Review of Arturo Espinosa Baro's PhD thesis

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1 Summary

Topological complexity was defined around the turn of the century by Michael Farber. For any space X, the number $\mathsf{TC}(X)$ depends only on the homotopy type of X, and quantifies the complexity of the task of navigation in X from a topological standpoint. Viewing X as the configuration space of a robotic mechanism or other mechanical system, knowledge of $\mathsf{TC}(X)$ may have practical implications for the motion planning problem for that system. Due to this, and due to its close connection with more classically studied invariants such as the Lusternik–Schnirelmann category $\mathsf{cat}(X)$, topological complexity has attracted a significant amount of attention from the homotopy theory community.

The definition of $\mathsf{TC}(X)$ is an instance of a more general concept called *sectional category.* Given a fibration $p : E \to B$, its sectional category $\mathsf{secat}(p)$ is the minimal k such that B admits an open cover $\{U_0, \ldots, U_k\}$, on each set of which p admits a local section. Replacing local sections by local homotopy sections, one arrives at the definition of $\mathsf{secat}(p)$ of an arbitrary map. (In fact, the notion is sufficiently homotopy invariant that we could also define the sectional category of a map to be the sectional category of a fibrational substitute.) Then $\mathsf{TC}(X)$ is defined to be the sectional category of the free path fibration

$$\pi : PX \to X \times X, \qquad \gamma \mapsto \big(\gamma(0), \gamma(1)\big)$$

which evaluates a path at its initial and final points. The connection to navigation in X becomes clear: local sections of π correspond to "motion planners" which determine a path from A to B for pairs (A, B) in a region of $X \times X$. Since π is a fibrational substitute of the diagonal map $\Delta : X \to X \times X$, we could equally define $\mathsf{TC}(X) := \mathsf{secat}(\Delta : X \to X \times X)$.

One of the main motivating questions in this area concerns the influence of the fundamental group on topological complexity. Given a discrete group G there exists a connected complex X with $\pi_1(X) \cong G$ and all higher homotopy groups trivial; such a space is called a $K(G, 1)$-space, and is unique up to homotopy type. We may therefore define $\mathsf{TC}(G) := \mathsf{TC}(X)$, where X is any $K(G, 1)$-space. A fundamental question posed by Farber, which remains unanswered despite having inspired a great deal of interesting research, is to describe $\mathsf{TC}(G)$ in purely algebraic terms. The corresponding question for Lusternik–Schnirelmann category is answered by the Eilenberg–Ganea theorem: if $\mathsf{cat}(G)$ is similarly defined, it agrees with $\mathsf{cd}(G)$, the cohomological dimension of the group G, whenever $\mathsf{cd}(G) \geq 3$ (and in fact this last restriction can be removed after work of Stallings, Swan and Dranishnikov–Rudyak).

One can observe that $\mathsf{TC}(G) = \mathsf{secat}(\Delta : X \to X \times X)$ is the sectional category of a map between aspherical spaces, whose homotopy class is then determined by the map on fundamental groups (in this case the diagonal homomorphism $d : G \hookrightarrow G \times G$). The jumping off point of this thesis is that the important problem of Farber from the last paragraph can be generalised. Given a homomorphism of discrete groups $\phi : G \to H$, we may define $\mathsf{secat}(\phi) := \mathsf{secat}(f : X \to Y)$ where X is any $K(G, 1)$-space, Y is any $K(H, 1)$-space, and f is any map inducing ϕ on fundamental groups. One can then ask for an algebraic description of $\mathsf{secat}(\phi)$. Farber's problem is the case $\phi = d : G \hookrightarrow G \times G$. Having this generalisation in mind can prove a useful sanity check when pondering potential conjectural descriptions of $\mathsf{TC}(G)$.

Another topic considered in the thesis is effective topological complexity. This is a variant of topological complexity which takes symmetries of the configuration space into account. There are several such variants, but the effective version is distinguished as the only variant which makes use of the symmetries to reduce the complexity of the motion planning problem. Given a group G (now regarded as a group of symmetries, as opposed to the fundamental group of a space) and a space X with G-action, the effective topological complexity $\mathsf{TC}^{G,\infty}(X)$ is defined similarly as $\mathsf{TC}(X)$, except that paths are now allowed finitely many "break points" where continuity is broken to jump to another point in the same G-orbit. It follows that $\mathsf{TC}^{G,\infty}(X)$ is bounded above by $\mathsf{TC}(X)$. Beyond computations made by Błaszczyk and Kaluba for $\mathbb{Z}_p$-spheres, there are relatively few calculations of effective topological complexity in the literature.

