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Report on the PhD thesis of Janik Kruse
Thesis title: ``Araki-Haag Detectors, Mourre Theory, and the Problem of Asymptotic Completeness in Algebraic Quantum Field Theory´´

The PhD thesis of Mr. Kruse is devoted to a circle of mathematical problems in the scattering theory in quantum field theory, in the mathematically rigorous and model-independent operator algebraic approach. The thesis is cumulative, in that it consists of an introductory (original) part on pages 1-30, and reprinted versions of published research journal articles published by Mr. Kruse (as single author), in Commun. Math. Phys. **405**, 236 (2024), and Lett. Math. Phys **114**, 106 (2024), where detailed descriptions of the results achieved in the thesis project as well as the complete proofs are presented.

As mentioned in the introduction of the thesis, one of the main objectives in the mathematically rigorous analysis of scattering theory in relativistic quantum field theory is to understand if (respectively, precisely under which conditions) asymptotic completeness holds, meaning that the Hilbert space of states decomposes into an orthogonal sum of bound states and scattering states. As is further outlined in the introduction, asymptotic completeness can be proved in non-relativistic quantum mechanics under fairly general and physically relevant conditions. One of the mathematical methods which have been of instrumental importance in the proof of asymptotic completeness in quantum mechanics are a class of operator estimates that have come to be known as Mourre´s commutator estimates or, more generally, Mourre theory.

While the proof of asymptotic completeness in scattering theory in quantum mechanics under general conditions (which also cover physically relevant cases) is quite demanding and technically intricate, the problem is considerably more complicated in relativistic quantum field theory when attempted at the level of full mathematical rigour. This is even more so the case in physical spacetime dimension since interacting quantum field theories that potentially admit bound states have only been rigorously constructed in lower spacetime dimensions (1+1 and 1+2), so there is also a sparcity of concrete mathematical models (admitting bound states) on which the scattering behaviour could be tested. Consequently, rigorous scattering theory in relativistic quantum field theory takes an axiomatic approach as the starting point (basically, by neccessity). This led to the Haag-Ruelle particle scattering theory. A refined

approach was given in the form of Araki-Haag detector models that allow a closer connection to the local structure of algebraic quantum field theory. These methods, particularly with regard to spectral analysis, were further developed by Buchholz, and by Dybalski.

The PhD thesis of Mr. Kruse is set in this framework. A novelty in the thesis (resp., in the journal publications cited above) is the adaptation of Mourre theory into the setting of relativistic algebraic quantum field theory, thereby broadening the methods available for scattering theory in that setting considerably. A central result is the convergence of Araki-Haag detector arrangements in states that belong to the absolutely continuous energy-momentum spectrum and that are energy-bounded. This is an interesting and important result which supercedes previous results, and comes close to establishing asymptotic completeness at the two-particle scattering level.

The presentation of the material in the thesis, as well as in the journal publications, is very clear and systematic. The mathematical research is of the highest mathematical level, as is also indicated by the fact that the journals in which the articles are published are the most highly regarded in the area of mathematical physics.

Therefore, the results achieved in the thesis of Mr. Kruse constitute a significant advance in the scattering theory of relativistic quantum field theory in the operator algebraic framework.

Thus, the thesis is certainly fulfilling the expectations for the requirements of a PhD. To give a more nuanced impression of where I see the overall quality of the thesis, in the German system, I would rank it at ``magna cum laude´´. It is certainly an excellent academic work; however, in my view, it is not fully deserving of a ``summa cum laude´´, or distinction, reserved for exceptionally outstanding theses.

(Prof. Dr. Rainer Verch)