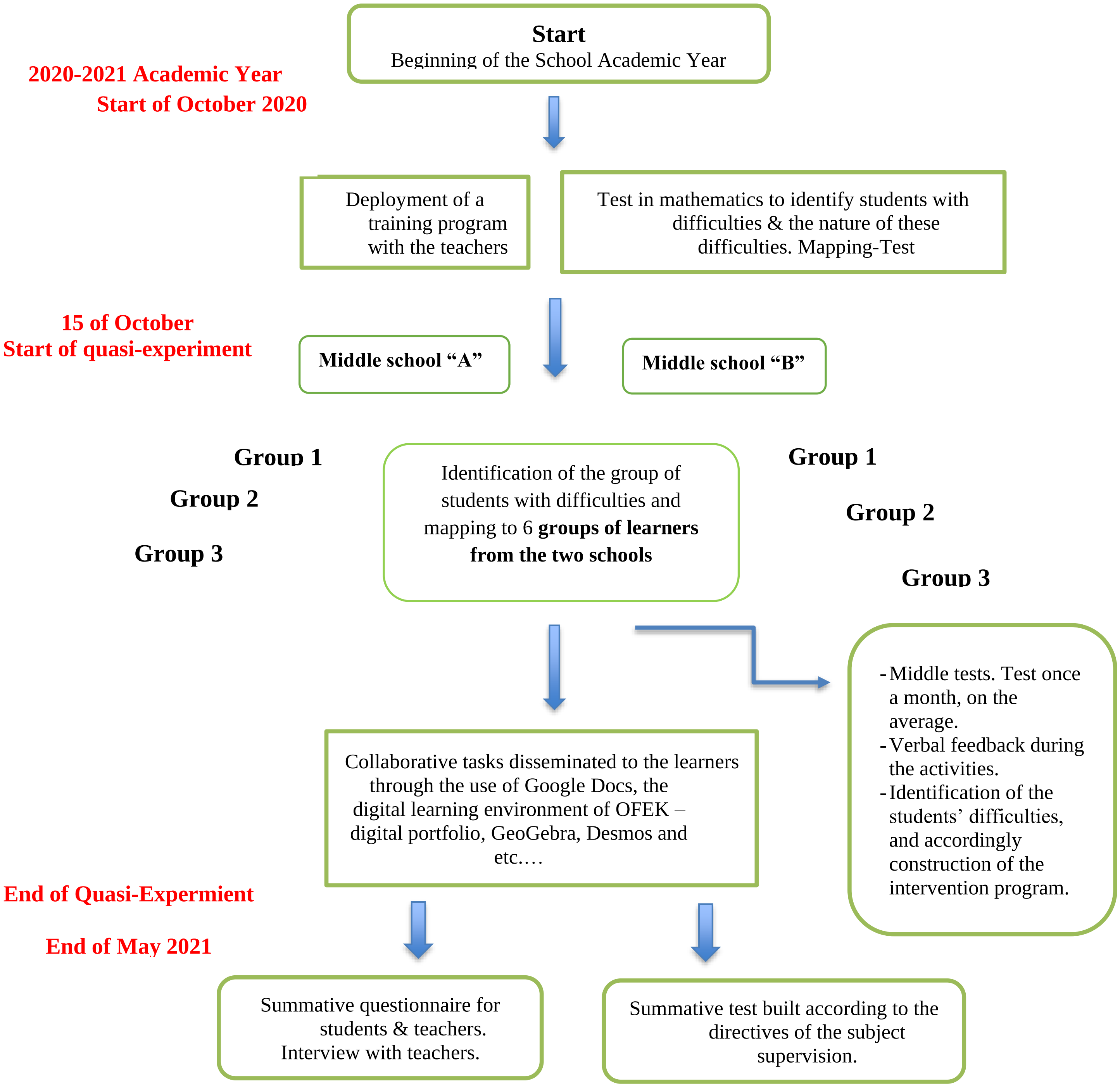
- To incorporate a number of learning topics that are needed by the learner who has difficulties in order to advance in the learning of mathematics in middle school.
- To implement and master basic mathematical skills in familiar and unfamiliar situations.
- To integrate different skills for the purpose of problem-solving.
- To base on collaborative work and mathematical discourse in order to solve the tasks.
- To foster a meta-cognitive discourse regarding the task learning process.
- To use technology in order to support the solving of mathematical problems in terms of the use of computational instruments in order to make it easier for the learner who has difficulties in terms of the cognitive load, simulations, uploading of the materials, and learning films for illustration.

The teacher's assessment of these tasks not only will be numerical but also will include formative feedback for the student about the mistakes, the type of mistakes, and the solution process (Ministry of Education, Culture, and Sport, 1995). (Formative assessment is a type of assessment that will be guided by the class teacher – an example of an authentic task. See Appendix Number 6.)

4.4.4 Time Schedule for the Performance of the Research Study

The following figure presents the time schedule for the performance of the research study.

Figure Number 12: Time Schedule of the Research Study (Source-own work).



4.4.5 Method of Analysis and Processing of the Data

Quantitative Findings

To determine the significance of the statements in the comparison tests according to the mathematics subject coordinator tests before and after the assimilation of the project among the students with difficulties in the researched school (experimental groups), the results of the mathematics subject coordination tests for the students with difficulties will be compared to those of the control group using the t-test.

Qualitative Findings

The answers to the open-ended questions (See Appendix Number 1) were coded as accepted (Sabar Ben Yehoshua, 2001) in a graduated process, ranging from open coding for the identification of the themes that describe the nature of the relationship of the online discourse and its impact on the students with difficulties in mathematics in particular and the learning environment in general.

In the second stage, the stage of axial coding, the researcher attempted to identify repetitions and their direction to topics and titles called “primary categories”. In the last stage, the “stage of the refinement” of the criteria or selective coding, the researcher attempted to increase the internal validity through the thickening of the data that belong to each category and by cross-checking with the quantitative analysis according to multiple frequency analysis (MFA) (Friedman, 2005, p. 255).

4.5 Selection of Research Sample

The sample included the quasi-experimental group and the control group, from a similar socioeconomic background (with a similar cultivation index – cultivation decile institution 8)[19]. The cultivation index of all the schools in Yafia Local Authority, whose residents number nearly 25,000 people, is similar to the mean of the researched schools (Ministry of Education, 2018d). In the quasi-experimental group, there are 30 students with difficulties in mathematics, while in the control group there are 28 students with math difficulties.

In addition, six teachers who teach the subject of mathematics and were trained to assimilate online collaborative discourse as a basis of the formative feedback in the researched school participated in the experimental group. In the control group, mathematics teachers in the seventh grade from two schools with similar socioeconomic characteristics were asked to distribute a comparative test at the end of the 2021 school year. It should be noted that the control group in the two schools did not have exposure to online collaborative discourse. The comparative test is based on the official curriculum of the Ministry of Education for the seventh grade ("The New Curriculum of the Seventh Grade in the Middle School of the Ministry of Education of Israel"), as it is expressed in the test of the supervisor of mathematics coordination (Ministry of Education, 2019). The use of the t-test attempted to confirm or disconfirm the hypothesis that the results of students with difficulties who were exposed to mathematical collaborative discourse would be higher than those of their parallel students in the schools in the control group.

[19]In Israel, local authorities and schools are divided based on care indicators by which the Ministry of Education determines the allocation of resources. The cultivation index consists of several components: The education of the most educated parent, the level of income per capita in the family, the periphery of the school, and a combination of immigration and a country of distress. The nurturing index is calculated for each student according to education stage and thus a "score" is created for each child. The care index scores create a continuum that is divided into ten equally frequent categories (deciles), from the lowest (decile 1 - strong population that does not need additional care) to the highest (decile 10 - weak population that needs additional care). The index value of a school is the average of the tenths of the students studying there. The value of the index in an authority, settlement or neighborhood is the average of the deciles of the students who live there.

Table Number 7: Data on the Two Schools and Groups (source: own work).

School	Number of Students in the School	Number of Students 7th Grade	Number of Students with Difficulties Chosen for the Quasi-Experiment	Number of Students with Difficulties in the Control Group	Number of Mathematics Teachers Who Participated in the Quasi-Experiment
School A	465	120	19	15	4
School B	441	108	11	13	2

4.6 Ethical Aspects

The quasi-experiment began, and the data was collected only after approval is obtained from the Chief Scientist in the Ministry of Education, the school principals, the students, and their parents (See Appendix Number 11: Consent Form for the Principal and Appendix Number 12: Consent Form for Parents). The objectives of the research study were explained to the teachers who participated in the carrying out of the research and the students who participated in the quasi-experiment. The teachers clarified to the students that their personal information and their responses to the questionnaires would remain confidential and would be used only for the purposes of this research study (Sabar Ben Yehoshua, 2001) and that they could end their participation in the quasi-experiment at any stage of the research.

Chapter 5: Research Findings

The research question asked about the influence of the use of online collaborative mathematical discourse as formative feedback that influences the advancement of the achievements of students with difficulties in mathematics in the seventh grade in the Arab sector. The model was mostly confirmed. This chapter presents the findings in relation to the model validity and the research hypotheses. The findings are presented in stages, according to the following divisions:

- Quantitative findings. Descriptive statistics of the entire sample regarding each one of the variables used in the research study. Means, standard deviations, and range of the variables were examined. The reliability and validity are also presented.
- Qualitative findings. Description of the qualitative findings with reference to the relationships found between the variables in the model.
- Summarization of the findings in relation to the research hypotheses and findings discovered beyond them.

5.1 The Quantitative Part: Data analysis of the impact of the online collaborative mathematical discourse on the achievement of students with difficulties

5.1.1 Findings and Analysis for the Students with learning difficulties in Math in Experimental and Control Group

5.1.1.1 Group Research

A total of fifty-eight Israel students participated in the research study (30 girls (51.7%), ages 12-13) when the range of the number of siblings in the family was 0-6 (mean = 3.39, standard deviation = 1.36). A total of thirty students participated in the experimental group (51.7%), while the rest were in the control group, and They didn't have the quasi-experimental method. The following table presents the distribution of the research participants according to the demographic variables.

Table Number 8: Distribution of the Demographic Variables among All the Research Participants (N=58), (source: own work).

	N	%	Min	Max	Mean	SD
Gender						
Male	28	48.3				
Female	30	51.7				
Number of Siblings in the Family			.00	6.00	3.39	1.36
0	1	1.7				
1	3	5.2				
2	9	15.5				
3	19	32.8				
4	12	20.7				
5	8	13.8				
6	4	6.9				
Group						
Quasi-Experimental	30	51.7				
Control	28	48.3				

5.1.1.2 Research Instruments for Students with Learning Difficulties in Mathematics in Experimental and Control Group

The questionnaire for students with learning disabilities in mathematics in the quasi-experimental and control group included many statements, all on a Likert scale of 1-5, which are divided into several topics. The first topic, self-efficacy, included 14 statements (1-14), which examine the respondent's degree of self-efficacy. The second topic, change in the learning strategy, included 13 statements (15-27), which examine the extent to which the respondent changed his learning strategy. These statements were affiliated to three sub-topics: memorization strategies (statements 15, 19, 24, 27), control strategies (16, 18, 22, 23, 25), and information processing strategies (17, 20, 21, 26). The third topic, intrinsic motivation and learning experience, included 22 statements (28-49), which examine the respondent's level of intrinsic motivation and learning experience. Statements 33, 38, 40, 41,45, 48 were written in language that characterizes a low degree of motivation and therefore the scale was reversed. These statements were affiliated to three sub-topics: sense of enjoyment and learning experience (statements 28, 32, 35, 37, 41, 42, 44, 47), sense of efficacy (statements 30, 31, 34, 38, 39, 43, 49), and sense of calm (statements 29, 33, 36, 40, 45, 46, 48).

For every respondent, the mean of responses to the statements of every topic and subtopic was calculated and thus the indices and sub-indices of the research were defined. The following table presents the general characteristics of the indices.

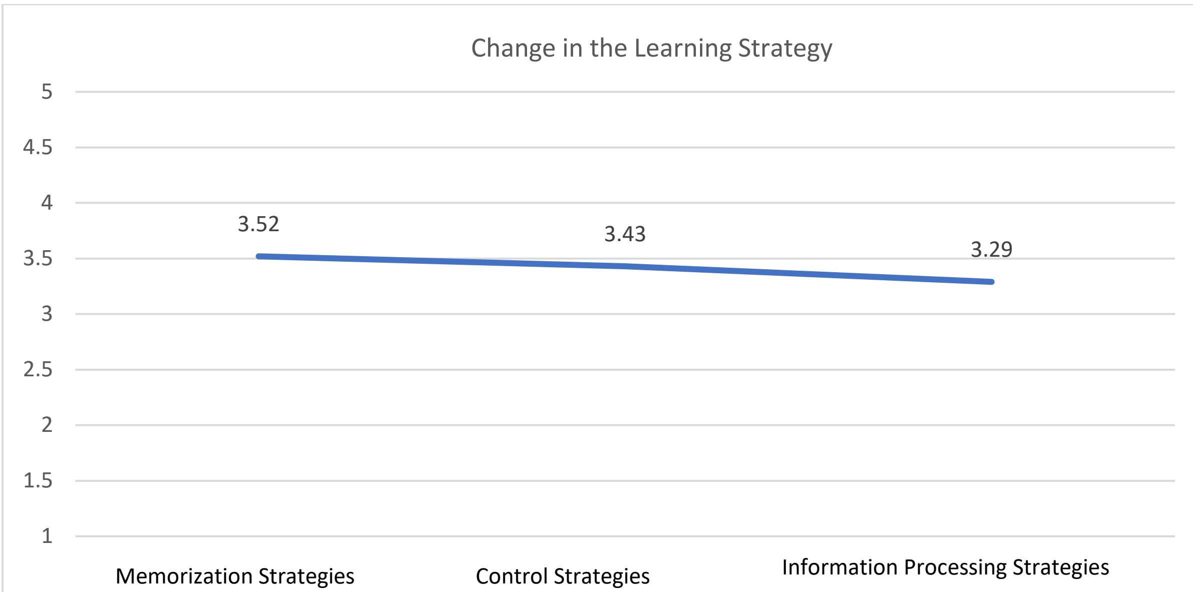
Table Number 9: General Characteristics, Mean, Standard Deviation, and Reliability of the Research Indices (N=58), (source: own work).

Index	Statement No.	Min	Max	Mean	SD	α
Self-efficacy	14	1.50	4.93	3.44	.98	.95
Memorization Strategies	4	2.00	5.00	3.52	.73	.59
Control Strategies	5	1.20	5.00	3.43	1.08	.87
Information Processing Strategies	4	1.00	5.00	3.29	1.01	.84
Change in Learning Strategy – General	13	1.38	4.92	3.42	.87	.91
Sense of Enjoyment & Learning Experience	8	1.75	5.00	3.29	1.09	.93
Sense of Efficacy	7	1.67	5.00	3.33	1.10	.92
Sense of Calm	7	2.00	4.50	3.29	.69	.53
Intrinsic Motivation & Learning Experience – General	22	1.90	4.82	3.30	.92	.95

The reliability of the indices, as examined according to Cronbach's alpha (α), was found to be high in most of the indices, a datum that characterizes a high degree of stability and consistency in the respondents' answers in the statements of every index. In the indices of memorization strategies and sense of calm, slightly low reliability was found, a datum that characterizes a slightly low degree of stability and consistency in the respondents' answers in the statements that belong to these indices.

To examine whether there are differences in the three indices of learning strategy (memorization, control, and information processing), repeated measurements tests were carried out.

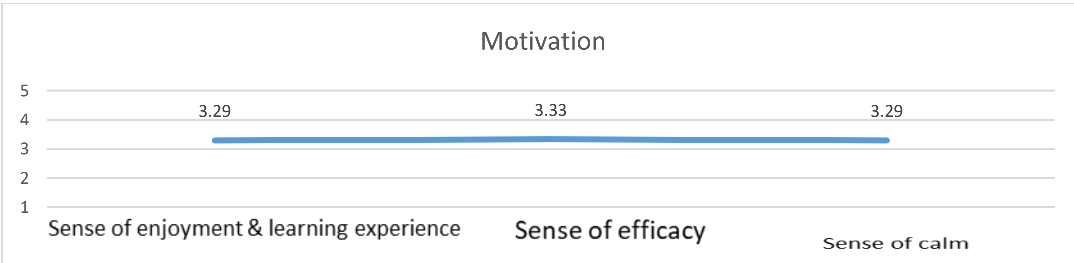
Figure Number 13: Change in the Learning Strategy, (source: own work).



In the tests it was found that there are no differences in the means of the three strategies: F(2,114)=2.81, $p>0.05$, in such a way that the mean of the respondents' answers in the index of control strategies (3.52 on a scale of 1-5), in the indices of memorization strategies (3.43 on a scale of 1-5) and the information processing strategy (3.29 on a scale of 1-5), are not significantly different Apart.

Furthermore, in order to examine whether there are differences in the three indices of motivation (sense of enjoyment and learning experience, efficacy, and calm), additional repeated measurement tests were carried out.

Figure Number 14: Indices of Motivation, (source: own work).



In the tests it was found that there are no differences between the means of the three indices of motivation, $F(2.114)=0.13$, $p>0.05$, in such a way that the mean of the respondents' answers in the index of sense of calm (3.29 on a scale of 1-5), the mean of the respondents' answers in the indices of sense of enjoyment and learning experience (3.29 on a scale of 1-5) and the mean of the respondents' answers in the sense of efficacy (3.33 on a scale of 1-5).

5.1.1.3 Quantitative Analysis of Self-Efficacy for Students in Experimental and Control Group

To examine the differences between the two research groups in the index of self-efficacy, t-test was performed for independent samples. The following table presents the means in the two groups and the test results.

Table Number 10: Self-Efficacy – Experiment vs. Control and t-Test Results (N=58), (source: own work).

	Experimental Group (N=30)		Control Group (N=28)		
Index	Mean	SD	Mean	SD	t
Self-Efficacy	4.26	0.42	2.57	0.55	13.19**

**p<0.01

It was found that the level of self-efficacy among the students in the experimental group is significantly higher than that among the students in the control group, t(56)=13.19, p<0.01. In other words, the students who participated in the research exhibited a higher level of self-efficacy than did the students who did not participate.

5.1.1.4 Quantitative Analysis of learning strategies for Students in Experimental and Control Group

To examine the differences between the two research groups in the indices of the change in learning strategy, additional t-tests for independent samples were performed. The following table presents the means in the two groups and the results of the tests.

Table Number 11: Indices of Change in the Learning Strategy – Experiment vs. Control and t-Test Results (N=58), (source: own work).

	Experimental Group (N=30)		Control Group (N=28)		
Index	Mean	SD	Mean	SD	t
Memorization Strategies	4.03	0.58	2.98	0.41	7.95**
Control Strategies	4.32	0.48	2.49	0.64	12.45**
Information Processing Strategies	3.97	0.79	2.57	0.68	7.22**
Change in Learning Strategy - General	4.15	0.41	2.64	0.45	13.23**

**p<0.01

It was found that the change in the general learning strategy among the students in the experimental group is significantly higher than it is among the students in the control group, t(56)=13.23, p<0.01. In addition, in the three indices of strategies (memorization, control, and information processing), the means of the students in the experimental group are significantly higher than the means of the students in the control group. In other words, the students who participated in the research study displayed a greater change in the learning strategies than did the students who did not participate.

5.1.1.5 Quantitative Analysis of Intrinsic Motivation and Learning Experience for Students in Experimental and Control Group

To examine the differences between the two research groups in the index of intrinsic motivation and learning experience, another t-test for independent samples was performed. The following table presents the means among the two groups and the results of the test.

