Syllogistic Validity and Existential Assumptions: A Modern Formal and Computational Reconstruction

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Abstract

The aim of this dissertation is to comprehensively describe the issue of the non-emptiness of terms in traditional logic (syllogistic) and its connections with modern formal logic, with a special focus on how the additional existential assumptions influence the validity of syllogistic proofs. It starts with a general overview of syllogistic and then proceeds to part one. The first part of the work is then concerned with, first, a thorough description of the historical views on syllogistic proofs, presented through an analysis of the development of the famous *Barbara, Celarent...* syllogistic mnemonic device; second, an analysis and study of historical sources and discussions on the issue of the non-emptiness of terms itself.

After answering how the issues under question were debated throughout history, part two is devoted to proposing new answers by formal analysis of syllogistic. Thus, it consists of demonstrating what additional existential assumptions need to be adopted in order for all 24 classically recognized syllogistic moods to be considered provable, both within the framework of modern formal logic and within the internal framework of syllogistic itself. To this end, first, a framework of modern formal logic, first order predicate calculus in particular, is employed, and the syllogistic moods are divided into several categories, based on the necessity of adopting additional existential premises in their proofs; next, a method of recognizing all the alternative direct and indirect proof cases within the framework of syllogistic itself is developed, and the computed proofs are arranged on several different sets of tables, from which conclusions pertaining to the mutual behaviors between the syllogistic moods are drawn. Through an analysis of all the alternative proofs displayed in the tables, the syllogisms are grouped into four different categories, differing from the usually recognized syllogistic figures; moreover, different sets of minimal inference rules sufficient to prove the whole system of syllogistic, different from the usual Aristotelian set, are proposed. Next, the necessity of adopting the additional ontological assumptions is traced back to the fundamental syllogistic relations of subalternation and conversion with limitation, and the framework of additional mnemonic names developed by Leibniz and covered in the first part of the dissertation is used to better understand spotted regularities.

The final part of the thesis aims to address the previously discussed issues using the latest available tools, namely the interactive proof assistants, and particularly Isabelle/HOL. Relying on the formalization of Aristotle's syllogistic carried out using this assistant, all alternative direct proofs of the 24 valid syllogistic modes are carried out, and then it is examined whether the proofs formalized in this way rely on the same additional existential premises as those identified earlier. Finally, all the conclusions are collected together and connected with contemporary debate in philosophy of logic, most importantly to the debates about the nature and significance of the transition between Aristotelian syllogistic and modern formal logic.

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Introduction

The aim of this dissertation is to comprehensively describe the issue of the non-emptiness of terms in traditional logic (syllogistic) and its connections with modern formal logic, with a special focus on how the additional existential assumptions influence the validity of syllogistic proofs. Accordingly, after a general (and necessarily brief) overview of syllogistic (Chapter 1), the first part of the work is concerned with, first, a thorough description of the historical views on syllogistic proofs, told through an analysis of the development of the famous *Barbara, Celarent...* syllogistic mnemonic device, which was used throughout the Middle Ages and beyond to codify the recognized proofs of syllogistic moods (Chapter 2); second, an analysis and study of historical sources and discussions on the issue of the non-emptiness of terms itself, taking into account ancient, medieval (both Arabic and European), and modern texts (Chapter 3).

After answering how the issues under question were debated throughout history, I proceed to propose my own answers. Thus, the second part of the dissertation consists of demonstrating what additional existential assumptions need to be adopted in order for all 24 classically recognized syllogistic moods to be considered provable, both within the framework of modern formal logic and within the internal framework of syllogistic itself (i.e., without using tools of modern formal logic). The result of this shall be first giving a novel *metatheoretical* account of Aristotelian system within the confines of the system itself and, second, rendering this new metatheoretical account and the problem of the use of empty terms using modern formal tools. To this end, first, a framework of modern formal logic, first order predicate calculus (first order logic) in particular, is employed, and the syllogistic moods are divided into several categories, based on the necessity of adopting additional existential premises in their proofs (Chapter 4). Next, in search for a minimal set of inference rules needed to prove all the syllogistic moods (both directly and indirectly), a method of recognizing all the alternative direct and indirect proof cases within the framework of syllogistic itself is developed, and the computed proofs are arranged on several different sets of tables, from which conclusions pertaining to the mutual behaviors between the syllogistic moods are drawn (Chapter 5). By analyzing all the alternative proofs displayed in the tables, I group the syllogistic moods into four different categories, differing from the usually employed syllogistic figures, as well as propose different sets of minimal inference rules sufficient to prove the whole system of syllogistic which are different from the usual Aristotelian set. Moreover, I trace the necessity of adopting the additional ontological assumptions back to the fundamental syllogistic relations of subalternation and conversion with limitation, and I use the framework of additional mnemonic names developed by Leibniz (and covered in Chapter 2) to better understand spotted regularities.

