Topics on Topological robotics

On topological complexity of Eilenberg-MacLane spaces and effective topological complexity.

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Abstract

In this dissertation we work on several problems concerning the relationship between topological complexity and sectional category, and groups. The work presented here can be divided in two main branches.

In the first part of the thesis, we investigate topics related with the description of the topological complexity of Eilenberg-MacLane spaces. First we develop the notion of sectional category of group monomorphisms, as a more general framework of study and which contains the original problem, and we provide a generalization of a characterization from Farber, Grant, Lupton and Oprea of TC of a group in terms equivariant maps to the classifying space of full families of subgroups.

We also develop a relative canonical class in this setting, and study its properties. Additionally, we introduce the notion of Adamson cohomology theory into the study of $\mathrm{secat}(H \hookrightarrow G)$. We will proceed as well to generalize the notion of essential cohomology classes to arbitrary group monomorphisms, and to build a more general version of the Farber-Mescher spectral sequence in order to get a new bound for $\mathrm{secat}(H \hookrightarrow G)$, which we will specialize to obtain new lower bounds of sequential and fiberwise TC. To finish this first part, we provide a characterization of TC of a group G in terms of the $\mathcal{A}$-genus in the sense of Clapp and Puppe.

In the second part, we switch our point of view, and consider, instead of $K(G,1)$-spaces, actions of groups over spaces, and so we investigate some properties of the effective topological complexity of Błaszczyk and Kaluba. First we develop a notion of effective LS-category, and then we observe the relationship between the effective topological complexity and category and the orbit map with respect to the action in some situations, giving several computations and examples. We will finish by providing cohomological arguments to determine cases in which such effective TC is non-zero in dimension two.

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