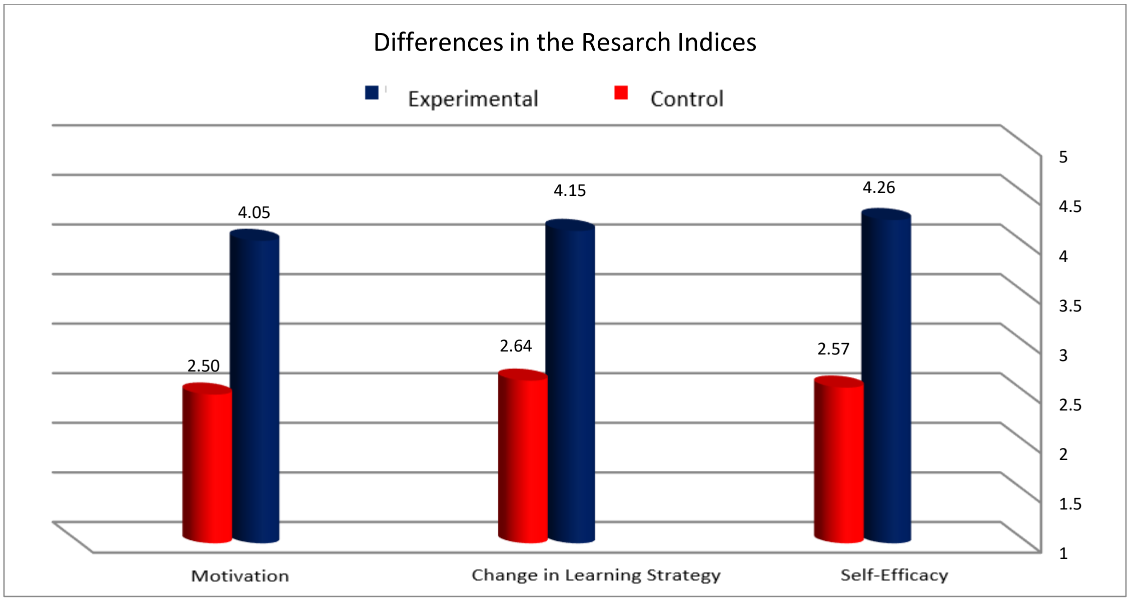
Table Number 12: Indices of Intrinsic Motivation and Learning Experience – Experiment vs. Control and t-Test Results (N=58), (source: own work).

	Experimental Group (N=30)		Control Group (N=28)		
Index	Mean	SD	Mean	SD	T
Sense of enjoyment & learning experience	4.15	0.68	2.36	0.55	10.95**
Sense of efficacy	4.24	0.49	2.35	0.61	13.11**
Sense of calm	3.75	0.55	2.81	0.46	7.01**
Intrinsic motivation & learning experience	4.05	0.51	2.50	0.46	12.02**

**p<0.01

It was found that the motivation and learning experience among the students in the quasi-experimental group are significantly higher than among the students in the control group: $t(56)=12.02$, $p<0.01$. In addition, in the three indices of motivation (sense of enjoyment and learning experience, sense of efficacy, and sense of calm) the means of the students in the experimental group are significantly higher than the means of the students in the control group (see Figure 15). In other words, students who participated in the research study exhibited intrinsic motivation and learning experience more than did students who did not participate.

Figure Number 15: Differences in the Research Indices, (source: own work).



5.1.1.6 Quantitative Analysis of the Mathematical Achievement for Students in Experimental and Control Group

To examine differences between the experimental group and the control group in the mathematical achievement, the types of scores were collected for comparison: the score of the MAFMAR test, and the score of the MAPPING test, and the scores in the numerical field, the score in the algebraic field, and the score in the geometric field of each test. Consequently, the scores collected at the beginning of the year and at the end of the year included a different number of questions. In each one of the four types of scores, the indices of the scores were defined as the ratio between the sum of the scores and the number of the questions.

To examine the differences between the two points of time (beginning of the year and end of the year) in each one of the four indices, t-tests for paired samples were performed. The tests were carried out separately for the thirty (30) students in the experimental group and the twenty-eight (28) students in the control group. The results are presented in the following table.

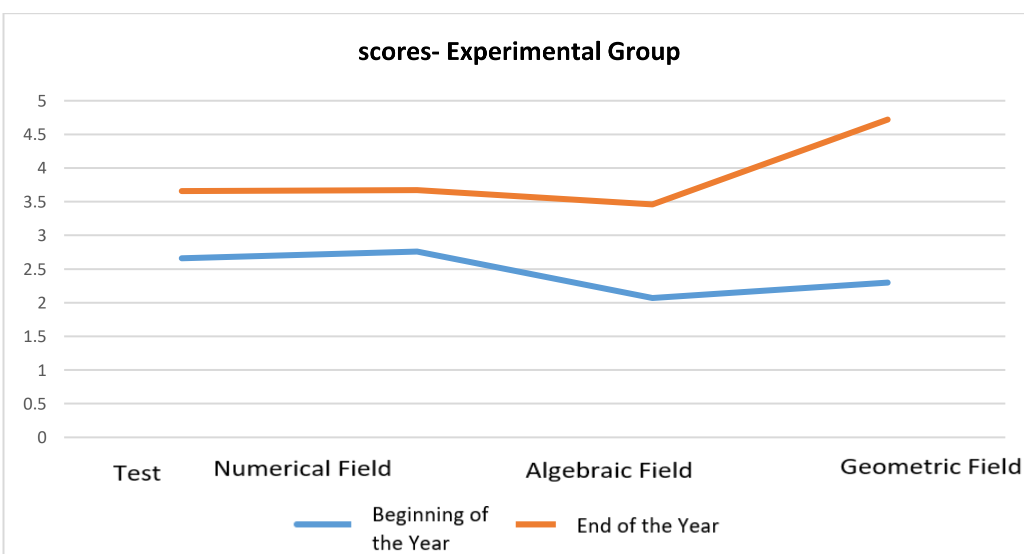
Table Number 13: Scores – Beginning of the Year and End of the Year in the Experimental Group and t-Test Results (N=30), (source: own work).

	MAPPING test beginning of the year		MAFMAR test end of the year		
Index	Mean	SD	Mean	SD	t
Test	2.66	.34	3.66	1.04	4.89**
Numerical Field	2.76	.31	3.67	1.13	4.33**
Algebraic Field	2.07	2.07	3.46	1.05	3.04**
Geometric Field	2.30	.98	4.72	1.78	6.71**

**p<0.01

It was found that the scores of the test were significantly higher at the MAFMAR test (3.66) compared to the MAPPING test (2.66): $t(28)=4.89$ $p<0.01$, the scores of the numerical domain were also significantly higher at the end of the year (3.67) compared to the beginning of the year (2.76): $t(28)=4.33$, $p<0.01$, additionally, the algebraic domain scores were significantly higher at the end of the year (3.46) compared to the beginning of the year (2.07): $t(28)=3.04$, $p<0.01$, and finally, the geometric domain scores were significantly higher at the end of the year (4.72) compared to the beginning of the year (2.30): $t(28)=6.71$, $p<0.01$.

Figure Number 16: Scores – Experimental Group, (source: own work).



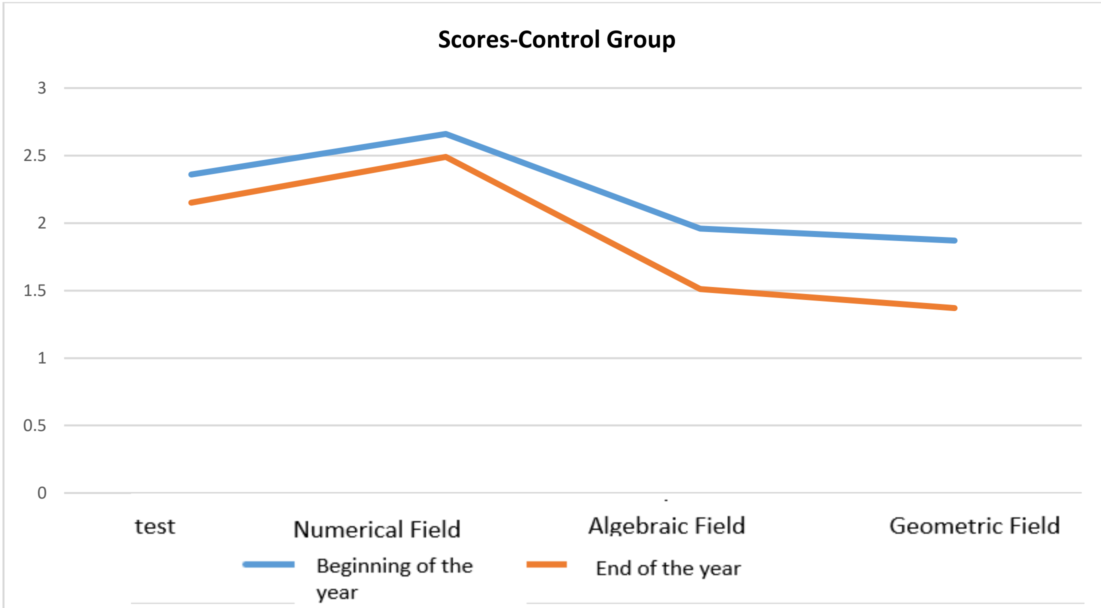
It was found that in each one of the indices the scores of the students of the experimental group were significantly higher at the end of the year than at the beginning of the year.

Table Number 14: Scores – Beginning of the Year and End of the Year in the Control Group and t-Test Results (N=28), (source: own work).

	MAPPING test Beginning of the Year		MAFMAR test End of the Year		
Index	Mean	SD	Mean	SD	t
Test	2.36	.33	2.15	.73	1.58
Numerical Field	2.66	.46	2.49	0.85	1.04
Algebraic Field	1.96	1.99	1.51	.91	1.22
Geometric Field	1.87	1.00	1.37	1.50	1.71

In contrast to the experimental group, in the control group no significant differences were found in any of the indices between the beginning of the year and the end of the year. In particular, no difference was found between the MAFMAR test scores (2.15) and the MAPPING test (2.36): $t(26)=1.58$, also, no difference was found in the numerical domain scores between the end of the year (2.49) and the beginning of the year (2.66): $t\ (26)=1.04$, , no difference was found in the algebra domain scores between the end of the year (1.51) and the beginning of the year (1.96): $t(26)=1.22$, and no difference was found in the geometric domain scores between the end of the year (1.37) for the beginning of the year (1.87): $t(26)=1,71$.

Figure Number 17: Scores –Control Group, (source: own work).



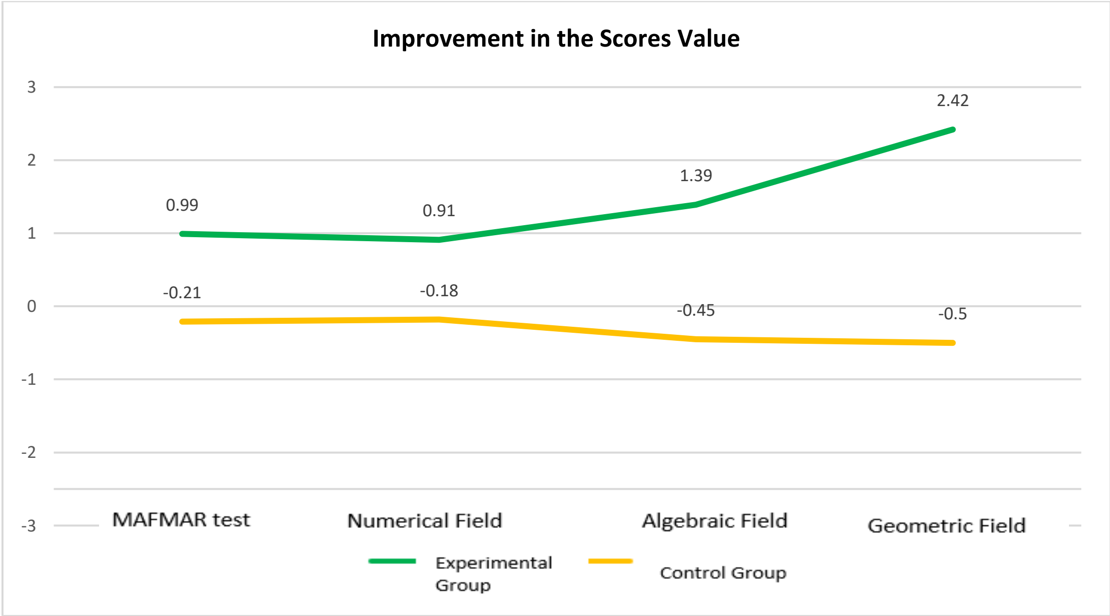
In addition, the differences between the two research groups were examined in the improvement carried out in every group, from the beginning of the year to the end of the year, in each one of the indices of the scores. The following table presents the means of the improvements in each group and the results of the t-test.

Table Number 15: Differences between the Research Group and Control Group in the Improvement of the Scores between the Beginning of the Year and the End of the Year and the t-Test Results (N=58), (source: own work).

	Experimental Group (N=30)		Control Group (N=28)		
Index	Mean	SD	Mean	SD	T
Improvement in the MAFMAR Test	0.99	1.09	-0.21	0.71	4.91**
Improvement in the Numerical Field	0.91	1.15	-0.18	0.89	4.03**
Improvement in the Algebraic Field	1.39	2.46	-0.45	1.96	3.13**
Improvement in the Geometric Field	2.42	1.98	-0.50	1.55	6.29**

$**p<0.01$

Figure Number 18: Improvement in the Scores Value (Source- own work)



As indicated, the experimental group displayed a greater improvement than did the control group. The improvement in the experimental group was significantly greater and more meaningful than in the control group in each one of the indices, in the numerical field, in the algebraic field and in the geometric field.

5.1.1.7 Correlations between Mapping Test and Summative Test Scores and Research Indices

To examine the correlations of the improvement between the Beginning and the End of the Year of the indices of the tests scores (Numerical, Algebraic, Geometric) and the indices of the research (the Learning Strategies, learning experience, Sense of efficacy, Sense of calm, Intrinsic motivation & learning experience), Pearson tests in the experimental group and in the control group were performed. The correlations are presented in the following tables.

Table Number 16.1: Pearson Correlations between the Score Improvement Indicators and the Research Indices in the Experimental Group (N=30), (source: own work).

	Score Improvement Indicators			
Index	Score Improvement Between Tests	Numerical	Algebraic	Geometric
Self-Efficacy	.418*	.216	.382*	.378*
Memorization Strategies	.257	.180	.382*	.174
Control Strategies	.354	.290	.315	.398*
Information Processing Strategies	.417*	.397*	.425*	.394*
Change in learning strategies – general	.444*	.383*	.495*	.403*
Sense of enjoyment & learning experience	.421*	.338	.456*	.394*
Sense of efficacy	.418*	.276	.683**	.428*
Sense of calm	.338	.389*	.360	.304
Intrinsic motivation & learning experience	.446*	.378*	.547**	.419*

$*p<0.05$ $**p<0.01$

It was found that in the experimental group, for the most part, there are significantly positive relationships between the research indicators and the score improvement indicators. That is, the improvement in scores between the end of the year and the beginning is associated with high levels of self-efficacy, a change in learning strategy and motivation. In particular, it was found that there is a moderately strong positive relationship between the general index of the change in the learning strategy and the improvement in the test scores between the end of the year and the beginning of the year (r=0.444, $p<0.05$), in such a way that the greater the change in the learning strategy, the greater the improvement in the score.

Table Number 16.2: Pearson Correlations between the Score Improvement Indicators and the Research Indices in the Control Group (N=28), (source: own work).

	Score Improvement Indicators			
Index	Score Improvement between Tests	Numerical	Algebraic	Geometric
Self-Efficacy	.286	-.030	-.297	.277
Memorization Strategies	.342	-.343	-.171	.039
Control Strategies	.327	-.023	-.210	.075
Information Processing Strategies	.206	-.012	-.125	-.003
Change in learning strategies – general	.338	-.176	-.148	.034
Sense of enjoyment & learning experience	.122	-.074	-.280	.336
Sense of efficacy	-.091	-.014	-.245	.232
Sense of calm	-.133	.132	-.140	.374
Intrinsic motivation & learning experience	-.027	.010	-.263	.363

It was found that, in the control group, the correlations were weak and not significant. In other words, the improvement between scores of the end and beginning of the year tests is not related to high levels of self-efficacy, change in learning strategy, and motivation.

5.1.1.8 Regressions for the Prediction of the Improvement of the Scores

To examine the influences of the background variables and the general index of the change in the learning strategy on the improvement in the scores, hierarchical linear regressions were carried out. These regressions were performed separately for the experimental group and the control group.

In the first regression, the dependent variable was the improvement in the score of the numerical field, when the predicting variables were gender and number of siblings – in the first step, and the general index of change in the learning strategy – in the second step.

It was found that in the first step, the gender and the number of siblings do not explain at all the improvement in the score in the numerical field. In the second step, it was found that in the experimental group the general index of change in the learning strategies explained significantly and positively the improvement, so that for every rise of one unit in the change in the learning strategy the improvement in the score rose by 1.525 units (see Table 17.1). In contrast to the experimental group, in the control group there was no significant effect of the change in learning strategy on the improvement in grades (see Table 17.1).

Table Number 17.1: Regression Correlations for the Prediction of the Improvement in the Score in the Numerical Field by Gender, Number of Siblings, and the General Index of Change in the Learning Strategies Experimental, vs. Control group, (source: own work).

	Experimental Group (N=28)					Control Group (N=28)				
Index	b	SE	β	t	p	b	SE	β	t	p
First Step										
Gender	.138	.412	.066	.334	.741	.196	.346	.112	.566	.576
Number of Siblings	-.122	.190	-.128	-.644	.525	.059	.111	.106	.537	.596
	$\Delta R^2 = 0.02, \Delta F = 0.26, p = 0.777$					$\Delta R^2 = 0.02, \Delta F = 0.29, p = 0.75$				
Second Step										
Gender	-.142	.367	-.068	-.387	.702	.283	.356	.162	.797	.433
Number of Siblings	-.096	.164	-.100	-.585	.564	.059	.111	.105	.530	.601
Change in Learning Strategy – General	1.525	.494	.545	3.086	.005	-.419	.402	-.212	-1.044	.307
	$\Delta R^2 = 0.28, \Delta F = 9.52, p = 0.005$					$\Delta R^2 = 0.04, \Delta F = 1.09, p = 0.307$				

In the second regression, the dependent variable was the improvement in the score in the algebraic field, when the predicting variables were gender and number of siblings – in the first step and the general index of change in the learning strategy – in the second step. It was found that in the first step the gender and number of siblings do not explain at all the improvement in the score of the algebraic field. In the second step, it was found that in the experimental group the general index of change in the learning strategy explained significantly and positively the improvement, so that for every increase in one unit in the change of learning strategy the improvement in the score rose by 3.210 units (see Table 17.2). In contrast to the experimental group, in the control group there was no significant effect of the change in learning strategy on the improvement in grades (see Table 17.2).

Table Number 17.2: Regression Correlations for the Prediction of the Improvement in the Score in the Algebraic Field by Gender, Number of Siblings, and the General Index of Change in the Learning Strategies Experimental, vs. Control group, (source: own work).