We now briefly summarize the contents of the thesis, which after the introductory Chapter 1 is split into 3 parts. **Part I** is preliminary, and contains a review of the material from homological algebra and the theory of sectional category which will be needed for the rest of the thesis. **Part II** concerns sectional category and topological complexity of $K(G,1)$-spaces, and consists of 3 chapters. **Chapter 4** concerns the sectional category of subgroup inclusions $H \hookrightarrow G$. A characterisation is given in terms of the existence of equivariant maps between classifying spaces, generalising a result of Farber–Grant–Lupton–Oprea. Then the Berstein–Schwarz class of G relative to H is introduced, which is a sort of universal cohomology class in $H^1(G; I(G/H))$ which simultaneously generalises the Berstein–Schwarz class of a group and the Farber–Costa canonical class for topological complexity, and is the main technical tool of the chapter. The main result here is that maximality of the sectional category, in the sense that $\mathsf{secat}(H \hookrightarrow G) = \mathsf{cd}(G)$, is detected by vanishing or otherwise of the appropriate power of the relative Berstein–Schwarz class. The chapter also contains a discussion of Adamson cohomology and its relationship to sectional category and Bredon cohomology. Most of the results of this chapter have been published in a joint article with Błaszczyk and Carrasquel-Vera in the Journal of Pure and Applied Algebra.

Chapter 5 is titled "Lower bounds of sectional category of subgroup inclusions", and picks up where the last chapter left off. The main tool is a generalisation of a spectral sequence due to Farber and Mescher, which can be used in favourable cases to detect when a cohomology class is *essential*, which means roughly that its weight when used to bound $\mathsf{secat}(H \hookrightarrow G)$ from below is equal to its dimension. The main result which can be derived from this spectral sequence is that $\mathsf{secat}(H \hookrightarrow G) \geq \mathsf{cd}(G) - \kappa_{G,H}$, where $\kappa_{G,H}$ is defined to be the supremum of $\mathsf{cd}(H_x)$ where $H_x = H \cap xHx^{-1}$ and x ranges over $G \setminus H$. Several applications to sequential topological complexity of $K(G,1)$-spaces and to parametrized topological complexity of group epimorphisms are given. The results of this chapter appear in a joint article with Farber, Mescher and Oprea which has been submitted for publication.

Chapter 6 presents an original interpretation of $\mathsf{secat}(H \hookrightarrow G)$ and $\mathsf{TC}(G)$ as an equivariant $\mathcal{A}$-genus. While still topological in nature, this does allow the author to refine some of the bounds derived from the characterisation in terms of equivariant maps between classifying spaces. A single-author article covering this material is in the latter stages of preparation.

Part III consists of a single chapter, **Chapter 7**, on properties of the effective topological complexity. There is a brief review of variants of topological complexities in the presence of symmetry, and basic properties of $\mathsf{TC}^{G,\infty}$. The main innovation of this chapter is the introduction of a new invariant of G-spaces, their *effective Lusternik–Schnirelmann category*, denoted $\mathsf{cat}^{G,\infty}(X)$. This is related to $\mathsf{TC}^{G,\infty}(X)$, and several conclusions are drawn. It is shown that when the orbit map $\rho : X \to X/G$ admits a strict section, then $\mathsf{cat}^{G,\infty}(X) = \mathsf{cat}(X/G)$ and $\mathsf{TC}^{G,\infty}(X) = \mathsf{TC}(X/G)$. When the orbit map ρ is a fibration, the above equalities become inequalities $\leq$, but one can also conclude that only one break point is required to obtain the minimum. Several examples are explored, focussing on the cases when G is a Lie group. The results of this chapter appear in a joint work with Błaszczyk and Viruel, which has been accepted for publication in the Proceedings of the Royal Society of Edinburgh, Section A: Mathematics.

The thesis concludes with an extensive bibliography, and a useful list of symbols and alphabetical index.