The final part of the thesis aims to address the discussed issues using the latest available tools, namely the interactive proof assistants, and particularly Isabelle/HOL (Chapter 6). Relying on the formalization of Aristotle's syllogistic carried out using this assistant, I identify, as in Chapter 5, all alternative direct proofs of the 24 valid syllogistic modes, and then examine whether the proofs formalized in this way rely on the same additional existential premises as those previously identified. Finally, I collect all the conclusions together and connect them to contemporary debates taking place in philosophy of logic, most importantly to the debates about the nature and significance of the transition between Aristotelian syllogistic and modern formal logic.

Research problem and questions

The problem of the non-emptiness of terms in syllogistic concerns the question of whether all (or, if not all, which) terms used in syllogistic reasoning must have a reference, i.e., be non-empty, in order for this reasoning to be valid. In modern formal logic, first order predicate calculus, rooted in the intuitions of Boolean algebra, operates with the concept of a domain of interpretation (universe of discourse) and does not allow for the construction of interpretations of formal languages for empty universes. The assumption of non-emptiness of sets is also significant as it plays a fundamental role in conducting formal proofs, and it is important from the perspective of contemporary research in the psychology of reasoning, where the interpretation of problems by a subject is decisive for the effectiveness and evaluation of their problem-solving attempts. Moreover, a decision of its adoption bears great consequences for the permissible structure of the syllogistic, as some of the relationships between the categorical statements can be argued to depend on this assumption. Since syllogisms are using natural language quantifiers and prompt natural language conclusions, they are a medium of interplay between formal logic and natural language and can be treated as a normative stance for modeling human communication, with the models being sensitive to the non-emptiness assumption as well.

In contemporary research on the history of logic, there is a widespread belief that the assumption of the non-emptiness of terms was also fundamental to Aristotle's syllogistic and, consequently, to the entirety of traditional logic (encompassing Arabic, medieval European, and modern texts). Nevertheless, more nuanced studies tell that this view does not reflect historical reality and have only emerged when attempts were made to translate syllogistic into the language of the recently developed formal logic. The study of the source texts, moreover, indicates that the issue of the non-emptiness of terms was historically widely debated in relation to categorical propositions, relations in the logical square, and syllogistic inferences themselves – yet it has not been thoroughly examined to date, and the question of the exact relationship between syllogistic and modern formal logic thus remains unresolved, largely due to the lack of consensus on the ontological commitments of the former. Moreover, although many historical debates were held about the non-emptiness of terms with regards to categorical statements as such and the relationships between them codified in the Logical Square (Square of Opposition), the issue has not been systematically examined in connection with syllogistic inferences, and particularly with the 24 classically recognized valid syllogistic moods, which shall be done in this work.

The main goal of the dissertation is thus to systematically analyze the presented issue. I pose and answer (among others) the following questions:

1. What ontological commitments must be adopted in syllogistic from the perspective of first order predicate calculus, and why? Also, what ontological commitments must be adopted in syllogistic from the perspective of the syllogistic itself, and why?

2. What is the relationship between various alternative proofs of syllogistic moods and the necessity of adopting additional ontological commitments?

3. How we can uncover this relationship, and what is the root cause of the necessity of adopting the additional commitments?

4. What various solutions to this problem were proposed throughout the ages?

5. What were the views on syllogistic proofs throughout the ages, how they have corresponded to the views on the ontological commitments, and how were they cataloged in syllogistic mnemonic devices?

6. How different views on the syllogistic proofs and on the necessity of adopting additional existential assumptions may have prompted certain scholars (e.g., Buridan, Leibniz) to propose their own extensions of syllogistic mnemonic devices?

7. What is the correspondence between formal proofs of syllogistic done using first order predicate calculus, inferences of syllogistic itself, and an interactive theorem prover

(Isabelle/HOL)? Also, whether the use of such a theorem prover can tell us anything more about the necessity of adopting additional ontological assumptions?