	Experimental Group (N=28)					Control Group (N=28)				
Index	b	SE	β	t	p	b	SE	β	t	p
First Step										
Gender	1.463	.933	.297	1.568	.129	-.591	.759	-.154	-.778	.444
Number of Siblings	-.309	.430	-.136	-.720	.478	.050	.243	.041	.206	.839
	$\Delta R^2 = 0.10, \Delta F = 1.45, p = 0.254$					$\Delta R^2 = 0.03, \Delta F = 0.33, p = 0.721$				
Second Step										
Gender	.874	.854	.177	1.023	.316	-.485	.792	-.126	-.612	.546
Number of Siblings	-.254	.381	-.112	-.666	.512	.049	.246	.040	.199	.844
Change in Learning Strategy – General	3.210	1.148	.485	2.796	.010	-.509	.894	-.117	-.570	.574
	$\Delta R^2 = 0.22, \Delta F = 7.82, p = 0.010$					$\Delta R^2 = 0.01, \Delta F = 0.32, p = 0.574$				

In the third regression, the dependent variable was the improvement in the score of the geometric field, when the predicting variables were gender and number of siblings – in the first step and the general index of change in the learning strategy – in the second step. It was found that in the first step the gender and the number of siblings does not explain at all the improvement in the score of the geometric field. In the second step it was found that in the experimental group the general index of change in the learning strategy explained significantly and positively the improvement, so that for every increase of one unit in the change in the learning strategy the improvement in the score rose by 2.388 units (see Table 17.3). In contrast to the experimental group, in the control group there was no significant effect of the change in learning strategy on the improvement in grades (see Table 17.3).

Table Number 17.3: Regression Correlations for the Prediction of the Improvement in the Score in the Geometric Field by Gender, Number of Siblings, and the General Index of Change in the Learning Strategies Experimental, vs. Control group, (source: own work).

	Experimental Group (N=28)					Control Group (N=28)				
Index	b	SE	β	t	p	b	SE	β	t	p
First Step										
Gender	.300	.718	.080	.418	.680	.183	.557	.060	.328	.745
Number of Siblings	-.491	.331	-.284	-1.484	.150	-.387	.178	-.398	-2.173	.039
	$\Delta R^2 = 0.09, \Delta F = 1.17, p = 0.328$					$\Delta R^2 = 0.16, \Delta F = 0.25, p = 0.107$				
Second Step										
Gender	-.139	.664	-.037	-.209	.837	.173	.585	.057	.296	.770
Number of Siblings	-.450	.297	-.260	-1.515	.143	-.387	.182	-.398	-2.129	.044
Change in Learning Strategy – General	2.388	.893	.473	2.674	.013	.046	.660	.013	.070	.945
	$\Delta R^2 = 0.21, \Delta F = 7.15, p = 0.013$					$\Delta R^2 = 0.00, \Delta F = 0.01, p = 0.945$				

5.1.2 The Quantitative Part: Analysis of the Data for the Teachers in Experimental and Control Group

5.1.2.1 Group Research

A total of twelve teachers, eleven women (91.7%), participated in the research study. Most have more than ten years of professional experience (66.6%), that is they are veteran teachers, all are academically educated, while most have a Master degree (75.0%). Six teachers participated in the experiment (50.0%) and six teachers belonged to the control group and did not participate in the experiment (50.0%). The following table presents the distribution of the respondents according to the demographic variables.

Table Number 18: Distribution of the Demographic Variables among All the Research Participants (N=12), (source: own work).

	N	%
Sex		
Male	1	8.3
Female	11	91.7
Experience		
1-5 years	2	16.7
6-10 years	2	16.7
11-20 years	4	33.3
More than 21 years	4	33.3
Education		
Bachelor degree	3	25.0
Master degree	5	41.7
Master of Education degree	4	33.3
Group		
Experimental	6	50.0
Control	6	50.0

5.1.2.2 Research Instruments for the Teachers in Experimental and Control Group

The questionnaire consisted of 26 statements, all on a Likert scale 1-5, divided into a number of topics. The first topic, the effectiveness of the process, included five statements (1-5), which examine the teacher's degree of agreement that online collaborative discourse is effective. The second topic, learning strategies, included six statements (6-11), which examine the extent to which the teacher believes that the learning strategy is effective. The third topic, motivation, included five statements (12-16), which examine the extent to which in the teacher's opinion online collaborative discourse gives the student motivation. The fourth topic, learning experience, included five statements (17-21), which examine the extent to which in the teacher's opinion online collaborative discourse gives a learning experience to the student with difficulties. The fifth topic, self-efficacy, included four statements (22-25), which examine the extent to which in the teacher's opinion the online collaborative discourse gives a sense of self-efficacy to the student. The sixth and last topic, achievement, included one statement (26), which examines the extent to which in the teacher's opinion the student's achievements in the subject of mathematics will increase if there is perseverance in the type of learning. For every respondent, the mean of his responses to the statements of every topic was calculated and the research indices were defined. The following table presents general characteristics of the indices.

Table Number 19: General Characteristics, Mean, Standard Deviation, and Reliability of the Research Indices (N=12), (source: own work).

Index	Number of Statements	Min	Max	Mean	SD	α
Effectiveness of the process	5	2.60	4.40	3.67	.66	.76
Learning strategy	6	2.50	4.67	3.68	.72	.77
Motivation	5	2.00	4.60	3.73	.73	.85
Learning experience	5	1.60	4.20	2.93	1.10	.94
Self-efficacy	4	2.75	4.50	3.77	.64	.76
Achievement	1	3.00	5.00	3.75	.62	--

The reliability of the indices, as examined according to Cronbach's alpha (α), were found to be high in all the indices, a datum that characterizes a high degree of stability and consistency in the respondents' answers in the statements of every index.

5.1.2.3 Quantitative Analysis of the Research Indices: Self-Efficacy, learning strategies, Motivation, Learning Experience, and achievement between Teachers in Experimental and Control Group

To examine the differences between the two research groups in the research indices, t-tests for independent samples were performed. The following table presents the means in the two groups and the results of the tests.

Table Number 20: Differences in the Research Indices – Experiment versus Control and t-Test Results (N=12), (source: own work).

	Experimental Group (N=6)		Control Group (N=6)		
Index	Mean	SD	Mean	SD	T
Effectiveness of the process	4.23	0.15	3.10	0.41	6.29**
Learning strategies	4.31	0.22	3.06	0.38	7.09**
Motivation	4.23	0.27	3.23	0.71	3.24**
Learning experience	3.97	0.20	1.90	0.21	7.61**
Self-efficacy	4.21	0.29	3.33	0.61	3.19**
Achievement	4.17	0.41	3.33	0.52	3.10**

**p<0.01

It was found that the level of process efficiency, especially among the teachers of the experimental group is significantly higher than among the teachers of the critique group: t(10)=6.29, p<0.01, the level of learning strategies among the teachers of the experimental group was also found significantly higher than among the teachers of the critique group: t(10)=7.09, p<0.01, the level of motivation among the teachers of the experimental group is found also to be significantly higher than among the teachers of the critique group: t(10)=3.24, p<0.01, finally, the level of educational experience among the teachers of the experimental group was accordingly significantly higher than among the teachers of the critique group: t(10)=17.61, p<0.01, the level of self-efficacy among the teachers of the experimental group is significantly higher than among the teachers of the critique group: t(10)=3.19, p<0.01, and the level of achievement among the teachers of the experimental group is significantly higher than among the teachers of the control group: $t(10)=3.10$, $p<0.01$.

In conclusion, it was found that in each one of the indices, the means of the teachers' experimental group are significantly higher than the means of the control group. In other words, teachers who participated in the experiment maintained more than did teachers who did not participate in the experiment that the online collaborative discourse is effective and instills motivation, experience, efficacy, and achievements for the student.

5.2 The Qualitative Part: Data Analysis of the online collaborative mathematical discourse effectiveness, and achievements

5.2.1 The Qualitative Part: Findings for the Teachers in Experimental and Control Group

The open-ended interview with teachers consisted of thirteen questions. The questions provided information about the changes that in the teachers' opinion occurred in the students with difficulties in mathematics after the deployment of the intervention program that lasted one academic year. The influence of the program on the effectiveness of the teacher's work with students who have difficulties was examined.

- The first group of questions (questions 1-5 and 8-9) examines the following characteristics: the effectiveness of the use of the online collaborative mathematical discourse as formative feedback with students who have difficulties.
  - Question 1: Did the discourse on collaborative learning help you as a teacher? In what way, please explain.

From the analysis of the teachers` answers, it was noted that the intervention program helped them in several respects:

- Knowing the students from closer, not only academically.
- Diversification in the examples that the teacher gave in order to tailor the learning to every learner differently.

- Accommodation of the learning to the learner's preferred style.
- One of the teachers reported that with her the self-awareness of the effectiveness of her manner of teaching increased.
- The teacher had more tools for dealing with every student differently.

- Question 2: How is the online collaborative mathematical discourse held with the students with difficulties different from the regular mathematical discourse that is held in the frontal class regarding the teacher and the student? Please describe.

From the analysis of the teachers` answers, it was noted that the teachers:

- Are more focused on the source of the difficulty.
- Are using visual communication frequently.
- Illustrated dynamically.
- Enables more space for learners in the expression of suggestions of solutions.
- The teacher learns about the students regarding the form of thinking through the solutions they provide.
- Saving exercises or forms of solution of different exercises in order to use them when necessary.
- The reduction of the 'grunt work' load on the teacher and the student, diagrams.
- Requires of the learner considerable attention.
- Requires of the teacher many preparations.

- Question 3: In your opinion as a teacher, can the use of collaborative mathematical discourse advance your work as a teacher of students with difficulties? Please explain.

The analysis of the teachers` answers, the teachers who participated in the intervention program, had a uniform answer that the mathematical discourse greatly helped them in their work with students who have difficulties. In the description of the answers, they noted:

- This is net teaching and learning of models in an illustrated manner through the drawing of a map between the concepts.
- There is illustrated repetition of the concepts.
- It is possible to examine among the student's automation in the giving of solutions immediately.
- I can as the teacher examine the correct naming of mathematical laws and concepts.
- They had the option of following up after the manner of thinking of each learner separately.
- Transforming the lesson from passive to dynamic.
- Removal of the 'grunt work' from me as a teacher and of course from the learner.
- One of the teachers emphasized that with the advantage there is also the difficulty of maintaining the role as an instructor who is not involved and who does not intervene.

- Question 4: Describe a case that you as a teacher were surprised to discover from the online mathematical discourse about a student with difficulties.

The analysis of the teachers` answers referred to:

- Ways of thinking through the different learning channels and knowing the different learning styles of the learners: visual, verbal, or auditory.
- The extent of the student's ability to perform manipulations between the various mathematical models, using mathematical concepts, using mathematical symbols, and constructing simple mathematical exercises.
- The differences in the student's ability to think and express their ways of thinking. The teachers identified students with difficulties who could do everything wonderfully in calculating orally, including the entire solution process. Still, in writing the process of solution, these students had great difficulty.

- Question 5: In online collaborative learning in mathematics, write what is the topic, how did you present it, and how did this influence the participation of the students in general and the students with difficulties in particular?

The analysis of the teachers` answers indicated that:

  - They presented learning in the online collaborative discourse through the use of films, games, mathematical digital learning environments (Desmos, GeoGebra), and use of dynamic regions.
  - Elements in the presentation of the topic: visual, dynamic, concrete, gradualness in presentation, possibilities of solution.

- Question 8: Did the perseverance in the collaborative mathematical discourse help you as a teacher to notice and characterize the students' difficulties in a different way from how you were accustomed in the characterization tests?
  - The analysis of the teachers` answers, indicated that they generally classify the learners into categories of good students, average students, and students with difficulties. Of course, the classification is done according to the grades, and they work with all the students with difficulties as one group. This time the classification is done according to the type of difficulty, and during the year every student his difficulties were identified in every learned topic, difficulties in previous material that are prerequisites of the learning of the new material, or difficulty in the understanding of the process of the solution, difficulty in the perception of the topic, and of course the teachers documented during the year the weak points of every learner and the focused the work on it.

- Question 9: What were your difficulties as a teacher in the implementation of the online collaborative discourse with the students?

The analysis of the teachers` answers, indicated that:

  - Preparation of mathematical activities that enable collaborative work because it is a new type of form of learning.

- The teachers lack expertise in the types of digital environments and the use of the different types of digital environments.
- Monitoring each student and preparing appropriate material requires mediation strategies that math teachers are not exposed to.

- The second group of questions (questions 6 and 11) examines the teacher's attitudes about the use of the online collaborative discourse and its impact on the structuring of the knowledge and the learning strategies.

- Question 6: To what extent, in your opinion as a teacher, did the intensive learning of mathematical content in an online collaborative manner increase the understanding of students with difficulties in mathematics and lead to the building of knowledge?

The analysis of the teachers` answers, indicated that:

- The online mathematical discourse raised the students' level of attention.
- The students worked on the conceptualization of ways of solution, there is a more correct use of concepts and laws. Teacher X said: "The moment that the students understand the teachers' statements and the students then it is possible to begin to teach the same material and work on what is missing, mathematics is a language, it is impossible to learn mathematics if you do not know basic concepts, laws, rules, and then it is possible to demand more of the student." Teacher Y said, "The understanding derives from logical thinking that is important to mathematics and in my opinion, this develops into a collaborative and concrete discourse that the technology can provide to us in different ways."

- Question 11: Did the use of the online collaborative mathematical discourse improve the learning strategies of the students with difficulties? Please describe with examples.
  - the analysis of the teachers` answers, hold that, in the mathematical discourse there was an improvement in the thinking regarding possible solutions to the exercises. The students became more effective in the

identification of the mistake when solving an exercise, they remember more concepts and mathematical laws. There is still no automation in the thinking and in the solution, they take more time to retrieve and examine the data. The students linked the exercise and the mathematical law, and there was an improvement in the thinking regarding possible solutions for exercises.

- The third group of questions (questions 7, 10, and 13) examines the influence on the learning experience, the students' motivation, and self-efficacy.
  - Question 7: To what extent did the participation of the students with difficulties increase in the lessons following the online collaborative discourse?
    - Based on the analysis of the teachers` answers, since this is an experiment that occurred under laboratory conditions the teachers were careful about the attendance in every lesson, they were in contact with the parents and the students in the beginning, they constantly polled the students regarding satisfaction with this learning method. The teachers upgraded the way of learning and resolved difficulties and tensions within the group.
    - There was also active participation in the lessons. The students participated actively in solving exercises or providing explanations and because the group was small the teachers say, we would ask from all the learners to participate and to make certain that every student will give an answer, even if this is a mistake, and we encouraged the analysis of the mistake as a part of the learning process.
  - Question 10: Did online collaborative discourse increase the students' motivation? How? Give examples.
    - According to the analysis of the teachers` answers, the small group gave the students a protected space, the student with difficulties copes when among students with difficulties like him and with a low number of students. But for the most part the students at first were afraid that the others would mock them and their manner of thinking about the solution formats they gave. The teachers reported that the intrinsic motivation rose

after the teachers told the students that they are found in a unique program in which the investment will cause an increase in the understanding of the subject. The students enjoyed learning mathematics without many exercises and learning with more emphasis on understanding, for them solving exercises was burdensome. The learning in the small group and the support the students received from one another and from the teacher helped boost motivation.

- Question 11: In your opinion, did the effective and intensive use of online collaborative learning improve the level of self-efficacy of the learners with difficulties?
    - The analysis of the teachers` answers holds that online collaborative mathematical discourse helped the students socially. They knew that there were other students who had difficulties, and they were not alone in dealing with this difficulty. The tasks were first built so that every student, even one with difficulties, can with a little help solve them. This gave the students the feeling that they were good at mathematics and that this was not unsolvable.

      The students came to the lessons without paper and pencil, in the lesson there wasn't the solving of many exercises but there was all the time practice and repetition of the concepts. There were for the most part easy questions that they knew to solve, and this boosted their self-image and the thought that they can challenge themselves more by asking questions that take them out of their comfort zone such as inquiry questions or complex questions.

- The fourth group of questions (question 12) examines the influence on the achievements. To what extent did perseverance in this type of discourse increase the achievements of the students with difficulties?

According to the analysis of teachers` answers:

- The teachers believe that perseverance in mathematical discourse will increase the learning achievements among students with difficulties. The teachers see that the student who has experienced intensive learning for a year with an emphasis on his weak points in a tangible manner developed for himself the ability to understand the new material. The use of online learning environments in a collaborative manner led to a deeper understanding of concepts and mathematical facts. The students think together, together advance understanding, and together learn from their mistakes. The student learned this year that the process of the solution goes through stages of understanding the question of data processing and then examination of the laws required for solution. They also reported that the follow-up after the school scores of the students with difficulties during the experiment showed a certain rise in the scores half a year after the beginning of the program.

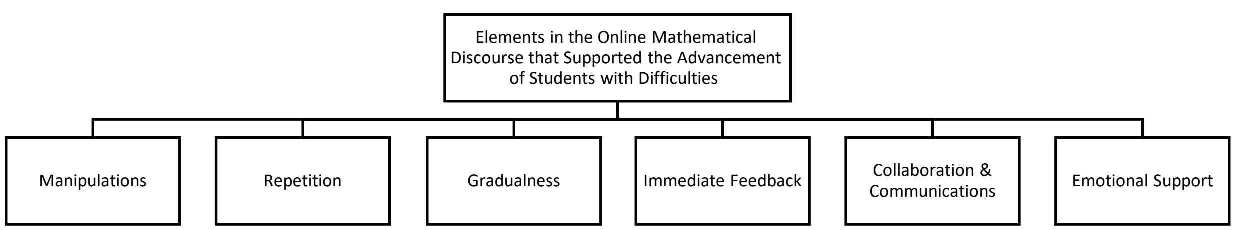
5.2.2 The Qualitative Part: Analysis and Discussion of the Qualitative Findings

The analysis of the teachers' responses indicates that most of the teachers maintain that online collaborative discourse promoted students with difficulties in cognitive, social-emotional, and academic terms.

The analysis of the teachers' responses addressed the following three dimensions: the elements of online mathematical discourse that supported the promotion of students with difficulties, the changes that occurred in students with difficulties following the intervention program, and the influence of the online collaborative mathematical discourse on the teacher's work.

1. Elements of Online Mathematical Discourse that Supported the Promotion of Students with Difficulties

Figure Number 19: Elements in the Online Mathematical Discourse that Supported the Advancement of Students with Difficulties as emerged from the teachers' responses, (Source - own work).



Manipulations. The teachers addressed the trait of dynamism in the digital environments and the demonstrations using illustrations, clips, and stories as an important attribute, because of the dynamism and the immediate and easy transition from one mathematical representation to another. In other words, the students with difficulties enjoyed learning while using digital environments and see it to be an effective tool that advances them in terms of their learning (Bouck, Park, Levy, Cwiakala, & Whorley, 2020; Ok & Bryant, 2016). There is attractiveness in the use of the digital learning environments both for the teacher and for the learner, the colors, the shapes, the dynamism, and the almost immediate transition between the screens. When a student does not understand a certain law or problem, the teacher can immediately transform the question into a graphic or algebraic (Oxman & Stupel, 2018). The teachers emphasized that this helped them examine the learning preferences of every learner and to train the students to be flexible in their manner of thinking.

Repetition. The teachers say that this is not memorization but the ability to repeat in every lesson the different concepts. According to the teachers, the repetition every time of facts, concepts, and laws in another way and in another context leads to the assimilation of the learning content, mathematical models, facts, and concepts. The repetition is carried out in two forms: lingual – orally and written.

Gradualism. The flexibility in asking questions and the large repertoire that mathematical digital software can offer helps the teachers address every group differently and every student in the group differentially. For example, teacher Y said: "*One of the students, called Rami, told me: I do not understand every time that I learn mathematics, I get exercises like the rest of the class, the teacher gives five questions and says that the more questions we solve the easier it will be on the test. But I always solve the five questions and, in every question, the same mistake, so how does this help me? This shows that it is very important to let the students solve easy exercises in the beginning until the establishment of the process of solving and then to advance to the more difficult questions."*

Immediate feedback. The students solve together, and everything appears online with the teacher, this promotes the teacher's immediate follow-up (Bouck & Flanagan, 2009) of the process of the solution and not only the giving of a correct / incorrect answer. The teacher immediately can learn about the thinking and stage of understanding of the material of the learner with difficulties. As a result, the teacher provides feedback for every learner according to his process of solution and helps the student establish correct logical thinking.