2 Assessment

I consider this thesis to be at the level of a good PhD thesis from a top 20 university in the UK. The original results, while not unexpected, still require a high level of technical mastery, and cover a broad range of techniques. The writing is excellent in general, despite the length of the attached list of corrections (which anyway is commensurate with the length of the thesis). There are places where I felt the prose could have been more brief, although I enjoyed the poetic style! My only small criticism was that the author seems reluctant to use categorical techniques. In particular, Lemmas 4.2.6 and 4.2.7 seem to say respectively that induction is left adjoint to restriction, and as a result restriction preserves injectives; Theorem 4.2.14 rests mainly on the fact that restriction is left adjoint to taking fixed points. In my opinion this viewpoint is not only useful for shortening proofs, but also for understanding the heart of the matter.

3 Conclusion

The thesis represents a significant body of original work, carried out both independently and jointly in collaboration with other researchers. It presents in a coherent narrative the results of 4 distinct scientific articles, which are either published, submitted for publication, or in the latter stages of preparation. There is no summary of the author's contributions to the joint works, although Chapter 6 is solely authored and demonstrates the candidates ability to work independently. The material presented clearly demonstrates the candidates breadth and depth of technical knowledge in the subject. The language criteria have been met.

In conclusion, my opinion is that the thesis meets all of the conditions of Article 187 of the Act of July 20, 2018, *Law on Higher Education and Science*, and justifies awarding the candidate a doctoral degree in the discipline of mathematics.

4 Corrections

page	location	correction
14	line -1	Reverse the order from $h_i \circ d'_{i+1}$ to $d'_{i+1} \circ h_i$
15	line -8	It's the other way around: covariant in the second, contravariant in the first
16	line 11	"there exist an unique R-linear map": You definitely don't want to require uniqueness here. Think about the case when P is free: there are potentially many ways to lift a basis to M
16	line 15	"$\alpha M \to I$" to "$\alpha : M \to I$" You don't want uniqueness here either: think about the case $M = 0$
16	line 16	It should be $\lambda : N \to I$ (and reverse the dotted arrow in the diagram)
17	line 18	"giving module" to "given module"
17	line -2	Missing) from the cohomology group
18	2.1.11(f)	The notation suggests you want to consider arbitrary collections, but inside the $\{\dots\}$ you have a finite collection
19	line 9	"ring group" to "group ring"
20	line 19	"G-action. and" to "G-action, and"
20	line -8	"$n \in \mathbb{Z}$": Or do you want $n \geq 0$?
20	line -5	The second $\mathcal{P}_\bullet$ in the display should be $\mathbb{Z}$
20	line -1	"second coordinate" to "first coordinate"
22	line -13	"such space $K(G, 1)$... and it is sometimes also denoted by BG": here you implicitly assume G is discrete. $K(G, 1)$ and BG can be very different for topological groups!
23	line -1	"For each $n > 0$" should be "For each $n \geq 0$"
27	2.2.15(6)	This is not quite true as written, as $u \cup v$ and $v \cup u$ live in potentially different groups. The correct statement involves the isomorphism $\tau : M \otimes N \to N \otimes M$
28	line -1	In the diagram U should be U_i. You should say what ρ is

29	line 6	"Every G-space" to "Every free G-space"
29	line 11	"$(x, y) \in X \times F$" to "$(x, y) \in P \times F$"
29	line 18	"Thye" to "The"
30	line -17	**Question:** "$E_{\mathcal{F}}G$ is also a model for $E_{\mathcal{F}\cap H}H$": can this be seen directly from the universal property, or do you need the characterisation in terms of fixed point sets?
31	line 12—	"$\mathrm{Or}_{\mathcal{F}}$" to "$\mathrm{Or}_{\mathcal{F}}G$"
32	(2.3.5)	Some notation is used here but not explained: presumably I_n indexes the (equivariant) n-cells, and $H_i \leq H$ is the isotropy subgroup of a cell e_i of X^H?
32	line -12	The sentence "Recall that $\langle H \rangle$ denotes the... of G containing H" seems misplaced
33	2.3.12	The term "semi-full" ought to be defined
33	line 18	"$(s, 1-s)$ of $(-s, s-1)$" to "$(s, 1-s)$ or $(-s, s-1)$"
35	line -10	"with $j'(i(x)) + d(E) \in E'$": Presumably this should say "with $j'(i(x)) = j(x) + d(E) \in E'$"
36	2.5.1	I've never heard the term "continuous group" used as a synonym for Lie group, and I would have thought it could be misleading
37	line -9	"if the subgroup" to "is the subgroup"
37	line -7	"simplectic" to "symplectic"
37	line -5	It looks like $\mathbb{F}^n$ should be $\mathbb{F}^{2n}$ (and twice more below)
38	line 7	"(n)" to "$O(n)$"
40	line -2	"authomatic" to "automatic"
43	line 3	Strictly speaking the composition $\pi \circ s_i$ equals the inclusion map $U_i \hookrightarrow X \times X$
44	line 2	The sentence "Originally defined... manifold." is not a sentence
44	line 3	"as a mean" to "as a means"
44	line -2	"Under such assumption, the maps s_i are (continuous) local sections": do you mean that they are homotopic to strict sections?
44	line -1	Again, the inclusion $U_i \hookrightarrow Y$ and not id_{U_i}
45	line 2	"lenguage" to "language"
45	line 9	"initial point" to "final point"
46	3.2.8(e)	You don't need reduced cohomology classes here. Perhaps you're thinking of the cup-length lower bound for LS-category, where you do need reduced since $\tilde{H}^*(X) = \ker(H^*(X) \to H^*(P_*X) \cong H^*(*))$
46	line -4	"by (d) and (g)": don't you mean by (a) and (d)?
46	3.2.9	I don't think paracompactness is necessary here
47	line 12	Again, you don't need reduced when talking about zero-divisors (note that $1 \in H^0(X \times X; R)$ is not a zero-divisor)
49	line 10	The class v lives in degree $2n$, not $4n$