As a whole, apart from its significance for understanding the history, the formal structure of syllogistic reasoning, and the interplay of both, the dissertation aims to bring valuable insights into how the syllogistic inferences can be satisfactorily used to model human communication. Since syllogisms, while comprising a formal system of reasoning, at the same time use natural language quantifiers and prompt natural language conclusions, they are a medium of interplay between formal logic and natural language. The non-emptiness assumption is in turn of significance from the standpoint of contemporary research in the psychology of reasoning, where the individual's interpretation of a given problem is decisive for both the effectiveness and the assessment of the quality of their attempted solutions. The usefulness of treating syllogistic as a normative framework for modeling communication has been acknowledged by scholars, with various models being proposed. The results of the project concerning the necessity of the use of empty terms and the alternative axiomatic bases, while providing a better knowledge of the internal structure of syllogistic, shall thus lay a foundation for rethinking and upgrading the models proposed so far. Moreover, the possibility of automation and formal verification of syllogistic reasoning provided by Isabelle/HOL and the developed formalization may prove to be useful in optimizing future empirical studies.

Chapter 1

The system of assertoric syllogistic

From the standpoint of modern formal logic, Aristotelian syllogistic is the subset of the first order predicate calculus (Corcoran, 1972a), (Andrade, 2007) and is not being considered to be a primary interest of modern logicians, to say the least. From the historical standpoint, however, it used to be contrary, and what is now termed “traditional logic” (Smith, 2022) was the primary[1] formal system considered for over two thousand years, up until the rise of mathematical logic in the late XIX century. Developed by Aristotle in *Prior Analytics*, the syllogistic was then commented on and interpreted by every major figure of European (and Arabic (Chatti, 2019)) philosophy, from first commentators of Aristotle such as Theophrastus (Barnes, 1985) and Alexander of Aphrodisias (Garin, 2019), to last great figures of medieval logic such as Ockham (Spade, 1999) and Buridan (King, 1985), and pretty much every major figure of later philosophy, including Leibniz (Gerhardt, 1962), Hobbes (1994), and Kant (1762). In the XVIII century, the situation in logic even resembled the situation in physics before the discovery of quantum mechanics and theory of relativity. Kant in his *Critique of Pure Reason* (BVIII; see also (Lu-Adler, 2016)) states that with Aristotle and his syllogistic, everything that could have been said on logic has already been stated and there is nothing more to add.

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[1] Together with the Stoic logic, which had, however, a much lesser impact (see (Bobzien, 2003) and (Frede, 1974)). We are also not considering logical traditions which originated in China or India.

In the Middle Ages, numerous interpretations of the Aristotelian system have been generated (with pretty much every commentator proposing his own reading) and then held up until the XIX century (with *Logique de Port-Royal*, Leibniz, and even *Logic* of Pierre Gassendi as possible examples). Nonetheless, two main trends in understanding what a syllogism is can be traced (Boger, 1998, p. 188). The first interprets it as an argument consisting of premises and conclusion whose validity is then determined by their mutual relationships (which are understood in the boundaries of the system), the second understands syllogism as an argument *pattern*, whose instances are then specific (valid or invalid) arguments.

Although it indeed seems like the two thousand years of tradition and commentaries have exploited the potential of Aristotelian system to generate novel perspectives (i.e., every major interpretation possible from *inside* of the system has already been proposed and investigated), the scholarship on "traditional logic" has not been completely abandoned with the advent of its modern alter-ego. On the contrary, needless to say that without the groundwork provided by Aristotle there would not be any formal logic (Tennant, 2014, p. 121), since Jan Łukasiewicz's "Aristotle's syllogistic: from the standpoint of modern formal logic" (Łukasiewicz, 1951), numerous perspectives of describing syllogistic reasoning using new formal apparatus have been proposed, ranging from an axiomatic reading developed by Łukasiewicz and held later by Patzig (1968) and Bocheński (1961), to a deductive reading advocated by Corcoran (1973) and Smiley (1973). While the first one stated Aristotelian syllogistic is to be thought of as a deductive axiomatic system modeled on the *Elements* of Euclid and specific syllogisms to be true conditional propositions, the other viewed it as a natural deduction system containing not axioms, but rules of inference.