Collaboration and communication. The teachers were satisfied with their reference to the cooperative work between the students with difficulties in the learning groups. The teachers mentioned three main factors that influenced the advancement of the collaboration: (1) the homogeneity of the groups in terms of the learning abilities, (2) the use of the digital environments that enable documentation of the work of every student separately and all the students together, or in other words, shared construction through the documentation of the processes, and (3) the brainstorming that develops between the learners and the manner of explanation when solving problems. According to Hwang and Wang (2015), collaboration, communication, and complex problem solving are important skills, the control of which is necessary since they challenge teachers and students and increase the effectiveness of the learning activities in the digital environments.

Emotional support. The analysis of the teachers' statements indicates that the personal relationship that developed between the teacher and the students and among the students themselves encourages the ability to listen among the students, coping with the surfacing of the learning material, and organization towards the solving of the problem.

Teacher X said: *"It was interesting to see how the students stimulate one another, help one another, if they got stuck on a certain concept and if a student did not participate then they would ask him what he thinks? Very nice group cohesion was formed."*

The teachers also supported the students who solved incorrectly, the teachers at first determined that every mistake is a source of joint learning in order to act together so as to identify and correct and is not a source of embarrassment or a source of fear of the material. In parallel, there was strong support from the students and friends. Teacher A noted: "*A type of trust among the friends is created, beyond the learning group there was good communication within the school, they felt strong together."*

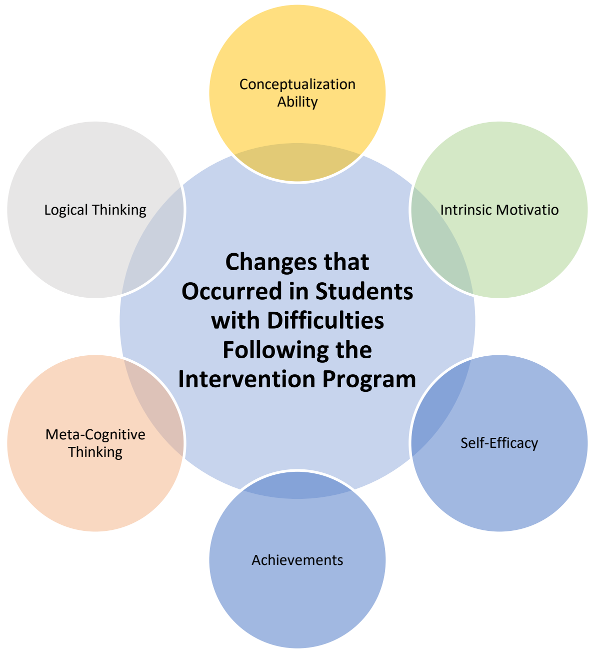
2. Changes that Occurred in Students with Difficulties Following the Intervention Program

In the analysis of the interview with the teachers, emphasis was placed on two main directions of change that occurred in the students with difficulties: the learning direction and the emotional direction.

The learning direction. The development of cognitive abilities, like meta-cognitive abilities, which advanced the ability to process data in a controlled manner and the development of logical thinking when solving a mathematical task that helped them remember more details. The teachers reported changes in the level, content, and frequency of use of concepts and mathematical facts. The use of mathematical language during the conversation advances deductive thinking. The language is an important instrument through which the learner explains, reasons, and justifies his arguments. Students with difficulties who do not have the necessary vocabulary find it difficult to phrase a response and explain it and therefore it is important to assimilate in students correct use of language in order to communicate mathematically correctly and automatically (Powell, Stevens, & Hughes, 2018). In the present research study, the follow up after the students with difficulties during the year in the classes indicated a change for the better in the scores.

Emotional direction. The program positively influenced the intrinsic motivation to learn and self-efficacy. Teacher S said: "*The students in the class began to speak and understand the mathematical language, and therefore they feel themselves to be a part of the class learning process.*"

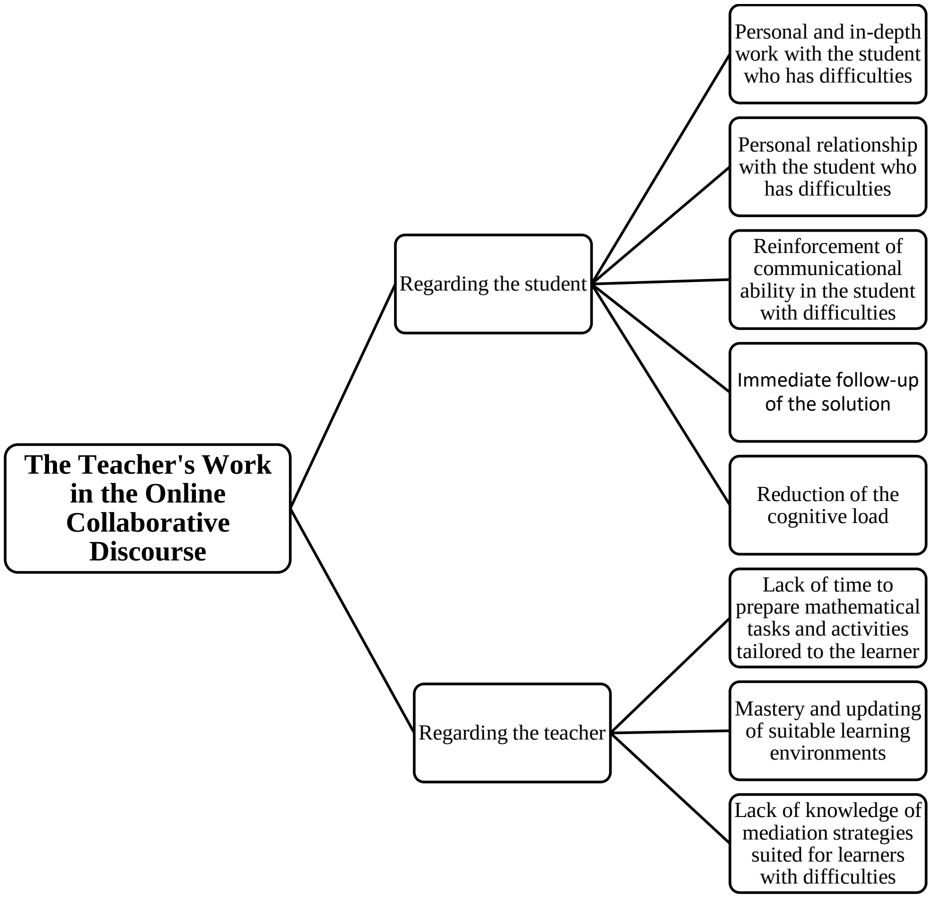
Figure Number 20: Changes that Occurred in Students with Difficulties Following the Intervention Program (Source - own work).



3. Influence of the Online Collaborative Mathematical Discourse on the Teacher's Work

The teachers that participated in the quasi-experiment noted two main influences, one aimed at the students and the other aimed at the teachers.

Figure Number 21: The Teacher's Work in the Online Collaborative Discourse (Source-own work).



The teachers who participated in the experiments declared several changes that took place and reshaped the holistic view regarding the subject of mathematics. The teachers claimed that the change that resulted from the quasi-experiment occurred in both the learners and the teachers. The teachers noted that in relation to the learner they could identify and understand in-depth the learners' difficulties and academic strengths.

Although there was always an attempt to help the teachers understand the students, to sense their strengths and weaknesses, the experiments were always limited to revolving about understanding whether the students understood the material or not. Furthermore, it was difficult to diagnose and analyze the problem in order to figure out a solution. Pursuantly, participating in the experiments gave the teachers more concrete tools to learn about the weaknesses, the student's preferred form of study, and the level of difficulty for the students, which is another element that was important in the assimilation of the material. It helped students and the teachers develop emotionally and socially. The teachers could identify difficulties, emotional, and social strengths. The learner became gradually more open, curious and asked questions, and the teachers in return succeeded each throughout using their personal methods to challenge the students.

According to the teachers, the use of digital learning environment helped in two manners. The first being, assimilating learning material in a more in-depth way, learning in a focused and tangible way; the second being, giving teachers richer and more diverse tools in order to impart educational content to students who find it difficult according to their academic preferences. Each teacher introduced learning and teaching in several new forms subjectively such as through using apps, videos, dynamic drawings, and illustrations, which strengthened the gradual learning of the material specifically. The digital learning environments enabled the teachers to present the areas of content in a richer and more interesting manner.

The teachers recognized the importance of online mathematical discourse, emphasizing that it also reshaped their teaching and improved it. Nevertheless, the teachers noted that this work required them to put in considerable effort and preparation. Pursuant to the conclusion above, the teachers realized they lack time, and found it difficult to prepare mathematical tasks and activities that required a considerable amount of time, and preparation of appropriate content suited for each student's ability accordingly. In addition, teachers had difficulty choosing the appropriate applications, since they looked for apps that had more experiential challenging learning environments, and for questions that are suitable to mediate collaboration, and of course these in return required hours of work or even in some cases was outside of their expertise or their knowledge. The teachers indicate that they lack tools and training (Martin & Madigan, 2006). This was the main reason for the difficulty in the development of the mathematical discourse work with students who have difficulties. As well, this is often the reason why many teachers withdraw from developing a mathematical discourse among students in general and among students with difficulties in particular.

5.3 Summary of the Data Analysis in Reference to Each Hypothesis

The analysis of the questionnaires regarding the students and teachers at the end of the intervention program of the online collaborative mathematical discourse for the students with difficulties, the analysis of the mapping tests and the summative test, and the analysis of the interviews that were conducted with the teachers who participated in the above experiments, provided an expanded and comprehensive answer to the main research questions and hypotheses.

Subsequently, the first question queries: how does the online collaborative mathematical discourse influence the building of effective learning strategies and the promotion of the academic achievements of students with difficulties in mathematics in the seventh grade?

The answer is provided through two-way response, demonstrated in two different trends of thinking. The first trend referred to the extent of the contribution of the online collaborative mathematical discourse to the promotion of learning strategies among students with difficulties in mathematics, whereas it emerged from the research that in the comparison between the experimental group and the control group, it is apparent that students with difficulties who participated in the research on online collaborative discourse improved significantly in all of the learning strategies aspects: memorization, control, and information processing. Highlighting that the main improvement was in the learner's ability to carry out control over his response, or in other words, his logical examination ability.

The results above illustrate that the critical thinking and brainstorming that took place in the groups, in addition to the ability to implement or apply moves in a concrete way, has developed new ways of thinking for the students. The different directions of thinking that have developed include the following: (a) the manner of absorbing the material in terms of remembering the mathematical laws and the relevant facts, and additionally the form of reading the mathematical problems; (b) processing the material, meaning translating the words into symbols and a mathematical exercise. (c) control strategies, where we found a noticeable improvement not only the mathematical solution skill, but also in the student sense of mathematical logic, which in turn indicates an improvement in relevant thinking process with an understanding of the question and the answer that was received. Overall, understanding a mathematical question requires a very strong process and logical thinking, subsequently, students with difficulties who carry out dialogue and brainstorming initiatives, come up with more ideas. Although sometimes the ideas are not relevant, they develop new ways of thinking and create new channels of thinking. Similarly, the accompaniment of friends and teachers through the dialogues and the brainstorming activities helps them mediate and differentiate between main points and minor issues in real time. Immediate correction enables the learner to assimilate correct ways of thinking and to advance onwards in the formation of correct relationships between the concepts and the correct analysis of the data.

Thus, to summarize the above results, the first hypothesis, which stated that students with difficulties who learn using online collaborative mathematical discourse develop higher order thinking abilities than do students who learn in the traditional method, *was confirmed*.

As stated above, sense of mathematics is not obvious, students who have difficulty do not have a mathematical sense, they do not have the ability of toggling between the laws and the facts, to choose the appropriate one to solve the question, and of course they cannot preserve these laws. In the study, the students were given the ability to relieve themselves of this cognitive burden by constant use of the digital environments, concrete demonstrations and intensive collaborative work environment with the students and the teacher. The immediate support and feedback, and the adaptation of the study materials to the student's level and abilities, has improved the conceptualization ability of the struggling student, and as a result there was an improvement in the ability to recall facts. Indeed, students with difficulties improved their ability to memorize data, laws, and mathematical facts. The online verbal discourse gave the students with difficulties time to internalize mathematical concepts and facts and to assimilate data. The use of digital environments improved demonstrations with which the students could build mathematical models and understand relationships between concepts, topics, and learning materials.

In conclusion, the second hypothesis, which stated that students with difficulties who learn using collaborative discourse based on the use of technology are quicker to remember, process, and retrieve facts than are students who learn using the traditional method, was also *confirmed*.

The improvements in the ability to process data and the ability to remember in the students who have difficulty in mathematics was done by digital illustrations, which work visually and auditorily. This improvement was important and significant for the students who had difficulty because they did not learn uniformly, but each one used his own strong channel. The digital environments provided the students with a building correct and meaningful mathematical model, in which they could see the manipulations between the verbal problem and the ability to represent it in a video or a graph or an appropriate mathematical exercise. Nevertheless, the dramatic effect was in their way of thinking and their ability to manipulate the different data, and in transformation in the learning of mathematics that promotes logical tangible thinking and emphasizes higher order thinking. Hence, it can be concluded that students with difficulties who participated in the research study have developed good logical thinking ability, compared to the rest of the strategies.

Their ability of logical thinking have advanced for a number of reasons: (a) they worked in small groups, in which the students held an in-depth mathematical discourse about different processes of solution, identified problems, discussed the solution of other students, and engaged in types of tasks that require estimation ability; (b) the students learned with the teacher to analyze the mathematical problems, to build a process of solution, and to advance mathematical thinking on data, facts, and relevant concepts. On the whole, we found that the third hypothesis, which stated that students with difficulties who learn using collaborative discourse based on the use of technology have greater control in the development of the stages of solution process, was additionally *confirmed*.

The second trend referred to the effect of the online collaborative mathematical discourse regarding the achievements of students who are struggling academically. The research revealed that students who participated in online collaborative mathematical discourse improved their scores significantly in comparison to students with difficulties who continued to learn in the traditional manner. The use of discourse assimilated higher order learning and thinking skills among the learners. Moreover, they learned to link between the data, to read data correctly, and to connect between the question and the relevant concepts. The students additionally developed logical and analytical thinking ability for the process of the solution, through the internalization of the concepts. In parallel, the cognitive burden of the calculations was removed from the students, and they learned to use correctly digital environments for calculations. The use of technology and digital environments in the field of education advanced the learning of mathematics among students with difficulties (Ok & Bryant, 2016).

The definition of educational achievements is not broad and aimed at learning specific knowledge and certain skills. Ultimately, the definition of educational achievements in mathematics is broad and includes the combination of three main components: first, remembering facts and laws and correct retrieval as required in the exercise; Second, correct reading of the mathematical problem and internalization of the mathematical concepts; Third, the ability to do mathematical manipulations and switch between different mathematical representations in a correct and focused way.

In the research, heavy and intensive work was done with the students on the quality of the work and not the quantity of questions, in which each teacher took a basic question and gave the students a task to solve, locate, identify the type of problem, their mistakes in the solution, and accordingly developing the exercise using the digital mathematical environments. The teachers built the questions using different representations, linking the question to the mathematical facts and laws needed for the solution, while using a rich discourse in mathematical concepts. Overall, we can conclude that there is a contribution of using online collaborative environments to the improvement of achievements of the learners with difficulties in mathematics, and the hypothesis was *confirmed*.

In the current research, the teacher was a key partner in the delivery of the learning material, part of the material was the result of the students' conclusions drawn from the data. The teachers developed in the students a more critical, analytical, and logical thinking while internalizing the demand for the students to interact and not just giving them an explanation on the board. The student's use of the auditory and oral channel, demonstrated in the communication between the learners, the mathematical discourse, and the collaborative brainstorming, gave support in terms of academic thinking and emotional ability to the struggling learner. As a result, the improvement was seen both in the achievements and in the correct organization towards solving a mathematical question, process organization and organization on the test page and in the form of writing the solution. Altogether, the hypothesis which stated that students with difficulties who learned mathematics using collaborative digital learning environments succeeded more than did students who learned in the traditional method, was *confirmed*.

As for the second question queries, how does the online collaborative mathematical discourse influence the process of identification and assessment of the learning abilities (weaknesses/strengths) of the students with difficulties and the promotion of emotional and social learning?

The answer to this question will, similarly to the first, be provided through two directions, the first relating to the teacher's feeling and perception of the online collaborative mathematical discourse as an effective means for the identification and assessment of the learning abilities (weaknesses/strengths) of the students with difficulties in mathematics. The answer relying in the first layer was provided by the teachers who all experienced the two types of teaching: first, the traditional teaching of acquiring the learning material in the classroom from the books and through a classroom exercise solution; second, the research method of acquiring the material in a collaborative online manner with extended daily use of the digital environments.

The analysis of the data for the secondary research question indicated conclusions regarding the influence of the online collaborative discourse on the effectiveness of the teacher’s work with students who have difficulties. According to the teachers, the collaborative mathematical discourse provided them with the tools to identify the skills and the pedagogical content that will help their students to get achievements. The teachers reported that performing a constant evaluation for each student in each lesson and in each exercise helped in locating the learners' difficulties and strengths. But they claimed that they lacked several things: the characterization of the content that was being developed required a relatively long time from them, the skills that must be sharpened, which in turn were defined from the evaluation process, were different and not uniform among the students. In order for such action from the teacher to have an effect on the student's achievements, it requires a lot of time and knowledge in the field, and sometimes there is no professional guidance for the teachers, that need sophisticated assessment skills so that they can identify what their students know and work in relation to the results expected of them.

Theoretically, the teachers saw optimal conditions in the research, but the field was different in their opinion, practically they needed a variety of ways in which they could evaluate the progress of their students. This variety consists, among other things, of tests, which they must prepare, and include in it interviews and discussions with students about their learning, and a systematic analysis of the student's work. Although the use of digital environments has raised awareness as to the causes of the difficulties, this was not a sufficient condition, it would always require the intervention of the teachers to build a mathematical discourse that includes concepts, facts and content in order to be relevant to the site of the difficulty. Hence, it can be concluded that the hypothesis about the effectiveness of the teacher’s ability to identify students with difficulties was *partially confirmed.*

The hypothesis about the ability of the teacher who integrates in the teaching the use of collaborative digital environments to broaden her knowledge about the reason for the difficulty of the learner with difficulties was *partially confirmed*.

Later on, the teachers reported that the immediate feedback made possible by the dynamic and interactive digital environments provided an answer to the development of the internal motivation of the struggling students. They added that using collaborative mathematical discourse in these environments promoted the learner's ability to solve mathematical problems. The collaborative work was accompanied by brainstorming and constant discussion among the struggling students, in which they used concepts, facts and laws when they gave their answers in a way that suited their way of thinking and by using the manipulation that was convenient for them, and of course the teacher supported the correction and promoted the process. Following the above, we conclude additionally that the hypothesis which stated that a teacher who incorporates in the teaching the use of collaborative digital environments can follow up dynamically after the development of the learner's abilities in solving mathematical problems, was *confirmed.*

The second direction that provided answers to the research, to what extent does online collaborative mathematical discourse help the development of the emotional-social learning ability among students who have difficulties in mathematics?

