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Report on the doctoral thesis
"Detektory Arakiego-Haaga, teoria Mourre'a i problem asymptotycznej zupełności w algebraicznej teorii pól kwantowych"
by Janik Kruse

The thesis is consists of 3 parts. The first is an introduction, which describes background material and the summary of main results. It is followed by reprints of two papers by Janik:

1. Mourre theory and asymptotic observables in local relativistic quantum field theory. Comm. Math. Phys. 405, 236 (2024)

2. Mourre theory and spectral analysis of energy-momentum operators in relativistic quantum field theory, Lett. Math. Phys. 114, 106 (2024)

Both papers use the Haag-Kastler framework of quantum field theory. This means, observables are described by a net of von Neumann algebras indexed by bounded open subsets of the Minkowski space, satisfying the postulates of isotony, locality and the spectral condition. In addition, it is assumed that the energy-momentum spectrum possesses a discrete mass shell. As is well-known, under these conditions one can define the so-called Haag-Ruelle in- and out-states. More precisely, one can define two natural isometric embeddings of the Fock space built on the spectral subspace of the above mentioned discrete shell into the physical space. These two embeddings describe states with a definite incoming and outgoing behavior and can be used to define the scattering operator. An important question arises: are the ranges of these two embeddings the whole physical space? If so, we say that the asymptotic completeness holds.

The paper 1. is devoted to Araki-Haag detectors. An Araki-Haag detector is a certain time-dependent operator, which is supposed to describe measurements in distant past/future. To some extent, the formalism of Araki-Haag detectors can be viewed as an alternative approach to scattering theory in QFT different from the Haag-Ruelle theory. It is a classic result of Araki and Haag that these asymptotic observables converge on incoming and outgoing states of bounded energy. Janik proves that certain Araki-Haag detectors converge (to zero) also on the orthogonal complement of in-/out-states. Unfortunately, the result is restricted to a rather special class of such vectors—in particular, their spectrum should be contained below the three-particle threshold. The paper complements results about this class of detectors obtained by Dybalski-Gérard. The proof uses the famous Mourre theory, developed in the 80's to study scattering and spectral properties of Schrödinger operators. It also makes use of estimates of Buchholz on Fourier transforms of observables in the Haag-Kastler framework. Finally, an important ingredient is partitioning the velocity space in non-overlapping patches, an idea introduced by Klaus Hepp to the Haag-Ruelle theory.

Paper 2 proves a certain form of the Limiting Absorption Principle (LAP) for the energy-momentum operator. The LAP is the statement about the existence of boundary values of the resolvent of a certain operator between appropriate weighted spaces. The weights in the LAP are defined using the generator of Lorentz boosts. The proof uses the Mourre theory, where the "conjugate operator" is the generator of boosts. The LAP is a technical result that often plays an important role in scattering theory. It implies absolute continuity of spectrum.

The thesis shows that its author understands very well the principles and main mathematical problems of quantum field theory in the algebraic formulation. He has a good command of various sophisticated techniques of modern operator theory. In particular, he makes an efficient use of the Mourre theory, one of the jewels of late twentieth century mathematical physics.

The author often stresses that an important source of his motivation is the desire to prove asymptotic completeness in quantum field theory. This is a very important goal. Unfortunately, in my opinion, his results, even if of interest, do not bring us much closer to proving asymptotic completeness. They use rather generic properties of Haag-Kastler nets, which are probably not sufficient to establish such a subtle property. Unfortunately, in my opinion, the situation of asymptotic completeness in relativistic quantum field theory seems very different from that of non-relativistic many-body systems, where asymptotic completeness has been proven under rather general assumptions long time ago.

In the introduction the author presents a panorama of the problem of asymptotic completeness, especially the results about nonrelativistic systems. These topics are well explained, and probably many readers of the thesis will find this part of the thesis quite interesting. Nevertheless, in my opinion the connection of the nonrelativistic properties to the main results of the thesis is not very strong. It seems to me that the hopes expressed by the author about extending asymptotic completeness to the relativistic setting are too optimistic.

Personally, I would prefer to see in the introduction a deeper analysis of some less standard tools used in the main results. For instance, it would be appropriate to sketch a proof of Buchholz's uniform bound on the Fourier transform of local observables, or the propagation of wave packets in under the Klein-Gordon evolution.

Anyway, in this thesis Janik showed his skills of a mathematician who is able to formulate and prove new results of interest for mathematical physics. He published these results in two papers in renowned journals. I am convinced that based on this work Janik Kruse deserves to be awarded the doctoral degree in Mathematics.

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