Author: Janik Kruse

Title: Araki–Haag Detectors, Mourre Theory, and the Problem of Asymptotic Completeness in Algebraic Quantum Field Theory

Summary

Scattering theory describes the asymptotic evolution of systems of interacting particles. A key concept in this area is asymptotic completeness, which asserts that every state can be decomposed into bound and scattering states. While asymptotic completeness is well-understood in non-relativistic quantum mechanics, it remains an open and challenging problem in local relativistic quantum field theory (QFT). In this dissertation, we adopt the axiomatic (model-independent) framework of algebraic QFT and the Haag–Ruelle scattering theory to investigate the problem of asymptotic completeness.

Modern proofs of asymptotic completeness in quantum mechanics rely on a Mourre estimate, propagation estimates, and the convergence of asymptotic observables, such as the asymptotic velocity. In QFT, Araki–Haag detectors, first introduced by Araki and Haag (1967) and later further developed by Buchholz (1990), are natural asymptotic observables. Controlling their convergence is an important prerequisite for asymptotic completeness in QFT.

We prove the convergence of Araki–Haag detectors on states of bounded energy that belong to the absolutely continuous part of the energy-momentum spectrum below the three-particle threshold. This result brings us closer to two-particle asymptotic completeness than the earlier work of Dybalski and Gérard (2014), who analysed the convergence of products of detectors with distinct velocities. Our proof shares similarities with proofs of the existence and completeness of wave operators in quantum mechanics. Notably, we apply Mourre's conjugate operator method to derive a local decay estimate, which marks the first application of Mourre's method in the relativistic QFT framework.

The conjugate operator method is a mathematical technique from spectral theory, which is based on a strictly positive commutator estimate. This method has been crucial to advance the spectral and scattering theory of quantum-mechanical many-body systems. Apart from proving the convergence of Araki–Haag detectors, as mentioned above, we also apply Mourre's method to derive a limiting absorption principle for the energy-momentum operators in relativistic QFT. The limiting absorption principle allows us to reproduce results on spectral properties of the energy-momentum operators, such as the absence of singular continuous spectrum.

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