This layer referred to the effect of the online collaborative mathematical discourse on the students who have difficulty in mathematics from a social emotional point of view. From the data analysis it is possible to conclude that the hypothesis which stated that math discourse that develops in online collaborative learning influences the motivation of the student with difficulties to solve math problems and participate in the tasks, was *confirmed*.

The hypothesis which stated that students with difficulties who learn with an online environment tend to have more positive learning experience than students who learn in the traditional method, was *confirmed*.

The hypothesis which stated that learners with difficulties who learn by online collaborative learning have better self-efficacy to cope with the accumulating math information, was *confirmed*.

The hypothesis which stated that there is a positive relationship between the use of collaborative social discourse based on digital environments and the promotion in the emotional aspect, or in other words, a positive learning experience and motivation to perform the task on the part of the learner with difficulties, was *confirmed*.

The analysis of the four hypotheses of the research question indicates that online collaborative mathematical discourse influenced broadly in social emotional terms. The discourse that develops in the online collaborative environment creates interaction between the learners and a feeling of social presence (Abedin, 2012; Resta & Shonfeld, 2013). The quantitative analysis indicated that the students who participated in the online collaborative mathematical discourse exhibited a high level of motivation, self-efficacy, and learning enjoyment (Bouck & Flanagan, 2009; Parkhurst, Skinner, Yaw, Poncy, Adcock, & Luna, 2010). The online discourse leveraged the students to advance in the learning of the subject of mathematics, apparently the work on the difficulties of each student differently in terms of the content and in terms of the manner of mediation of this content that the learner finds it difficult to understand helped the students be more active mathematically and advanced their understanding. The discourse in the collaborative environment and in the small group gave the teacher more time to focus on each learner. The students and the teachers built both academic and personal connections; the teacher with the students constituted a support network for one another. This encourages the students emotionally and influences their intrinsic motivation. The motivation was expressed in their participation in the solving of problems and in good communication with other students.

Chapter 6: Conclusion

6.1 Discussion

Mathematics as a language requires the students to use it in every stage of thinking: in the input of data, in the processing of data, and in the output. Correct use of language, which includes storage and effective retrieval of the information, leads to cognitive accuracy (Philosof, 2007). Mathematical conceptualization and acquisition of mathematical facts is perceived as one of the abilities that predict success in mathematics. The skill of the use of language enables communication among people, expression of ideas, and development of the ability to think at a higher level (Peng, Lin, Ünal, Lee, Namkung, Chow, & Sales, 2020). The teaching of mathematics has always aspired to develop an independent learner, who has the potential to build his knowledge, build mathematical models independently and autonomously, to enable him to integrate the knowledge when solving a high order of mathematical problems (Asor, Kaplan & Kent-Maimon, 2001).The aspiration is to give the learner the ability to think about thinking, critical meta-cognitive thinking, and to perform correct integration between the mathematical contents, despite the difficulty in the inculcation of this strategy, but this is important to students with difficulties in order to advance them scholastically (Gidalevich & Kramarski, 2019; Mevarech & Kramarski, 2014).

This research study investigated the influence of the online collaborative mathematical discourse on the mathematical achievements of students with difficulties, in two middle schools during the 2020-2021 school year. In the research we attempted to examine the influence of the types of discourse that develop in the digital collaborative learning environments on the learning strategies and the influence of the elements of the online collaborative discourse on the teaching effectiveness of teachers of students with difficulties. The mathematical discourse is a part of the requirements of the mathematical curriculum. On the basis of the discourse, a collaborative mathematical discussion develops around a problem through which the learners build knowledge and develop ways of thinking, when the teacher guides the learners throughout the discourse and directs them to bring up different ideas and to explain their solutions (Rawding & Wills, 2012).

Teaching that advances collaborative discourse can broaden the students' thinking and advance the learning and their understanding (Alexander, 2017).

In the era of the use of digital environments, we sought to examine the impact of the implementation of the online collaborative mathematical discourse and its influence on the way in which the learner with difficulties expresses his knowledge, draws his map of thinking, and builds knowledge. The incorporation of the digital environments improves the manner of teaching of mathematics and the students' understanding and leads to in-depth learning. Researchers propose different ways of representation of a mathematical problem (Wassie & Zergaw, 2018). The use of computerized technology enables the learner with difficulties to attempt the examination of the correctness of the solution in visual terms and in the continuation, this makes it easier for him to provide suitable explanations (Oxman & Stupel, 2018; Wassie & Zergaw, 2018). The learning based on digital environments is personal and customized to the level of understanding of the material of the student who has difficulties (Slavin & Lake, 2008), taking into consideration every learner's personal learning style (Armstrong & Gutica, 2020; Olisama et al., 2018). This challenges students, facilitates the development of mathematical abilities, and increases the students' learning motivation and satisfaction (Harasim, 2017; Palloff & Pratt, 2005; King & Zucker, 2009).

The promotion of the use of technology and digital environments led to an innovative form of pedagogy in which the teacher is no longer the dominant figure with knowledge but rather her/his role is to guide and lead the students to build their own knowledge, on the basis of the new curriculum, the integration of collaborative digital environments is proposed, as well as the building of a discourse that promotes learning. The question that arose in the research is whether the integration of the three elements in the framework of the teaching of students with difficulties in mathematics in middle school has impact. If so, then what are they taught? How are the three elements integrated? How do the students build their knowledge? Is there influence of the integration of these elements on the learning strategies and learning experience, in parallel to the examination of the work effectiveness of the teachers who teach these students?

So that we can go in-depth in the analysis of the data, we chose to hold the research in the quasi-experimental approach (the research was held in two middle schools that are similar in terms of the learners' background and the cultivation index, the research students and the control group were chosen from the two schools). For the purpose of the collection of the data in the two schools a number of parallel and identical processes were performed – in each one of the schools seventh grade students were examined as well as the teachers who teach the students who participated in the research. On the research level, this holistic process enabled a look at the topic of the teaching of mathematics to students with difficulties through online mathematical discourse from two perspectives: from the perspective of the mathematics teachers and from the perspective of the students who have difficulties and in three levels – social, emotional, and academic.

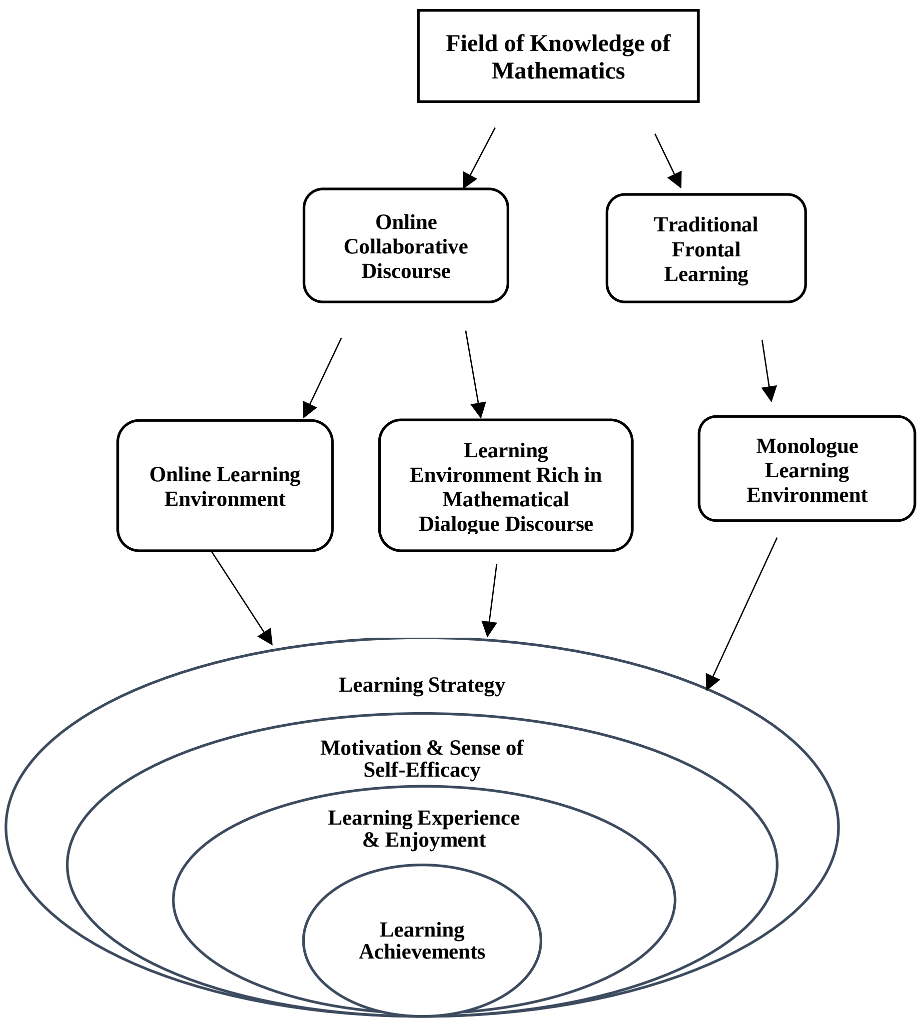
The focus on two schools in the Arab sector alone is in order to focus on the learners with similar cultural and characteristics. This provides a broad perspective of the students who have difficulties and allows us to compare between the teachers who cope with students who have the same background characteristics. The open-ended interview with the teachers and the documentation of the types of difficulty among the students participating in the research created qualitative information, the analysis of which helped to propose a model that illustrates the system of relationships between the variables of the quantitative research and to indicate the relationship between the use of online mathematical discourse in the teaching of students with difficulties in mathematics and their ability to improve the academic achievements.

The present research study emphasized the collaborative mathematical discourse that occurs with the mediation of technology that has the potential to increase the involvement of the students with difficulties in their learning. The teachers re-defined the traits of the mathematical discourse of the mathematical learning activities mediated by technology, the main components in the online collaborative discourse and its influences on the participation and involvement of the students who have difficulties, and the components in the online mathematical discourse that influence the process of learning and the achievements of the students with difficulties. The research study indicated the importance of collaborative mathematical discourse in relation to the learner who has academic, social, and emotional difficulties. Although the mathematical discourse praised the teacher's work and supported the promotion of the teacher's ability in professional terms with the students who have difficulties, the teachers reported difficulty in the implementation of the mathematical discourse with the students who have difficulties. One of the difficulties is that it is necessary to hold the mathematical discourse in the learning classes and as a part of the curriculum but in parallel there is a lack of guidelines, and the guidelines are decentralized and not in uniform language. There is insufficient reference on the part of the Ministry of Education regarding how it is possible to hold the mathematical discourse online, appropriate types of activities, and determining the level of collaboration. To overcome the difficulty, all the teachers who participated in the research study also participated in a special training program of four sessions.

All the teachers who participated in the research study recognize the contribution of the use of digital environments for the advancement of the teaching and learning. They indicated many advantages both for the learner and for the teachers. The teachers emphasized the importance of the use of mathematical discourse and digital environments as a learning means that promotes the teaching/learning of students with difficulties. They maintain that the incorporate of online discourse provides enrichment and support in all respects for the students who have difficulties and that every teacher can do this. However, a descriptive analysis of the teachers interviewed about the teaching methods they adopt in actuality indicated that the mathematics lessons do not have a "culture of mathematical discourse" and what is done in the classes or on the screens for the most part is unplanned, a monologue that arises as necessary after the student's question or the development of lack of understanding of the material on the part of one student or a group of students. The mathematical discourse despite its importance as a part of the mathematical curriculum is not included in a structured manner in the learning classes and the lessons. The use of digital environments was another means for the explanation of the learning material and not as a collaborative means through which it is possible to build a process of learning discourse that includes the building of knowledge, change of learning strategies, and development of high order thinking.

This research study attempted to explain the issue of what the elements in the online collaborative discourse are that influenced the ability of students with difficulties in academic, social, and emotional terms and the contribution of this structured discourse to the effectiveness of the teacher's work. The analysis of the findings led to the creation of a model that describes the desired learning process for the development of abilities that contribute to the changes in the learning strategies among students with difficulties in mathematics and lead to the increase in the achievements.

Figure Number 22: Model of the Research Learning Process Based on Mathematical Discourse (Source-own work)



The model shows that the advancement of the process of online collaborative mathematical discourse process leads to a change in the learning strategies among students with difficulties and influences them in social and emotional terms. Regarding the teachers, although there is complexity in the process of assessment and identification of the learners' difficulties, this contributed to the advancement of the effectiveness of their work and the focus of their work with the students who have difficulties. The further contribution of the model is the creation of understanding in the teachers that there is differentiation between the students not only on the background of the scores but also on the background of the type and nature of the problem. Here the teacher's work is differentiated; the teacher moves from the building of a work plan for a group of students with difficulties with the same range of scores to the writing of a personal work plan for students on the basis of the type of problem and the specific difficulty, whether this is in the stage of the data processing, analysis, or ability of conceptualization. The research findings indicate that teachers generally build personalized teaching plans for students with difficulties according to the range of scores and not according to the skills in which the student is not skilled or has not yet acquired. While the two categories are called students with difficulties, every category has different mediation and treatment. It is necessary to dedicate attention to the assessment of the learners' type of difficulties and whether the difficulty is environmental or developmental, to clarify what are the points of weakness and strength in this field, and to evaluate the suitable mediation in order to develop the learning abilities.

The findings indicate that it is desired to address the online collaborative mathematical discourse as an important means in the teaching of students with difficulties in mathematics. So that the teachers will succeed in being partners in the development of the process of the teaching of mathematics based on online mathematical discourse, they need to be exposed to the types of discourse, the components of the discourse, and the types of the learners' difficulties and the mediation strategies that increase the effectiveness of the learners' understanding and lead to the advancement of the effectiveness in their work. Most of the teachers today in the classes hold a monologue mathematical discourse, when the teachers speak, explain, build the mathematical model, and offer the learners to ask questions. A small number of teachers hold a mathematical discourse with a small circle of students in the class, generally the “good students”, but this is still a discourse and not a mathematical discourse. The manner of teaching offered today may indicate the weakness of teaching in general and for students with difficulties in the subject of mathematics in particular. The teachers need orderly and consistent training in the entire topic of mathematical discourse in a collaborative online format, its components, the elements that compose the discourse in general, and of course the construction of tasks suited for the development of an online discourse. To develop an online mathematical discourse, it is necessary to have suitable mathematical activities that integrate a number of mathematical topics in the discourse, advance gradual learning, and advance collaboration and critical thinking (thinking on thinking) among the learners, and regarding the teachers enable processes of evaluation and feedback that are built according to the curriculum. In addition, it is necessary to incorporate special teaching hours in order to promote mathematical discourse that does not depend on paper and pencil and the solving of questions in a repetitive fashion but advances comprehension and gradual construction of mathematical models. Therefore, this also requires a proper technological infrastructure and an appropriate number of means and learning environments in order to enable the students with difficulties to practice the learning online within the school walls.

6.2 Research Uniqueness and Contribution to the Field of Mathematical Education

From the research conclusions and model of the intervention program deployed for the cultivation of the abilities of the learner who has difficulties during the solving of problems in mathematics, it is possible to derive the educational contribution for the field. The intervention program proposed a range of authentic problems that are rich in the collaborative mathematical discourse that integrates learning material in a range of topics. The mathematical discourse was opened on the basis on digital learning environments that enable immediate feedback and formative feedback in a collaborative manner. For the mathematics teachers, an operative program is proposed for their work with students in practice, especially among students with difficulties, with low achievements in the middle school, when the educational principles that build it are taken from the research studies and models for the learning of mathematics.

This innovative research illuminates the importance of the collaborative discourse in technology rich environments on the building of knowledge among students who have difficulties in the field of mathematical knowledge in comparison to the regular class discourse. The research examined the nature of the mathematical discourse that advanced the effectiveness of the teachers' work, thus leading to more effective learning. The research study emphasized the importance and contribution of the creation of online collaborative discourse to the promotion of the motivation, the self-efficacy, the change in the learning strategies, and consequently the increase in the academic achievements among the students with difficulties.

6.3 Research Limitations

The primary research limitations are associated with the sample size and representation.

- There is a limitation in terms of the research validity. The present research study includes a relatively low number of participants (teachers and students) in a homogeneous learning environment.
- The research we will be discussing is a quasi-experiment in which students who have difficulties in mathematics participated. The process of selecting the participating students with difficulties was strictly conducted, using criteria that were selected beforehand, and additionally by performing an educational mapping test and learning skills. Generally, the number of students who have difficulties is between 10-20% of all school students, so it was difficult to locate students with difficulties in mathematics It required intensive work by the teachers, and a lot of time.
- There is a limitation in terms of the ability to generalize the research findings, in light of the low number of participants. Although the differences in the achievements of the students with difficulties in mathematics between the

experimental group and the control group are statistically significant, it is difficult to offer research generalizations beyond the researched population.

6.4 Recommendations for Further Research

The nature of the research study is experimental, and the intervention program is pioneering and limited to the seventh grades in the two specific schools in Arab urban communities in the Northern District of Israel. Therefore, it is proposed to carry out a similar research study among larger groups of students. A similar research study should be held among students from other age groups, both in middle school and in the high school.

From the findings of the present research study, it is possible to advance future directions of research that were derived from the limitations of the present research study. Recommendations for directions of future research include:

- A research study that copes with the limitations of the validity of the present research study, and therefore the recommendation is to hold continuation research that will examine in-depth the nature of the online mathematical discourse. The research study will be carried out with the participation of a larger number of mathematics teachers and a larger number of students with difficulties through the development of the research field to the different grades in middle school.
- The assimilation of the online collaborative discourse as formative feedback is a long process that offers accommodations and pedagogical and didactic changes. Therefore, the statistical conclusions after a year are partial, but in any event they are an important part of the process of the construction of a long-term intervention project. Therefore, in these research studies it is necessary to examine after a certain period of time the degree of retention for the long-term of the skills learned among the students and the extent to which they are able to implement the abilities that were learned during the experiment and during the solving of problems in mathematics.
- The research will examine at length whether the learning strategies inculcated in the collaborative discourse has impact on the general learning strategies of the learner with difficulties, or in other words, the efficacy of the student with

difficulties to implement the skills learned in the subject of mathematics in learning other subjects.

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Appendices

Appendix Number 1: Interview with the Teachers at the End of the Experiment

Please answer the open-ended questions and explain clearly your arguments.

1. Did the discourse on collaborative learning help you as a teacher? In what way, please explain.

   ____
   ____
   ____
   ____
   ____

2. How is online collaborative discourse that is held with students with difficulties different from regular mathematical discourse that is held in the frontal class regarding the teacher and the student? Please describe.

   ____
   ____
   ____
   ____
   ____

3. In your opinion as a teacher, can the use of collaborative mathematical discourse advance your work as a teacher of students with difficulties? Please explain.

   ____
   ____
   ____
   ____
   ____

4. Describe a case that you as a teacher were surprised to discover from the online mathematical discourse about a student who has difficulties.

   ____
   ____
   ____
   ____
   ____

5. In online collaborative learning in mathematics, write what is the topic, how did you present it, and how did this influence the participation of the students in general and the students with difficulties in particular?

____
____
____
____
____

6. To what extent, in your opinion as a teacher, did the intensive learning of mathematical content in an online collaborative manner increase the understanding of students with difficulties in mathematics and lead to the building of knowledge?

____
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____

7. To what extent did the participation of the students with difficulties increase in the lessons following the online collaborative discourse?

____
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____

8. Did the perseverance in the collaborative mathematical discourse help you as a teacher to notice and characterize the students' difficulties in a different way from how you were accustomed in the characterization tests?

____
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____
____

9. What were your difficulties as a teacher in the implementation of the online collaborative discourse with the students?

____
____
____
____
____

10. Did the online collaborative discourse increase the students' motivation? How? Give examples.

11. Did the use of the online collaborative mathematical discourse improve the learning strategies of the students with difficulties? Please describe with examples.