50	Section 3.3	Throughout the section you seem to switch back and forth between using G or π to denote an arbitrary discrete group ("abstract" means "discrete", right?)
50	line -12	"excelent" to "excellent"
52	line -5	$1 = n = k$?
52	line -4	"lector" means "reader"?
58	line 4	"equipped with a canonical G-action": Surely "**the** canonical G-action"?
58	line 14	Misplaced bracket in the definition of σ
59	line -15	"Due to homotopy invariance of sectional category, $\mathsf{secat}(H \hookrightarrow G)$ depends only on the conjugacy class of H in G": Right, but isn't this more to do with change of base points?
60	4.1.3(b)	Remove "reduced" and the tildes. "$u_1 \cup u_2 \cdots$" to "$u_1 \cup u_2 \cup \cdots$"
62	(4.1.3)	Change d_p to ∂_p (as it was denoted on p.24)
63	line 1	"so we can rewrite 4.1.3 as" to "so we can rewrite the left-hand side of 4.1.3 as"
63	line 2	"d_p" to "∂_p". "x_{p-1}" to "x_p"
63	line 7	"it follows that this coincides with 4.1.3" to "it follows that this coincides with the right-hand side of 4.1.3"
63	line 9	**Question:** Why does this diagram commute?
66	line 10	Should it not be $\mathbb{Z}[G/H] \otimes I^k \cong (\mathbb{Z} \otimes I^k) \oplus (I \otimes I^k)$? The point being that the extension of G-modules $0 \to I \to \mathbb{Z}[G/H] \to \mathbb{Z} \to 0$ splits as an extension of H-modules
67	line -8	"Let $\mathbb{Z}[G] \otimes M$ be equipped" to "Let $\mathbb{Z}[G/H] \otimes M$ be equipped"
67	line -3	"$(H \otimes x)$" to "$(gH \otimes x)$"
68	line 1	"$(gH \otimes m)$" to "$(gH \otimes x)$"
69	line 1	"previous two corollaries" to "previous two lemmas"
74	line -12	Should $H_n(E_{\langle H \rangle}(G)_{n+1}, E_{\langle H \rangle}(G)_n)$ be $H_n(E_{\langle H \rangle}(G)_n, E_{\langle H \rangle}(G)_{n-1})$?
74	line -8	Should the $\mathbb{Z}[(G/H)^n]$ be $\mathbb{Z}[(G/H)^{n+1}]$ (and several instances below)?
76	line -1	"fruithful" to "fruitful"
77	line -7	"with basis in one-to-one correspondence with the generators of H_3": this doesn't sound right, as I think you want F to have rank 2 but the given presentation of H_3 has 3 generators
78	line -10	"how good" to "how well"
78	line -8	"as discussed at the end of section 3": I couldn't find such a discussion, could you give a more precise reference?
79	line 5	"analogues" to "analogous"
79	line -8	"$\mathrm{Or}_{\langle F \rangle} G$" to "$\mathrm{Or}_{\langle H \rangle} G$"
82	line 17	"proof the equalities" to "prove the equalities"
82	line -13	Elements of $\delta(G)$ are of the form (a^m, b^m), rather than (a^m, b^n)

