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Dear Madam/Sir,

I would like to report on the doctoral thesis of Janik Kruse "Araki–Haag Detectors, Mourre Theory and the Problem of Asymptotic Completeness in Algebraic Quantum Field Theory."

Janik Kruse's PhD dissertation addresses the longstanding and fundamental problem of asymptotic completeness in local relativistic quantum field theory (QFT), within the rigorous and model-independent framework of algebraic quantum field theory (AQFT). This concept, central to scattering theory, asserts that all physical states can be understood as combinations of bound states (localized systems like atoms) and scattering states (freely moving particles after interactions). While well established in quantum mechanics, asymptotic completeness remains an open question in QFT due to issues such as particle creation and the rich superselection structure of states. Unlike quantum mechanics, QFT allows for processes where particles are created or annihilated during interactions, so the particle number does not remain fixed. The existence of superselection sectors means that in QFT there are many inequivalent representations of observables, and classification of such representations, especially in the presence of long-range interactions is a difficult open problem in its own right.

Asymptotic completeness has been established in the low-coupling regime of the $P(\phi)_2$ theory at the level of two and three particles. Recently, asymptotically complete integrable models have also been constructed in low dimensions, where the scattering operator is specified as part of the input data defining the model. Asymptotic completeness has also been established for certain wedge-local models. Crucially, some counterexamples to asymptotic completeness exist within the framework of axiomatic QFT, as there exist models with states that do not allow for particle interpretation. Part of the challenge in proving asymptotic completeness is to formulate additional axioms that would exclude such models and would be physically motivated.

The situation in non-relativistic QFT looks much better, though. Asymptotic completeness has been proven for Pauli–Fierz Hamiltonians by Dereziński and Gérard, for the confined Nelson model by Ammari, for the $P(\phi)_2$ model in finite volume by Dereziński and Gérard, for Rayleigh and Compton scattering by Fröhlich, Griesemer, and Schlein, and for the translation-invariant Nelson model below the three-boson threshold by Dybalski and Møller. In all these models, it was possible to employ techniques similar to those used in quantum mechanics.

The thesis addresses the problem of proving asymptotic completeness within the framework of AQFT. In this axiomatic formulation, a QFT model is specified by a net of algebras interpreted as local algebras of observables. Scattering theory is describe by means of Haag-Ruelle scattering theory. Roughly speaking, one starts defining creation operators one-particle states from the vacuum that have good localisation properties. Then one proves the existence of incoming and outgoing asymptotic scattering states by applying such creation operators to the vacuum and taking the limit of their localisation regions to minus and plus infinity. Using these scattering states one defines incoming and outgoing Hilbert spaces H^{in} and H^{out} as their respective spans. These spaces could in principle be very different and one says that an AQFT model *is asymptotically complete* if they in fact coincide, $H^{in}=H^{out}=H$.

The strategy to address asymptotic completeness in this setting has been proposed by Buchholz and Haag and is based on the concept of particle detectors.

Araki-Haag detectors are asymptotic observables introduced by Araki and Haag in 1967. They act as "particle counters" by measuring deviations from the vacuum state in scattering processes. Their convergence (step 1) is a key prerequisite for proving asymptotic completeness in QFT. After proving their convergence, one needs to verify that these particle detectors are only triggered by scattering states (step 2) and that every non-zero quantum state in H must trigger at least one detector (step 3).

Proving convergence on arbitrary states (not just scattering states) is difficult due to the presence of pathological states and charged particles. Earlier work by Dybalski and Gérard analyzed products of detectors sensitive to particles with distinct velocities but could not address single detectors due to missing low-velocity estimates. Recently, in [Kr24a], Kruse succeeded in proving the convergence of a single Araki–Haag detector on states below the three-particle threshold and also showed that the detectors are triggered only by the scattering states at the two-particle level. This is Theorem 1.3.3 of the thesit. Section 1.3 of the thesis sketches the proof of this theorem. The remaining open problem is showing that every non-zero quantum state in H must trigger at least one detector (step 3), but this could be very hard, due to the existence of superselection sectors.

In section 1.4, the thesis introduces Mourre's theory. This theory, developed in the context of non-relativistic quantum mechanics, provides tools for analyzing the spectrum of self-adjoint operators. It is based on positive commutator estimates and is used to prove the absence of singular continuous spectrum and establish limiting absorption principles. The dissertation applies Mourre theory to relativistic QFT for the first time, using it to prove the convergence of Araki–Haag detectors and analyze the spectral properties of energy-momentum operators.

The thesis is original and uses some novel techniques, notably, it incorporates Mourre's conjugate operator method, marking its first successful application in the relativistic QFT setting. Through this method, Kruse establishes a local decay estimate for quantum field theoretic Hamiltonians, enabling the limiting behavior of detector observables to be controlled. As a secondary contribution, he also proves a limiting absorption principle for energy-momentum operators in AQFT, yielding results about the absence of singular continuous spectrum in certain energy regions.

Overall, these results contribute substantial progress toward the goal of proving asymptotic completeness in QFT, specifically achieving step 2 of a three-part program originally proposed by Haag and Buchholz. Step 3, concerning the accessibility of all states via detectors, remains open but is discussed in detail with plausible strategies involving energy-momentum tensors.

Janik Kruse demonstrates a high level of originality, depth, and technical competence in addressing an extremely challenging problem in mathematical physics. The thesis shows:

- Mastery of advanced mathematical techniques from spectral theory, particularly Mourre theory, and their adaptation to new contexts.
- Deep understanding of the AQFT framework, including its axioms, scattering theory, and spectral structures.
- Technical innovation, notably in proving convergence results under less restrictive conditions and in adapting methods from non-relativistic quantum mechanics to the relativistic setting.
- Clarity and rigor in mathematical exposition, with careful attention to both physical motivation and mathematical detail.

The candidate has published (or prepared for publication) significant parts of the thesis, further indicating the maturity and relevance of the work.

This thesis represents an excellent contribution to the field of mathematical physics and meets, or exceeds, the standards for a PhD dissertation. I strongly recommend the award of the doctoral degree to Janik Kruse.

Yours faithfully,

Kasia Rejzner