12. To what extent did perseverance in this type of discourse increase the achievements of the students with difficulties?

13. In your opinion, did the effective and intensive use of online collaborative learning improve the level of self-efficacy of the learners with difficulties?

Thank you very much.

Appendix Number 2: Questionnaire for the Teacher in the Experimental Group

Dear teacher,

The questionnaire before you is a part of a research study carried out in the Faculty of Education of Adam Mickiewicz University in Poznań, Poland. The goal of the research is to examine the attitudes and considerations of the mathematics teachers about the intensive integration of online tasks in a collaborative manner in the classrooms and its influence on the class students in general and the students with difficulties in particular, in terms of the learning abilities and skills and achievements.

The questionnaire is composed of three parts. You are asked to answer open-ended and closed-ended questions. The first part asks about personal and demographic information, which will remain confidential. The names of the participants and their schools will not be published anywhere. The second part includes statements, when every statement has five answers (1 – do not agree at all, 2 – do not agree, 3 – do not know, 4 – agree, and 5 – greatly agree). The third part presents open-ended questions.

Part 1: Background Variables

1. Sex:

A. Male
B. Female

2. Experience in teaching:

A. 1-5 years
B. 6-10 years
C. 11-20 years
D. 21+ years

3. Academic degree in the subject of mathematics:

A. Student teacher (student in a teacher training college)
B. Bachelor degree B.A./B.Sc./B.Ed. (university/college – recognized in Israel)
C. Master degree M.A./M.Sc.
D. Doctoral degree Ph.D.

4. What is the number of computers in the computer room? __________

5. How many in-service training courses on the topic of the use of the computer and the integration of the computer in teaching did you take in the past five years?

A. 0 B. 1 C. 2 D. 3 E. More than 3

6. When was the last in-service training course?

A. About a year ago
B. About two-three years ago
C. More than three years ago

7. How many hours were in the last in-service training course you participated in?

A. Up to 56 hours
B. 56-112 hours
C. More than 112 hours

Part 2

In the statements before you, please choose the answer (from 1-5, when 1 – do not agree and 5 – agree very greatly) that in your opinion is the most suitable. (If you have an explanation, please write a few words under the statement.)

	Statement	1 Do not agree	2 Agree slightly	3 Agree moderately	4 Agree greatly	5 Agree very greatly
1	The online collaborative discourse gave me information about the student's process of solving in the tasks.					
2	My participation as a teacher in the online mathematical discourse strengthened my knowledge about the learner's weaknesses.					
3	The follow up in the online discourse about the learner's manner of participation increased the awareness of his preferred learning style.					
4	The follow up after the learning processes of the student who has difficulties in the online mathematical discourse enables the development of new ways in the learning method.					
5	In the online mathematical collaborative discourse the follow up after the processes of learning and thinking in the learner is intensive and in real time, so that the teacher is aware of the learner's knowledge gaps.					

6	My mediation as a teacher in the online discourse served as formative assessment regarding the continuation of the solution among the students with difficulties.					
7	The online mathematical discourse offers the learner time for thinking and organization of the answers.					
8	The online collaborative mathematical discourse requires the ability to reason and explain, and this strengthens the student’s thinking.					
9	The online mathematical discourse between the learners forces the use of mathematical concepts, and this influences the understanding.					
10	The collaboration in the online mathematical discourse between the learner with difficulties and his classmates or the teacher in real time makes the task performance easier.					
11	The online collaborative discourse enables exposure to different ways of thinking and learning styles.					
12	The online mathematical discourse enables the use of different means, such as software programs and illustrations, and this challenges the students with difficulties.					
13	The mediation in the online collaborative mathematical discourse allows the students with difficulties to ask questions.					
14	The online mathematical collaborative discourse causes the students with difficulties to act more because of the encouragement they receive from their classmates.					
15	Controlled online mathematical collaborative discourse encourages students with difficulties to participate more since this protects them from mockery of “mistakes”.					
16	The learner’s ability to perform different manipulations and calculations using online software in problems in a short time and without much mental effort increases the					

	learner's participation in the performance of online tasks.					
17	The student with difficulties expresses enjoyment participating in the online tasks.					
18	The student with difficulties participates in the lesson and gives new ideas.					
19	The student with difficulties frequently enters the discourse and likes participating in it.					
20	The student with difficulties does the homework and in the class enjoys solving the problems put on the board.					
21	The student with difficulties is more caring about success in the learning material and dedicates time to the learning of mathematics.					
22	The online mathematical discourse enables the phrasing of the answer in different ways, and thus encourages weak students to present solutions in a concrete manner.					
23	The online mathematical discourse influences the degree of interest in the lesson of learners with difficulties.					
24	The collaborative dialogue in the online discourse develops the thinking and increases the ability to ask questions among the students who have difficulties.					
25	The caring of the teachers and the classmates about the learner's answers encourage the learner to be more active in the discourse.					
26	The achievements in the subject of mathematics will increase if we persevere in this type of learning in our classes.					

Thank you very much

Appendix Number 3: Questionnaire for the Teacher in the Control Group

Dear teacher,

The questionnaire before you is a part of a research study carried out in the Faculty of Education of Adam Mickiewicz University in Poznań, Poland. The goal of the research is to examine the attitudes and considerations of the mathematics teachers about the mathematical discourse in the learning classes and its importance to the student with difficulties, in terms of the learning abilities and skills and achievements. The questionnaire is composed of two parts. The first part asks about personal and demographic information, which will remain confidential. The names of the participants and their schools will not be published anywhere. The second part includes statements, when every statement has five answers (1 – do not agree, 2 – do not agree slightly, 3 – agree moderately, 4 – agree greatly, and 5 – agree very greatly).

Part 1: Background Variables

1. Sex:

A. Male
B. Female

2. Experience in teaching:

A. 1-5 years
B. 6-10 years
C. 11-20 years
D. 21+ years

3. Academic degree in the subject of mathematics:

A. Student teacher (student in a teacher training college)
B. Bachelor degree B.A./B.Sc./B.Ed. (university/college – recognized in Israel)
C. Master degree M.A./M.Sc.
D. Master degree M.Ed.
E. Doctoral degree Ph.D.

4. What is the number of computers in the computer room? __________

5. How many in-service training courses on the topic of the use of the computer and the integration of the computer in teaching did you take in the past five years?

A. 0 B. 1 C. 2 D. 3 E. More than 3

6. When was the last in-service training course?

A. About a year ago
B. About two-three years ago
C. More than three years ago

7. How many hours were in the last in-service training course you participated in?

A. Up to 56 hours
B. 56-112 hours
C. More than 112 hours

Part 2

In the statements before you, please choose the answer (from 1-5, when 1 – do not agree and 5 – agree very greatly) that in your opinion is the most suitable. (If you have an explanation, please write a few words under the statement.)

	Statement	1 Do not agree	2 Agree slightly	3 Agree moderately	4 Agree greatly	5 Agree very greatly
1	The online collaborative discourse gave me information about the student's process of solving in the tasks.					
2	My participation as a teacher in the mathematical discourse in the class strengthened my knowledge about the learner's weaknesses.					
3	The follow up in the class mathematical discourse about the learner's manner of participation increased the awareness of his preferred learning style.					
4	The follow up after the learning processes of the student who has difficulties in the class mathematical discourse enables the development of new ways in the learning method.					
5	The class mathematical discourse enables the follow up after the processes of learning and thinking in the learner, intensively and in real time, so that the teacher is aware of the learner's knowledge gaps.					
6	My mediation as a teacher in the class mathematical discourse served as formative assessment regarding					

	the continuation of the solution among the students with difficulties.					
7	The class mathematical discourse offers the learner time for thinking and organization of the answers.					
8	The class mathematical discourse requires the ability to reason and explain, and this strengthens the student's thinking.					
9	The collaborative mathematical discourse between the learners forces the use of mathematical concepts, and this influences the understanding.					
10	The collaboration in the class mathematical discourse between the learner with difficulties and his classmates or the teacher in real time makes the task performance easier.					
11	The class collaborative discourse enables exposure to different ways of thinking and learning styles.					
12	The class mathematical discourse enables the use of different means, such as software programs and illustrations, and this challenges the students with difficulties.					
13	The mediation in the class mathematical discourse allows the students with difficulties to ask questions.					
14	The class mathematical collaborative discourse causes the students with difficulties to act more because of the encouragement they receive from their classmates.					
15	Controlled class mathematical collaborative discourse encourages students with difficulties to participate more since this protects them from mockery of "mistakes".					
16	Class mathematical discourse about different possible solutions for mathematical problems encourages the learner with difficulties to perform different and increases the learner's participation in the performance of class tasks.					

17	The student with difficulties expresses enjoyment in participating in the class tasks.					
18	The student with difficulties participates in the lesson and gives new ideas.					
19	The student with difficulties frequently enters the discourse and likes participating in it.					
20	The student with difficulties does the homework and in the class enjoys solving the problems put on the board.					
21	The student with difficulties is more caring about success in the learning material and dedicates time to the learning of mathematics.					
22	The class mathematical discourse encourages weak students to present solutions in a concrete manner and enables phrasing of the answer in different ways.					
23	The class mathematical discourse influences the degree of interest in the lesson of learners with difficulties.					
24	The collaborative dialogue in the class discourse develops the thinking and increases the ability to ask questions among the students who have difficulties.					
25	The caring of the teachers and the classmates about the learner's answers encourage the learner to be more active in the discourse.					
26	The achievements in the subject of mathematics will increase if we develop learning based on class mathematical discourse.					

Thank you very much

Appendix Number 4: Questionnaire for Students Who Particpated in the Experiment

Dear student,

Please answer the questions below clearly and according to what you think. In the second part of the questionnaire there are statements, when each statement has 5 answers (1 – do not agree, 2 –not agree slightly, 3 – agree moderately, 4 – agree greatly, 5 – agree very greatly). You must choose the answer that in your opinion is most suitable. (If you have an explanation, please write a few words under the statement.)

Background Variables

1. Student name: _____________

2. Student sex:

   A. Male
   B. Female

3. Number of siblings at home: ______________

Part 1: Self-Efficacy Questionnaire

Below there are statements, when each statement has 5 answers (1 – do not agree, 2 – agree slightly, 3 – agree moderately, 4 – agree greatly, 5 – agree very greatly). You must choose the answer that in your opinion is most suitable. (If you have an explanation, please write a few words under the statement.)

	Statement	1 Do not agree	2 Agree slightly	3 Agree moderately	4 Agree greatly	5 Agree very greatly
1	After using tasks, I believe more that I can be effective in different roles.	1	2	3	4	5
2	I can achieve most of the goals I set for myself.	1	2	3	4	5
3	Everything is possible if I truly try.	1	2	3	4	5
4	When I am facing difficult tasks, I am certain I can accomplish them.	1	2	3	4	5
5	In general, I think that I can achieve what is important to me.	1	2	3	4	5

6	I can succeed in every task when I am set in my opinion.	1	2	3	4	5
7	I can meet many challenges successfully.	1	2	3	4	5
8	I am sure that I can perform well most of the tasks.	1	2	3	4	5
9	I always can find a way to achieve my goals, even if they are very difficult.	1	2	3	4	5
10	I can perform things well under very difficult conditions.	1	2	3	4	5
11	I am certain that I can perform many tasks successfully.	1	2	3	4	5
12	I can perform most of the tasks well, in comparison to other people.	1	2	3	4	5
13	I am confident in myself when I perform new tasks.	1	2	3	4	5
14	Even when the situation is difficult, I can perform rather well.	1	2	3	4	5

Part 2: Learning Strategies Questionnaire

In the statements below there are 5 answers (1 – do not agree, 2 – agree slightly, 3 – agree moderately, 4 – agree greatly, 5 – agree very greatly). You must choose the answer that in your opinion is the most suitable. (If you have an explanation, then please write in a number of words under the statement.)

	Statement	1 Do not agree	2 Agree slightly	3 Agree moderately	4 Agree greatly	5 Agree very greatly
15	When I learn, I try to memorize the material so that I will know to answer each question asked about it.	1	2	3	4	5
16	When I learn, I try to examine myself and to check whether all the concepts are clear to me.	1	2	3	4	5
17	When I learn, I try to find relationships between the material I learned and the material I learned in other subjects.	1	2	3	4	5
18	When I learn, I try to understand first what exactly I must learn.	1	2	3	4	5
19	When I learn, I memorize as many details as possible.	1	2	3	4	5

20	When I learn, I try to think how the learned material connects with what I have already learned.	1	2	3	4	5
21	When I learn, I stop periodically to examine whether I remember.	1	2	3	4	5
22	When I learn, I try to think how it is possible to utilize the learned knowledge in a practical manner.	1	2	3	4	5
23	When I learn and do not understand, I search for additional information that will clarify what is not clear.	1	2	3	4	5
24	When I learn, I memorize every new detail so that I can repeat it orally.	1	2	3	4	5
25	When I learn, I examine whether I remember the main points.	1	2	3	4	5
26	When I learn, I attempt to find relationships between what I have already learned and the learned material so as to understand better.	1	2	3	4	5
27	When I learn, I repeat the material aloud again and again.	1	2	3	4	5

Part 3: Intrinsic Motivation and Learning Experience Questionnaire

In the statements below there are 5 answers (1 – do not agree, 2 – agree slightly, 3 – agree moderately, 4 – agree greatly, 5 – agree very greatly). You must choose the answer that in your opinion is the most suitable. (If you have an explanation, then please write in a number of words under the statement.)

	Statement	1 Do not agree	2 Agree slightly	3 Agree moderately	4 Agree greatly	5 Agree very greatly
28	When I performed the online collaborative task, I thought how much I am enjoying it.	1	2	3	4	5
29	I was not agitated during the performance of the online collaborative tasks.	1	2	3	4	5
30	I felt that this was my choice to perform the tasks.	1	2	3	4	5
31	I think that I am rather good in these types of tasks.	1	2	3	4	5
32	The tasks were very interesting.	1	2	3	4	5
33	I felt tense when performing the tasks.	1	2	3	4	5
34	I think that I succeeded rather well in this activity in comparison to the members in the group.	1	2	3	4	5

35	I performed the tasks "with fun".	1	2	3	4	5
36	I felt calm during the performance of the tasks.	1	2	3	4	5
37	I greatly enjoyed doing the tasks.	1	2	3	4	5
38	I did not truly choose to perform these tasks.	1	2	3	4	5
39	I am satisfied with my performances in these tasks.	1	2	3	4	5
40	I was "pressured" during the performance of the tasks.	1	2	3	4	5
41	The tasks were very boring.	1	2	3	4	5
42	I felt that I am doing what I want to do while performing the tasks.	1	2	3	4	5
43	I felt that I am rather skilled in these tasks.	1	2	3	4	5
44	I think that the tasks were very interesting.	1	2	3	4	5
45	I felt "pressured" during the performance of the tasks.	1	2	3	4	5
46	I felt that I must do the tasks.	1	2	3	4	5
47	I would describe the tasks as very enjoyable activities	1	2	3	4	5
48	I did the tasks since I did not have a choice.	1	2	3	4	5
49	After I performed the tasks, I felt that I am rather talented.	1	2	3	4	5

Thank you very much.

Appendix Number 5: Questionnaire for Students Who Participated in the Control Group

Dear student,

Please answer the questions below clearly and according to what you think. In the second part of the questionnaire there are statements, when each statement has 5 answers (1 – do not agree, 2 – do not agree slightly, 3 – agree moderately, 4 – agree greatly, 5 – agree very greatly). You must choose the answer that in your opinion is most suitable. (If you have an explanation, please write a few words under the statement.)

Background Variables

1. Student name: ______________

2. Student sex:

   A. Male
   B. Female

3. Number of siblings at home: _______________

Part 1: Self-Efficacy Questionnaire

Below there are statements, when each statement has 5 answers (1 – do not agree, 2 – agree slightly, 3 – agree moderately, 4 – agree greatly, 5 – agree very greatly). You must choose the answer that in your opinion is most suitable. (If you have an explanation, please write a few words under the statement.)

	Statement	1 Do not agree	2 Agree slightly	3 Agree moderately	4 Agree greatly	5 Agree very greatly
1	I really like participating in mathematics lessons.	1	2	3	4	5
2	In the mathematics lessons I feel that I can achieve most of the goals I set for myself.	1	2	3	4	5
3	In mathematics everything is possible if I truly try.	1	2	3	4	5
4	When I face difficult mathematical tasks, I am confident I can perform them.	1	2	3	4	5
5	In general, I think I can achieve what is important to me in the mathematics lessons.	1	2	3	4	5

6	I can succeed in every mathematical task when I am set in my opinion.	1	2	3	4	5
7	I can meet many mathematical challenges successfully.	1	2	3	4	5
8	I am sure that I can perform well most of the tasks.	1	2	3	4	5
9	I always can find a way to achieve my goals in the mathematics lessons, even if they are very difficult.	1	2	3	4	5
10	I can perform things in mathematics well under very difficult conditions.	1	2	3	4	5
11	I am certain that I can perform many tasks successfully.	1	2	3	4	5
12	I can perform most of the tasks well, in comparison to other people.	1	2	3	4	5
13	I am confident in myself when I perform new tasks.	1	2	3	4	5
14	Even when the mathematical assignments are difficult, I can perform them rather well.	1	2	3	4	5

Part 2: Learning Strategies Questionnaire

In the statements below there are 5 answers (1 – do not agree, 2 – agree slightly, 3 – agree moderately, 4 – agree greatly, 5 – agree very greatly). You must choose the answer that in your opinion is the most suitable. (If you have an explanation, then please write in a number of words under the statement.)

	Statement	1 Do not agree	2 Agree slightly	3 Agree moderately	4 Agree greatly	5 Agree very greatly
15	When I learn, I try to memorize the mathematical material so that I will know to answer each question asked about it.	1	2	3	4	5
16	When I learn, I try to examine myself and to check whether all the concepts are clear to me.	1	2	3	4	5
17	When I learn, I try to find relationships between the material I learned and the material I learned in other subjects.	1	2	3	4	5
18	When I begin to learn mathematics, I try to understand first what exactly I must learn.	1	2	3	4	5
19	When I learn mathematics, I memorize as many details as possible.	1	2	3	4	5

20	When I learn mathematical material, I try to think how the learned material connects with what I have already learned.	1	2	3	4	5
21	When I learn, I stop periodically to examine whether I remember the mathematical concepts.	1	2	3	4	5
22	When I learn mathematics, I try to think how it is possible to utilize the learned knowledge in a practical manner.	1	2	3	4	5
23	When I learn mathematical material and do not understand, I search for additional information that will clarify what is not clear.	1	2	3	4	5
24	When I learn mathematical material, I memorize every new detail so that I can repeat it orally.	1	2	3	4	5
25	When I learn mathematical material, I examine whether I remember the main points.	1	2	3	4	5
26	When I learn mathematical material, I attempt to find relationships between what I have already learned and the learned material so as to understand better.	1	2	3	4	5
27	When I learn mathematical material, I repeat it aloud again and again.	1	2	3	4	5

Part 3: Intrinsic Motivation and Learning Experience Questionnaire

In the statements below there are 5 answers (1 – do not agree, 2 – agree slightly, 3 – agree moderately, 4 – agree greatly, 5 – agree very greatly). You must choose the answer that in your opinion is the most suitable. (If you have an explanation, then please write in a number of words under the statement.)