83	line -3	“action action”
84	line 1	“which subsumes the” what?
84	line -12	Below you also use ρ to denote the restriction of $\mathbb{Z}[G] \to \mathbb{Z}[G/H]$ to $K \to I$, which seems worth mentioning here
85	5.1.2	The proof seems to involve unnecessary steps: $\delta(u) = \delta(1 \cup u) = \delta(1) \cup u = \omega \cup u$
86	line 5	The final S in the display should be A
86	line -12	The first $(xH \cdot f)$ in the display should be $(xH \otimes f)$
86	line -10	Both instances of $F(x \otimes f)$ on the final line of the display should be $F(xH \otimes f)$
86	line -9—	All the M’s until the end of the page should be A’s
88	line -10	“$(x)(y)$” to “$(x \otimes y)$”
91	line -12	“Remark 5.1.8” to “Theorem 5.1.7”
93	5.2.1	**Question:** What is the precise relationship between i_0 and the Bockstein δ of Proposition 5.1.3?
98	line -11	“It is interesting to remark as well that if $H^{\mathrm{cd}(N)}(N, \mathbb{Z}[N])$ is free abelian”: As is the case if N is a Poincaré duality group
100	5.4.4	Having made this definition it might be worth observing that if $H \leq G$ is malnormal then $\mathsf{secat}(H \hookrightarrow G) = \mathsf{cd}(G)$, as follows directly from your Theorem 5.3.3 and Corollary 5.3.5
106	line 9	Two instances of X_i should be U_i
107	line 7	“an analogous” to “an analogue”
107	line 10	“imputs” to “inputs”
107	line 13	Did you really mean to exclude $p_X^*(\alpha_0)$ from the product? If so, it’s unclear what role A_0 and α_0 are playing in the definition
107	line 17	“$\cdots \leq \sum_{i=1}^{k} l(A_i)$”: Again, are you sure the sum starts at $i = 1$ and not $i = 0$? And then, to get $l(X) \leq \mathcal{A}$-genus(X) don’t you need $l(A_i) = 1$ for $A_i \in \mathcal{A}$? Every way I try to understand the definition of length given above, it seems to me that $l(A) = 0$ for $A \in \mathcal{A}$. This whole passage could be made clearer
108	6.2.2	Add a full stop
109	line 2	“In this caseFor the case”
109	line 5	“π^k” to “π^r”
111	line 5	“there exists an n-dimensional model of the space $E_{\langle K \rangle}(G)$”: yes, for $n = \mathsf{cd}_{\langle K \rangle}(G)$
113	6.2.9	There is of course a much easier proof: K is a retract of $H \rtimes K$, so $K(H \rtimes K, 1)$ dominates $K(K, 1)$
113	line -1	Finish with a full stop. In fact, is this line of reasoning finished?
115	line 5	“thoroughful” is not a word
115	6.2.13	$\mathcal{F}in$ to me would suggest the family of finite subgroups $H \leq G$, whereas I think you mean the collection of orbits G/H