	Statement	1 Do not agree	2 Agree slightly	3 Agree moderately	4 Agree greatly	5 Agree very greatly
28	When I am performing the mathematical task, I think about how much I am enjoying it.	1	2	3	4	5
29	I am not agitated during the performance of the mathematical tasks.	1	2	3	4	5
30	I feel that this is my choice to perform the mathematical tasks.	1	2	3	4	5
31	I think that I am rather good in the class mathematical tasks.	1	2	3	4	5

32	The mathematical tasks are very interesting.	1	2	3	4	5
33	I feel tense when performing the mathematical tasks.	1	2	3	4	5
34	I think that I always succeed in the class mathematical tasks in comparison to the members in the group.	1	2	3	4	5
35	I perform the mathematical tasks “with fun”.	1	2	3	4	5
36	I feel calm during the performance of the mathematical tasks.	1	2	3	4	5
37	I like doing the mathematical tasks.	1	2	3	4	5
38	I do not really like carrying out mathematical tasks.	1	2	3	4	5
39	I am satisfied with my performances in the mathematical tasks.	1	2	3	4	5
40	I am generally “pressured” during the performance of the mathematical tasks.	1	2	3	4	5
41	The mathematical tasks are very boring.	1	2	3	4	5
42	I like performing the mathematical tasks since then I understand the material better.	1	2	3	4	5
43	I feel rather skilled in the performance of mathematical tasks.	1	2	3	4	5
44	I think that the mathematical tasks are very interesting.	1	2	3	4	5
45	I feel “pressured” during the performance of the mathematical tasks.	1	2	3	4	5
46	I feel that I must do the mathematical tasks.	1	2	3	4	5
47	I would describe the mathematical tasks as very enjoyable activities.	1	2	3	4	5
48	I did the mathematical tasks since I did not have a choice.	1	2	3	4	5
49	After I perform the mathematical tasks, I feel rather talented.	1	2	3	4	5

Thank you very much.

Appendix Number 6: Example of an Online Collaborative Task

Target Practice[20]

Dear students,

The task before you is the order of operations in arithmetic. The student must express his solution while using words and presenting a continuum of a solution, referring to the solution of his fellow student.

Task	Student 1	Student 2	Student 3	Student 4	Student 5	Section
Solve the two problems: (12 - 3) - 2 = 12 – (3 - 2) = Why in your opinion are the results of the two problems different?						**1**
Solve: (16 - 6) x 2 - 1= 16 - (6 x 2) - 1= 16 - 6 x (2 - 1) = In which problems, in your opinion, is it possible to erase the parentheses and the result will not be different?						**2**

[20] The task is taken from the database of tasks in mathematics for the fourth grade, the Ministry of Education, Culture, and Sports, 1995. The task is intended for fourth grade students, but it can be used for seventh grade students who have difficulties, since the topic is a part of the curriculum of the beginning of the seventh grade (since the curriculum in mathematics is spiral by nature).

Every student will choose to add parentheses differently and will solve one problem: 17- 3 x 4 + 2= 17- 3 x 4 + 2=						**3**
Is it necessary to add a pair of parentheses so that the exercise will be correct: 20 – 5 – 2 = 17 16 – 6 x 2 = 4						**4**
Is it necessary to add a pair of parentheses so that the exercise will be correct? 7 + 3 x 4 – 2 =13 8 + 16: 4 x 2 = 12 If yes, note where.						**5**
Add a pair of parentheses in each problem, if necessary, to obtain the **largest possible answer**. 7 x 4 + 5 – 3 = 10 – 4 x 2 + 1 =						6
Write in the box a number 1-9 and if necessary, add parentheses so as to obtain an answer as close as possible to 10. □ -4 x 3 =						7
Write in the boxes a number 1-9 and if necessary, add parentheses so as to obtain as large an answer as possible. 25 □ x △ =						8

Characteristics of the Task

Topic: Measure – natural numbers

Actions – natural numbers

Rank of difficulty of the questions:

- Question 1 easy
- Question 2 easy
- Question 3 moderate
- Question 4 easy
- Question 5 moderate
- Questions 6-8 are difficult and require inquiry.

There is no dependence between sections.

The task will be disseminated to the students through the use of Google Docs

Description of the Nature of the Questions

In the task there are three parts:

- Questions 1-3. The students must solve problems that are different only in the position of the parentheses (order of operations in arithmetic).
- Questions 4-5. The students must determine the order of operations suitable on the basis of a given answer.
- Questions 6-8. The students must determine the appropriate order of operations on the basis of a given condition regarding the desired result.

Appendix Number 7: Test to Identify Difficulties among Seventh Grade Students in Mathematics-Mapping Test

Student name: ________________ **Date: ___________________**

Question 1:

Solve the following questions:

1. $7{,}048 + 54 =$
2. $5{,}030 - 405 =$
3. $403 * 12 =$
4. $714 : 7 =$

Characterization of the question:
- A standard question at the level of an algorithm that tests the learner's ability to solve an addition exercise and a subtraction exercise with breaking down the number, while correctly arranging the exercises vertically and correctly writing the numbers below each other according to the value of each digit. Therefore, the learner requires a thorough understanding of the principles of the decimal structure. (The learner must be able to organize himself).
- Exercises of this kind are addressed in the elementary school starting from the third grade.

Question 2:

Complete each of the exercises with the missing number so that the equation is correct:

1. $546 + 34 =$ ________ $+ \ 54$

2. $9{,}804 - 39 = 9{,}800 -$ ______

Characterization of the question:
- A slight question of insight, which examines the concept of equality. Equations of this kind are addressed in elementary school in grades 4 and up.
- To solve such a question, the student must understand that there is equality between the left side of the right side and the right side. So he will have to find the value of the expression on the left and then use the laws of reduction and enlargement of addition and subtraction operations.

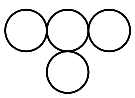
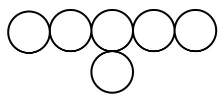
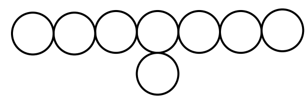
Question 3:

Ruthie had 35 marbles.
During the break she played marbles and lost 2/7 of the marbles.
How many marbles were left for Ruthie after the break?
Introduce the solution.

Characterization of the question:
Standard application-level question.
- The question examines the student's understanding of the comprehension of the whole and the parts. In the process of solving the question, there are two stages: the first part is understanding which part is left for Ruthie from all the marbles, and the second part is to calculate the value of the part and find the remaining marbles.

Question 4:

Eitan assembles the structures of circles according to a fixed law:

		
Structure 1	Structure 2	Structure 3

A. Eitan continues to construct circular structures with the same rules.
Complete the table with the number of squares in each of the buildings.

Structure	Structure 1	Structure 2	Structure 3	Structure 4	Structure 5	Structure 8	Structure 10
Number of circles	4						

A. The 20th building has 42 circles.
How many circles are there in the 21st?________________
B. The 28th building has 58 circles.
How many circles are there in the 26th?______________
C. How many circles are there in n? Write an algebraic expression ___________________

Characterization of the question:
- This is a standard inquiry question at the level of insight.
- A relative question requires a connection between several elements in order to find rules.
- The student is required to express the rule by using algebraic expressions (starting to measure algebra only at the beginning of seventh grade).

Question 5:

Solve the following questions:

1. $30 - (15 - 5) : 5 =$
2. $17 - (3 + 6 * 2) =$
3. $39 - 9 : 3 =$
4. $5 : (12 - 2 * 6) =$

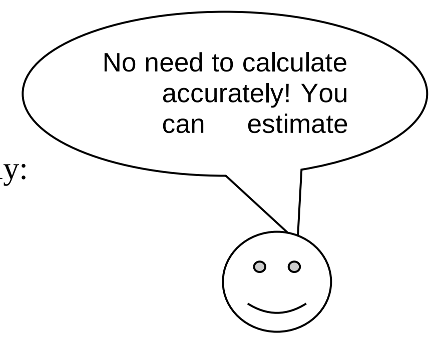
Characterization of the question:
Standard question at the algorithm level.
- Calculation of the answer according to the rules of the order of operations and the parentheses (material taught in grade school, third grade).

Question 6:

A. 10 notebooks cost NIS 35
How much do 40 notebooks cost?
B. 10 notebooks cost NIS 35
How much do 25 notebooks cost?

Characterization of the question:
Standard application level question. The solution is based on an understanding of the multiplication operation in a complex situation. The solution can be represented in an arithmetic manner by calculation or visual form by illustration in the drawing or words.
- A solution to the type of questions of this kind is in the elementary school in the fifth grade.

Question 7:



When dividing 22,017 by 22, the resulting result is approximately:
A. 10
B. 100
C. 1,000
D. 10,000

Characterization of the question:
- An insightful question that examines the ability to estimate the outcome of a large numbering exercise, this exercise examines the learner's ability to sense the numbers and the ability to reorganize the exercise to find the closest answer to the real answer.
- Such questions are addressed in the elementary school starting in the fourth grade.

Question 8:

Solve the following questions:

A. $542 * 56 < \_\_\_\_\_\_ * 56$

B. $5943 : 3 > 5943 : \_\_\_\_\_\_$

C $0.25 * \_\_\_\_\_\_ = 10$

D. $\frac{3}{4} * \_\_\_\_\_\_\_ = 1$

Characterization of the question:

- Question at the level of insight, a question that requires understanding the relationship between numbers and understanding the meaning of multiplication and division (section A and B), and understanding the meaning of the symbols <,> and the ability to read the exercise correctly.
- Section C and D are two sections requiring a level of understanding of the decimal fraction (section C) and regarding the fraction and the various representations and the multiplication characteristics of the opposite fractions (section 4). These questions are addressed in the sixth grade elementary school.

Question 9:

Circle the largest number:

A .$\frac{2}{3}$ B .$\frac{3}{4}$ C .$\frac{4}{5}$ D .$\frac{5}{6}$

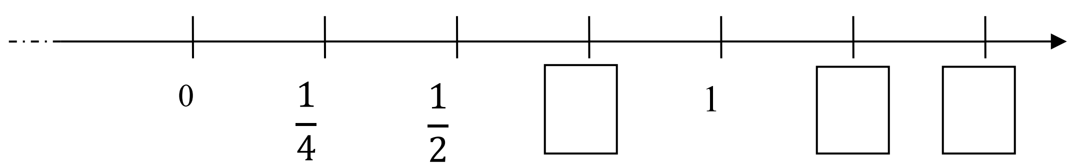
Characterization of the question:

-A standard level of insight question that checks for comparison of fractions.

-A question of this kind is in the third grade elementary school.

Question 10:

Fill in the blank:



Characterization of the question:

-A simple question at the application level that examines the understanding of the concept of fraction, and the recognition of simple fractions as points on the number line.

-This type of question is addressed in the elementary school - from the fifth grade.

Question 11:

The price of a package of butter cookies is 5.15 NIS, and the price of a package of chocolate cookies is 6.55 NIS.
Ronit bought 6 packages of butter cookies and one package of chocolate cookies.
How much did Ronit pay for butter cookies and chocolate cookies?

Characterization of the question:
-A question at the level of implementation that is addressed in the sixth grade.
-The solution consists of two stages (the first is a multiplication exercise in which the necessary knowledge is a multiplication exercise solution of a decimal number in the whole and the second a computation exercise of two decimal numbers).
-Students who didn't do well in the exercise: It is necessary to distinguish between students who know the stages but find it difficult to multiply and connect decimal numbers and students who do not understand the required question.

Question 12:

Solve the following questions. Show the way:

1. $\frac{2}{5} + \frac{2}{3} =$
2. $3.84 + 5\frac{16}{100} =$
3. $2\frac{1}{5} - 1\frac{3}{5} =$
4. $38 - 2.3 * 10 =$

Characterization of the question:
The question of understanding at the level of an algorithm in which the addition, subtraction, and multiplication of mixed numbers is examined. In addition, the ability to switch from decimal representation to fraction representation is examined.
This type of question is addressed in the fourth and fifth grades.

Question 13:

Several times larger than 1/3
Answer____________________:
In a large number 1 from 1/3
Answer____________________:

Characterization of the question:
A question at the level of understanding that examines the meaning of the simple fraction and the containment relationship between the whole and the parts.
In both sections, the understanding of the containment and comparison ratios is examined using large terms.
Such questions are addressed in the elementary school starting in the third grade.

Question 14:

In the seventh grade, 50 students study.
22 students from these classes participate in the chess department.
Which percentage of these students participate in the chess class?

Characterization of the question:
A question at the application level that examines the concept of the percentage.
This topic is taught in the sixth grade in the elementary school.

Question 15:

Solve the following questions. Show the way:

1. $6 : \frac{1}{5} =$
2. $\frac{2}{5} : \frac{3}{4} =$
3. $\frac{6}{11} : 6 =$

Characterization of the question:
Easy question of understanding.
-The question examines the learner's ability to solve division exercises in simple fractions.
-The solution of fractional division exercises is in the elementary school, sixth grade.

Question 16:

A bag with 100 round chewing gum pieces costs NIS 20.
What is the price of each gum?

Characterization of the question:
A question at the application level that examines the meaning of the operation of the division.
-The difference is between a small number and a large number - so only the learner's in-depth understanding of the concept of division will lead to a suitable placement, because dividing a small number in a large number is not natural for the students.

Question 17:

In what exercise is the answer 0.5 ?

A. 100: 200

B. 200: 1000

C. 20: 1000

D. 20: 10,000

Characterization of the question:

-Easy question at the level of knowledge.

-A question that examines the understanding of decimal fractions This topic is taught in the fifth and sixth grade.

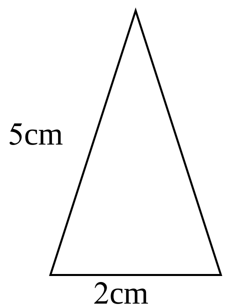
Question 18:

Here is a small illustration of an equilateral triangle.

A. What is the perimeter of the triangle? _______________ cm.

B. What is the length of the side of a square whose circumference is equal to the perimeter of the triangle?

___________ cm.

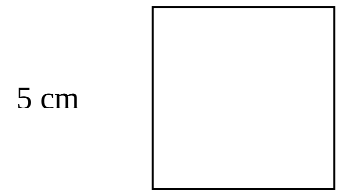


Characterization of the question:

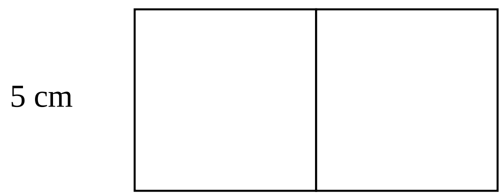
-A question that deals with the concept of the perimeter and characteristics of an equilateral triangle in Part A and in the square in Part B.

-The two sections are at the application level / the student can solve this in a tangible arithmetic manner so that he completes in the figure the length of the missing side based on the characteristics of the equilateral triangle, and in section B to draw a square.

Question 19:



Below is a small diagram of a square whose side is 5 cm.
A. Consider the area of the square: _____ cm^{2}.
B. Calculate the size of the square: _____ cm.
Attach another square to the square in the same size as shown in the figure:

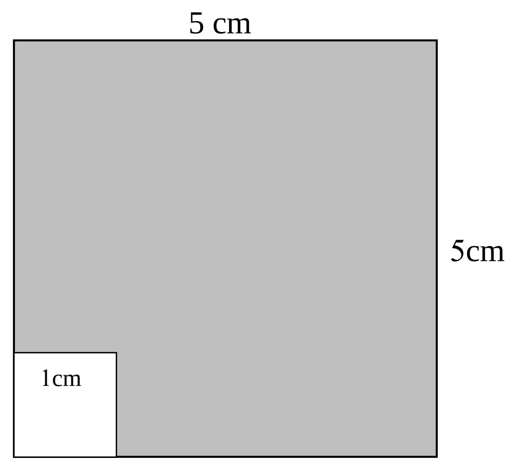


C. Calculate the resulting rectangle area: _____ cm^2.
D. Calculate the resulting rectangle: _____ cm.
If a rectangle is attached to the rectangle, another square is obtained in the same dimensions:
E. What will the new rectangle area received? _________ cm^2
F. What will be the perimeter of the new rectangle received? _________ cm

Characterization of the question:
-Question at different levels of thinking: Section A and B at the level of the algorithm / section C and D at the application level / and sections E and F at the level of insight.
-The question examines the understanding of the concepts: scope / area / characteristics of the length of the ribs in the square (dealing with these concepts already from the first grade).
-Questions on the calculation of the perimeter and area of a square are addressed in all classes in the elementary school as early as the second grade.
-Sections 5 and 6 have an understanding of rules (high level of thinking), dealing with this type of question from the fifth and sixth grades.
-You can solve this type of exercise by visual illustration such as: drawing to the form obtained.

Question 20:

Here is a drawing of a square in a square.
A. What is the area of the gray-colored shape?
______ cm^2
B. What is the perimeter of the shape painted in gray?
____ cm



Characterization of the question:
- a question dealing with the area and perimeter of squares.
- The first section is an easy question at the algorithm level.
- The second section is also a standard question of insight.

Good luck students!

☺

Appendix Number 8: Interview with the Students after the Identification of Their Difficulties after the Characterization Test

Summative table – every teacher fills out the table for the purpose of the analysis of the data and the advancement of the learning for the student individually.

Student Name	Reading Comprehension (Understanding the Question)	Manner of Solution	Use of Concepts	Process Thinking	Conceptualization	Memory of Facts
Student 1						
Student 2						
Student 3						
Student 4						
Student 5						

Appendix Number 9: Summative Test after the Quasi-Experiment

This test was constructed at the end of the experiment according to the material learned in the classrooms and under the guidance of the school instructor of the subject of mathematics on behalf of the Supervision of Mathematics in the Ministry of Education.

Summative Assessment Task – Mathematics, Grade 7 (Ministry of Education, 2015)

Student name: ___________________ Group number: __________________

School name: ____________________

Question 1

Using the laws of operations and the combination of like terms, match each expression in column A with the same expression in column B.

Column A			Column B
2(3x + 4)	•	•	–2b + 2b
4 + 6x	•	•	6x + 4
3a – 3a	•	•	7x + 6x
2x + 5x + 6x	•	•	6x + 8

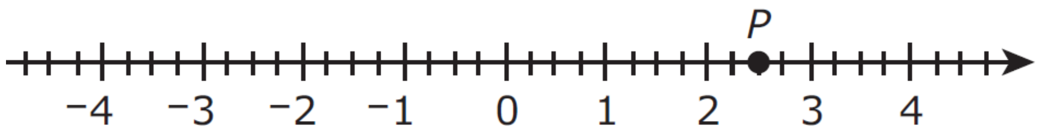
Question 2

Which numerical expressions are equal in size to $-3 \cdot \frac{4}{-5}$? (Mark all the possibilities)

i $\frac{-3}{-5} \cdot 4$ .ii $-\frac{3}{5} \cdot 4$ .iii $\frac{-3 \cdot 4}{-3 \cdot (-5)}$ .iv $-3 \cdot 4 \cdot \frac{-1}{5}$ .v $\frac{3}{5} \cdot 4$ .vi $\frac{3 \cdot 4}{5}$.

Question 3

A number line is given, upon which point P is marked:



A. Mark point Q on the opposite number to the number represented by point P.

B. Mark the number $\frac{5}{4}$ on the number axis as point T.

C. Which point represents a smaller number, P or T? ____________

Question 4

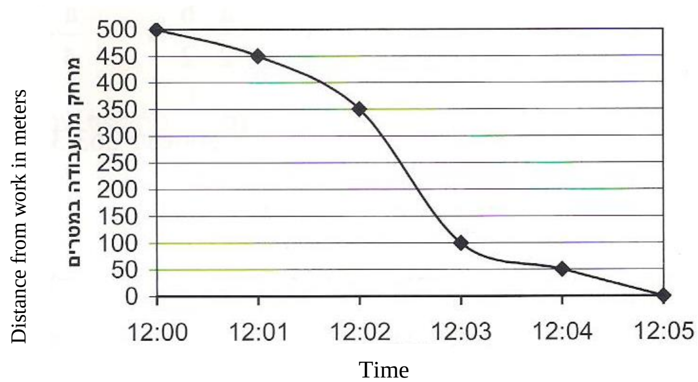
The function **y = 2x – 3** is given.