121	line 14	"can defines" to "can define"
121	line -9	"Althoug" to "Although"
123	7.1.6(b)	"nrmal" to "normal"
124	7.1.7(c)	I think you want the sectional category of the map $(\rho_X \times \rho_X) \circ \pi : PX \to X/G \times X/G$ here
124	7.2	Remove full stop from section heading
124	line -17	"As such, We" to "As such, we"
124	line -15	"on a pointed CW complex": As far as I can tell the base point doesn't play a role until Section 7.4, so it seems weird to insist on it here
126	7.2.4	In the second bullet point, $\mathsf{TC}^{\mathbb{Z},\infty}(S^n)$ should be $\mathsf{TC}^{\mathbb{Z}_p,\infty}(S^n)$
127	line 1	"In[23]" to "In [23]"
127	line -14	"TC may fall non-trivially below dimension 2": do you mean "above stage 2"?
127	line -9	"which coincides, at dimension two, with $\mathsf{TC}^G_{\text{effv}}$": The word dimension seems wrong here, but I don't know what to call it—"sequential index"?
127	line -4	"defined by $\iota_0(x) = c_x \in PX$": You should probably define $\mathcal{P}_0(X)$ as X first
128	line 4	"The case $k = 0$ is straightforward": However, isn't the argument exactly as given 6 lines below ("Now, consider... and, therefore, closed")? I suggest giving the argument for $k = 0$ at the start of the proof
128	lines 10, 11	"Given that X is taken to be a Hausdorff space... we assumed X to be compact...": I don't see the compact Hausdorff assumption made anywhere. Maybe it should be in the statement of Lemma 7.3.1
128	line -7	I couldn't grasp the meaning of this diagram. Is the line over $\mathcal{P}_{n-1}(X)$ supposed to denote closure? If so, closure where (given that it's closed in $\mathcal{P}_n(X)$)? If not, what else?
129	line 6	"sequence of integers" is incorrect. "that that"
129	line -13	"J is equipped with the discrete topology which...": seems to show that $S(F)$ is not *sequentially compact*. Do you want some metrizability condition somewhere?
130	line 9	"The map $\overline{s_k}$ restricted to U_i": But U_i is not a subset of the domain of $\overline{s_k}$. I think you mean the composition $\overline{s_k} \circ s_i \circ (\rho_X \times \rho_X)$. This could be explained better
130	line 13	"$\pi_k \circ \xi_i$" to "$\pi_{k+2} \circ \xi_i$"
131	line 4	Don't you mean "with the property $\mathcal{TC}(X) \leq \mathsf{TC}(X)$"?
131	line 14	"property (1)" to "property (a)"
132	line 7	"(3)" to "(c)"
132	lines 9, 10	By q_k you seem to mean $q_k \times q_k$, or maybe $q_k(X \times X)$
132	line -14	"Theorem 3.2.8(4)" to "Theorem 3.2.8(g)"
132	lines -10,-9	Should be Theorem 7.4.2 and Proposition 7.2.4

132	7.4.3	With the exception of $\mathbb{Z}_2$ acting linearly and orientation-reversing with an $(n-1)$-dimensional fixed points set (see your Example 7.5.1)!
133	line 10	It seems odd not to write the definition of $H(x,t)$ (but only $H(x,0)$ and $H(x,1)$)
133	line 11	"a a"
134	7.5.2	This follows immediately from Theorem 7.4.2 and Proposition 7.4.4
135	7.5.3	**Question:** Could you explain this example in a little more detail? Presumably Σ means *unreduced* suspension? Why is the action linear?
136	line -4	"$U_i = (s \times s)^{-1}$" to "$U_i = (s \times s)^{-1}(V_i)$"
137	line 1	"Let explore" to "Let us explore"
138	line 3	"Let G any" to "Let G be any"
138	line 6	"Example 7.6.2(3)" to "Example 7.6.2(5)"
139	line 2	The target of $\overline{\rho_X}$ should be $P(X/G)$ rather than $X/G \times X/G$
141	line -2	"≤ 2" to "≤ 23"
144	7.6.8(a)	$\mathsf{TC}(M)$ is a stronger upper bound than $2\dim(M)$, so should be included in the statement
145	line 11	Define P somewhere
145	7.6.10	The argument seems to assume that X, or at least X/G, is connected
146	7.6.12(1)	Should "non-nilpotent" be "nilpotent" here?
146	7.6.13	Remark finishes mid-sentence. I assume the ending was "so $\mathsf{cat}^{G,\infty}(M) = \mathsf{cat}(M/G) = 0$, while $\mathsf{cat}(M) = \dim(M)$"
147	line -1	The display should start with "$\iota \circ f(\gamma_1, \gamma_2) = \cdots$"
148	line -6	Placement of full stop in display
149	line -7	Delete "$G' = \{g_1, \ldots, g_{n-1}\}$ a subgroup of order $n-1$ and"
150	line -4	"However, $H^{2n}(X \times X) \neq 0$": What is n here? If the claim is that $\mathsf{cd}(X \times X) = 2\mathsf{cd}(X)$, then this requires some justification
151	7.7.5	**Question:** Isn't it always the case that $\mathsf{TC}^{G,2}(X) \geq \mathrm{nilker}(H^*(X \times X; R) \to H^*(\neg X; R))$, and that the freeness hypothesis is only required to get $\mathsf{TC}^{G,\infty}(X) = \mathsf{TC}^{G,2}(X)$? In which case Corollary 7.4.4 feels more like an application of this in a specific instance, rather than a generalisation
152	7.7.7	Are you assuming n even?
153	[5]	I believe the last author goes by Sánchez Saldaña
154	[15]	To appear in Proc. Roy. Soc. Edinburgh Sect. A
157	[68]	Title should be "Equivariant topological complexities"
157	[70]	To appear in Israel J. Math

Signed by:

Mark Grant

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Initial

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