Complete the table:

X	Y
–4	
	3
0	
$\frac{1}{2}$	

Question 5

Uri went from his home in a straight line to his workplace. The graph before you describes the distance Uri is from his workplace, beginning at the time when he left his home until the time he reached his workplace.



A. What distance did Uri travel from his home to his workplace? _____ meters

B. How many meters did Uri travel from 12:01 to 12:02?

C. At what time interval (of one minute), was Uri's average speed the highest?

Question 6

The length of a train track is $8\frac{9}{10}$ km.

They placed at the start of the track, at the edge of the track, and throughout the track 6 signs found at equal distance. What is the distance between each pair of signs in meters?

i. 1780 meters ii. 1483.33 meters   iii. 178 meters   iv. 148 meters

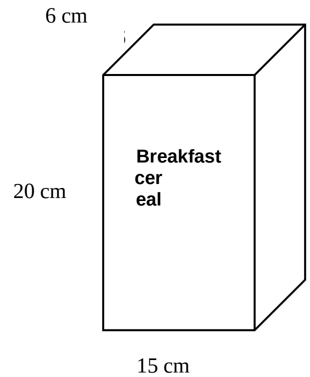
Question 7

Before you is a box of breakfast cereal. The dimensions of the box are written on the box. The box contains 8 identical small boxes, which together fill the entire volume of the large box.

A. What is the volume of the small box? Present the method of solution.

B. Which of the following option can fit the small box (the dimension are in cm – mark all the possible answers)?

i. 3 X 5 X 15 ii. 3 X 3 X 25 iii. 6 X 5 X 7.5 iv. 4 X 5 X 11.25 v. 3 X 10 X 7.5



Question 8

Before a school ceremony the school students entered the sports hall. At first 40 students entered together. Then the principal decided to have every minute a set number of students enter the hall. The table presents some of the data.

Number of minutes that passed	0	1	2	3	4	5
Number of students in the hall	40	48	56			

A. Fill in the table the number of students who were in the hall after 3, 4, 5 minutes.
B. How many students were in the half after 7 minutes?
C. How many students were in the half after n minutes?
D. After how many minutes were there160 students in the hall? Present the method of solution.

Question 9

Solve the following exercises. Present the method of solution.

A	B	C
$24 + 6 : 3 =$	$(-2 + 5) : 6 \cdot 3 =$	$\frac{3 \cdot 2 + 5 \cdot 2}{-8 : 2} =$

Question 10

Solve the exercise:

$$-4^2 + 2(2 - 5 \cdot 4) : 6 =$$

Question 11

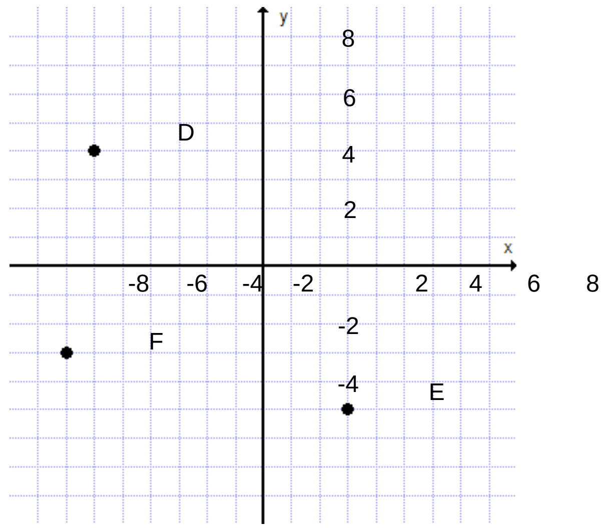
Arnon practiced walking and swimming for 5 days. In every day he did exactly the same practice.

- The practice includes both walking and swimming.
- Every day the part of the walking is 45 minutes.
- All total, Arnon practiced in the five days for 375 minutes.

How much time did Arnon swim every day during these five days? Present the method of solution.

Question 12

The following coordinate system is given:



A. Mark in the coordinate system the points A (4,5), B (-5,-2), C (0,6).

B. Write the coordinates of the points in the coordinate system:

D (___, ___) E (___, ___) F (___, ___)

C. Mark point G whose x coordinate is shared with the x coordinate of point F and whose y coordinate is shared with the y coordinate of point E. Write the coordinate G (___, ___).

D. Add point T so that the rectangle FGET will be created.

Calculate the perimeter of the rectangle: ____________

Calculate the area of the rectangle: ____________

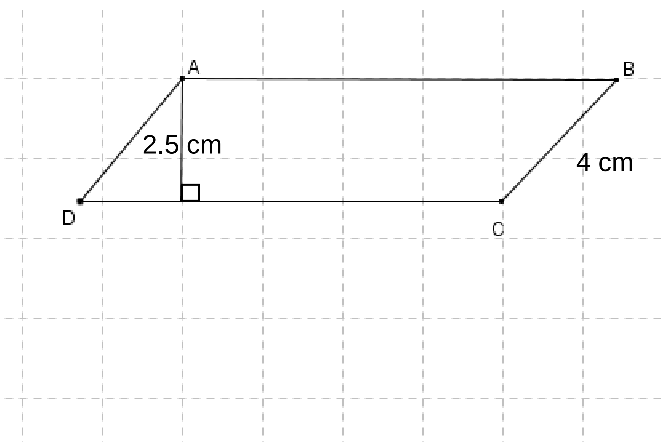
Question 13

The altitude of the parallelogram is 2.5 cm. The area of the parallelogram is 30 cm^2. (The measurements in the diagram are not precise).

A. What is the length of side AB?

B. What is the length of the altitude to side BC?

C. Draw the altitude to side BC.



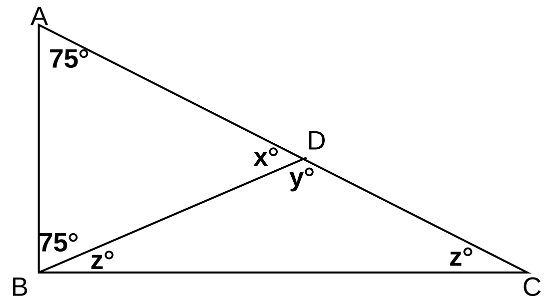
Question 14

A. Calculate the size of the angles indicated by x, y, and z. Present the method of solution.

Answer: x = ________ y = ________ z = ________

B. What type of triangle is ABC?

C. What type of triangle is BDC?



Question 15

Solve the following equations and present the method of solution.

A	B
4(x + 2) – x = –1	3m – (1 – m) = 2m

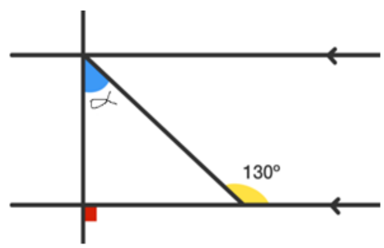
Question 16

Note the number from the numbers written below, for which we will obtain a positive result if we insert it into the algebraic expression $\frac{1+3x}{1-2x}$:

i. – 2 ii. -1 iii. 0 iv. 1

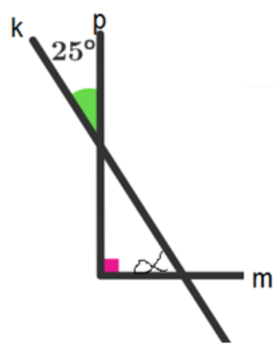
Question 17

A What is the value of angle α ?



B The two lines k, p intersect and the lines k, m also intersect. Given: m ⊥ p.

What is the value of angle α ?



Question 18

Calculate the value of the expression $\frac{3m - 4y}{2m}$ for m = 4 and y = -2.

i. 2 ii $\frac{1}{2}$. iii $\frac{5}{2}$. iv $\frac{2}{5}$.

Question 19

The function $y = 4x - 1$ is given.
Find the value of y if $x = -2$? Explain your answer.
Find the value of x if $y = 9$? Explain your answer.

Good luck!

Appendix Number 10: Specifications of the Teacher Training Program

The teacher training program included:

- **Types of discourse held in the lessons.**
  - Mathematical discourse focused on collection of information (informative).
  - **Gradual** directed discourse.
  - In the discourse the teacher / student illustrates the building of the mathematical model in the learner with the use of three representations – verbal, algebraic, and illustrative.
  - Dialogic discourse – teacher/student and student/student.
  - Reflexive discourse about the data and about the learning process and ways of collaboration.

- **Components of the mathematical discourse for the teachers**
  1. Work on effectiveness
  - Creation of motivation and interest in the learner (the teacher presents problems from easy to difficult, enables questions of clarification, and the learners phrase similar questions).
  - Involvement of the learners in the educational goals (the learner plans his learning, determines in an initial manner the mathematical learning material for which the lack of understanding causes difficulties with his advancement in mathematics).
  - Mediation of relevance (connects between the theoretical learning material and the everyday life through concrete mathematical problems related to the learner's life).
    a. Identification of points of strength / weakness among the students with difficulties.
  - The teacher develops a mathematical discourse with emphasis on the learners' processes of thinking and learns about the preferred forms of learning (writing/auditory-verbal/visual-tangible).
  - The teacher develops questions that require processual thinking in order to understand about the learners' previous knowledge - difficulties/strengths.
    a. Providing mathematical problems from a higher order.
  - Questions on integration between the previous material and the new material.
  - Questions that require verbal analysis / explanation / justification.
  - Theoretical questions of the implementation of mathematical law and questions of implementation of mathematical laws in a practical way through the integration of problems from everyday life.
    a. Development of mathematical discourse between the learners focused on a goal / topic.
  - The teacher will enable an encouraging and safe learning atmosphere, set boundaries, and block responses of mockery, condescension, and encourage the learners to express their opinion about possible ways of solution.
  - The teacher will make certain that the students will understand what is required in the tasks.

- The teacher will ask the learner to conceptualize what is required in correct mathematical language.
  a. The inculcation of learning strategies that include critical thinking and metacognition.
- Mediation that supports the construction of knowledge and development of thinking through dialogic discourse.
- The development of the estimation ability and criticism ability of answers.
- The learner will explain and justify the manner of solution through the representation of the solution in different ways (information processing).
- The linkage of the question to the knowledge and concepts required in order to solve the mathematical problem.
- The development of the ability to ask questions and provide justifications verbally and in calculations (drawing conclusions).

- **Type of information that the teachers are required to collect.**
  - Learning knowledge and learner status (on the basis of the information collected about the learner in the mapping test)
  - Strengths / weaknesses (understanding major and minor points, understanding the part and the whole, the ability to divide a task into sub-tasks)
  - Ability of mathematical conceptualization and ability of correct use of mathematical concepts
  - Ways of thinking
  - Ways of organization for solving a mathematical task
  - Ability to organize for the solution of a mathematical task
  - Ways of solution and different representations of the solution process (verbal, graphic, and algebraic)
  - Ways of learning preferred by the learner
  - Ability to understand / explain / justify a mathematical problem

- **Main topics of learning on which the teachers will focus – the seventh grade**

The teachers will work on seven main subtopics according to the curriculum proposed for the seventh grade.

1. Order of operations in arithmetic
2. Laws and algebraic expression
3. Solving equations and word problems
4. Positive and negative numbers
5. Coordinate system
6. Angles and triangles
7. Rectangle and box (area and volume)

Appendix Number 11: Consent Form for the Principal

To the Principal of the ______________________School

Date: __________________

Re: Research on the Topic of the Influence of the Online Collaborative Discourse as Formative Feedback for the Improvement of the Achievements of Students with Difficulties in Mathematics in the Seventh Grade in the Northern Urban Arab Sector

In the framework of my studies for the doctoral degree in the school of education at AMU, Adam Mickiewicz University in Poznań, Poland I am performing a research on this topic. The objective of the research study is to improve the achievements of students who are having difficulties with the subject of mathematics. Mathematics is one of the main subjects of studies in the education system in Israel, at all the age groups. The subject has considerable importance, since it is a basis for quantitative logical thinking and infrastructure for the studies of science and technology. It is known today that most of the students find it difficult to understand and internalize processes in the context of numbers, concepts, and use of symbols, which increases the level of difficulty and develops anxiety among many students with regard to all that pertains to mathematics in particular and subjects connected to mathematics in general.

For the purpose of the research, the following actions are planned:

1. With your consent as the school principal, for the purposes of the research study two tests will be held, the first for all the students of the seventh grade at the start of September, a mapping test for students, and a test at the end of the school year towards June, the MAFMAR[21] test.
2. For the purpose of the triangulation of the information that will be collected from the student at the two points of time, the respondents will be instructed to note in the mapping test at the two dates of administration an identical code, when the deciphering of this code is known only to the respondent himself.
3. The importance of the collection of the achievements of the students who did not participate in the intervention program and who continued to study as regular will be for the purpose of the comparison between their achievement and those of the group of students who participated in the intervention program, while examining the changes that occurred.
4. The information above will be collected anonymously. In addition, the participants will be instructed not to note identifying information of any other person.
5. All information related to the research will be kept in computer files to which the access is limited, through a password, only to me and my work adviser. It will further be clarified that the publication of the research findings will be carried out in a way that will enable the identification of the respondents and/or their school. The collection of this information was authorized by the Chief Scientist's Bureau

[21] The word is an acronym meaning Supervisor Coordinating the Subject of Mathematics.

in the Ministry of Education, subject to the conditions presented in the approval document.

For every question that pertains to the research study it is possible to contact me at the phone number: 052-658-3456 or by email: maha.abu.hatoum@gmail.com

I will be grateful if you agree to have your school participate in the research study and fill out the informed consent form, sign it, and bring it to the school as soon as possible.

Sincerely,

Maha Abu Hatoum

Informed Consent Form for the Collection of the Non-Identifying Information

Dear Maha Abu Hatoum,

Since you are carrying out the research on the topic of "influence of Online Collaborative Discourse as Formative Feedback for the Improvement of the Achievements of Students with Difficulties in Mathematics in the Seventh Grade in the Northern Urban Arab Sector" and since you asked for my consent as a principal for the collection of non-identifying data from the ____________________ school (please write the name fully) from the seventh grade.

Therefore, after in your letter:

- You explained the objectives of the research study and the topics that will be examined in its framework,
- You described all the actions, its content, in which the school students will participate, seventh grade in the framework of this research study,
- You committed that the collection of the information will be carried out in a way that will not identify the respondents or any other person,
- You obligated that the research findings will be published in a way that will not allow the identification of the respondents,

I confirm that I have understood all that was said above and hereby give my consent for the collection of the above information by you from the school.

In witness whereof I have signed:

________________ ______________________________ ____________________

Date Name of the School Principal Signature

Appendix Number 12: Consent Form for the Parents

2020-2021 Academic Year

Date: ______________

Dear Parents,

Re: Research on the Topic of the Influence of the Online Collaborative Discourse as Formative Feedback for the Improvement of the Achievements of Students with Difficulties in Mathematics in the Seventh Grade in the Urban Arab Sector in the North

The management of the school where your son/daughter is studying was asked to distribute this letter to the parents of all the seventh-grade students. In the framework of my doctoral studies in the School of Education at Adam Mickiewicz University in Poznan, I am carrying out a research on this topic under the guidance of Professor Hanna Krauze-Sikorska and Dr. Tomasz Przybyla. The objective of the research study is the improvement of the achievements of the students who have difficulties in the subject of mathematics. Mathematics is one of the central subjects studied in the education system in Israel, at all age groups. The great importance of the subject is that it is a basis for quantitative logical thinking and an infrastructure for the studies of science and technology. It is known today that most of the students have difficulties with understanding and internalizing processes in the context of numbers, concepts, and use of symbols, thus increasing the level of difficulty and developing anxiety among many students in all that pertains to mathematics in particular and the subjects related to mathematics in general.

For the purpose of the research study, the following actions are planned:

1. With the consent of the school principal, for the purpose of the research study the intervention program will be deployed. It will be held once a week during one academic hour (individualized study hour) in the framework of the study day. The program will be carried out during the 2020-2021 academic year and will be held in a group framework. Every group will consist of five students.
   During the program the mathematics teachers will be trained to teach the students the study materials in mathematics in an online manner, through the holding of an online mathematical discourse in which they can identify the students' difficulties – weaknesses, through the use of authentic mathematical tasks that were constructed especially for the purpose of this research study. These group meetings will be performed in parallel to the study material that is learned in the homeroom class, so as to help the learners bridge the content gaps and develop strategies of learning and creative thinking in a collaborative manner. The program will be performed through the use of the individualized learning hour that is implemented in the schools and will continue during the week in an online fashion. This program will be implemented by the mathematics teachers of the school where the student learns, who will receive from me as the researcher of the appropriate instruction.

2. Before the deployment of the intervention program and at its end, all the participating students will be asked to take a class mapping test, which will enable the learner's achievements in the topic of mathematics at the beginning and at the end of the year to be determined. To triangulate the information that will be collected from this student at two points of time, the respondents will instructed to note in the mapping test at the two dates it is held an identical code, which only the respondent himself will know to decipher. In addition, the research includes questionnaires that examine the learning strategies, the self-efficacy, and the degree of motivation. The filling out of the questionnaires is planned to take up to 45 minutes. The filling out of the questionnaires will be anonymous, when for this purpose the students will be instructed not to note on the questionnaire, they will fill out their name or any other identifying information about themselves. The above information will be collected anonymously. In addition, the respondents will be instructed not to note the identifying details of any other person.

3. All the information related to the aforementioned research study will be kept in computer files, when the access to these files will be restricted, using a password, only to me and the instructor of my research. It will further be clarified that the publication of the research findings will be held so that it will not be possible to identify the respondents. The collection of this information is permitted by the Office of the Chief Scientist in the Ministry of Education, subject to the conditions presented in the permission document from it (a copy of the permission document was given to the school management), and it is possible to view it upon request.

It is important to note that the students who will not participate in the research study, whether because their parents did not consent to their participation or because they themselves refused to participate, will not be harmed in any way. The right of the respondents whose parents consented to their participation in the research study to decide themselves to leave the intervention program and the actions of the collection of information in the middle will be respected, and they will not come to any harm because of this. These rights will be clarified to the students themselves, in clear language, before the beginning of the actual performance of the research study.

For any question pertaining to the research, I can be reached by phone at: 052-658-3456 or by email at: maha.abu.hatoum@gmail.com.

If you agree for your son/daughter to participate in the discussed research, I will thank you to fill out the attached letter of consent, to sign it, and to return it to the school as soon as possible.

Thank you.

Maha Abu Hatoum

Letter of Consent for the Collection of Non-Identifying Information

Dear Maha Abu Hatoum,

Since you are conducting a research study on the topic of "The Influence of Online Collaborative Discourse as Formative Feedback for the Improvement of the Achievements of Students with Difficulties in Mathematics in the Seventh Grade in the Urban Arab Sector in the North" and since you asked for my permission to collect non-identifying information from my son/daughter _________________________ (please fill out the complete name) from grade _____________________ (please fill out the grade designation, for instance grade 7 A) who is studying in the school _________________.

Therefore, after in your letter you:

- Explained about the research objectives and topics that will be examined in the framework of the research study,
- Described all the actions, with the content, in which my son/daughter will participate in the framework of the research study,
- Obligated that the collection of the information will be carried out in a way that does not identify the respondents or any other person, and
- Obligated that the research findings will be published in a way that will not allow the respondents to be identified

I hereby confirm that I have understood the above statements and grant my consent for this information to be collected by you from my son/daughter.

In witness whereof I affix my signature:

_________________ _________________ _________________

Date Parent name Signature