Aristotle himself in *Prior Analytics* defines the syllogism (gr. συλλογισμὸς) as follows (for comparison of various ancient definitions of the syllogism see (Novaes, 2017)):

> A deduction is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so. I mean by the last

> phrase that it follows because of them, and by this, that no further term is required from without in order to make the consequence necessary.[2] (24b18–22)

A similar statement can be also found in *Topics*:

> Now a deduction is an argument in which, certain things being laid down, something other than these necessarily comes about through them.[3] (100a25)

And in *Sophistical Refutations*:

> For a deduction rests on certain statements such that they involve necessarily the assertion of something other than what has been stated, through what has been stated.[4] (165a1)

Just from these fragments, one thing is apparent: there is no one concrete definition of the syllogism. As it is the case in many places of *Prior Analytics*, Aristotle's comments are often both imprecise and (naturally) not fitting our conceptualizations anymore. Here, it is doubtful whether a syllogism is to be understood as a *metalogical* structure by which a performed inference is ruled (as it is suggested by the first definition), or as a logical structure, i.e., inference (respectively, argument) itself (as it is portrayed by the second one). It is further doubtful whether it is meant as a process of going from presupposed statements to a necessary following assertion (as in the third definition), or as an assemblage of premises, or even of premises together with the conclusion (as in the first one).[5]

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[2] συλλογισμὸς δέ ἐστι λόγος ἐν ὧι τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῶι ταῦτα εἶναι. λέγω δὲ τῶι ταῦτα εἶναι τὸ διὰ ταῦτα συμβαίνειν, τὸ δὲ διὰ ταῦτα συμβαίνειν τὸ μηδενὸς ἔξωθεν ὅρου προσδεῖν πρὸς τὸ γενέσθαι τὸ ἀναγκαῖον. For the Greek text see (Aristotle, 1938). This, as well as the translations from Aristotle's works below, is taken from (Barnes, 1984).

[3] Ἔστι δὴ συλλογισμὸς λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει διὰ τῶν κειμένων. For the Greek text see (Aristotle, 1960).

[4] ὁ μὲν γὰρ συλλογισμὸς ἐκ τινῶν ἐστὶ τεθέντων ὥστε λέγειν ἕτερόν τι ἐξ ἀνάγκης τῶν κειμένων διὰ τῶν κειμένων [...]. For the Greek text see (Aristotle, 1955).

[5] For more general considerations about inferences and their instantiations, see (Salto, 2025).

For sure, one thing can be noted: a syllogism has a notion of *necessity* (gr. ἀνάγκη) inscribed in it. As of yet, those definitions do not tell us anything about the structure of the syllogism, respectively, syllogistic (i.e., the number of premises, moods, figures etc. – anything that is traditionally associated with syllogistic), but they certainly state that whatever follows from the premises, follows from them necessarily (or: out of necessity, gr. ἐξ ἀνάγκης; note that it can be applied whatever the interpretation from previous paragraph we decide to stick to). From that follows, in turn, that both of the traditional pre-formal interpretations mentioned above are not Aristotelian in a pure sense, while their definitions of syllogisms allow it either to be invalid or to have specific invalid cases.

Moreover, is it worth to note the Aristotelian concern with discussion and proper argumentation (in a discussion). Thus, in *Topics*, the definition of a syllogism appears in the context of finding

> a line of inquiry whereby we shall be able to reason from reputable opinions about any subject presented to us, and also shall ourselves, when putting forward an argument, avoid saying anything contrary to it. First, then, we must say what deduction is, and what its varieties are, in order to grasp dialectical deduction; for this is the object of our search in the treatise before us.[6] (100a20–24)

From which then follows the position of Smiley (1973) and Corcoran, (1972b) that syllogisms are not only a pair of premises and a conclusion, but also the reasoning done to ensure that the information in conclusion is present already in the premises. Thus, apart from valid arguments which were counted as syllogisms in the traditional views, a *deduction* establishing the validity of them is also accounted for as a syllogism, and Aristotle is said to have developed an underlying deductive structure (Corcoran, 1974, p. 281) for next both discussing sciences

[6] [...] τῆς πραγματείας μέθοδον [...], ἀφ' ἧς δυνησόμεθα συλλογίζεσθαι περὶ παντὸς τοῦ προτεθέντος προβλήματος ἐξ ἐνδόξων, καὶ αὐτοὶ λόγον ὑπέχοντες μηθὲν ἀφοῦμεν ὑπεναντίον. Πρῶτον οὖν ζητητέον τί ἐστι συλλογισμός, καὶ τίνες αὐτοῦ διαφοραί, ὅπως ληφθῇ ὁ διαλεκτικὸς συλλογισμός· τοῦτον γὰρ ζητοῦμεν κατὰ τὴν προκειμένην πραγματείαν. See (Aristotle, 1960).

which are organized axiomatically (as in *Posterior Analytics*) and modeling arguments (as in *Sophistical Refutations*).

The present work has two main themes: the first is giving an account of how the syllogistic proofs and the use of empty terms in syllogistic reasoning were treated historically throughout the centuries; the second is giving a novel *metatheoretical* account of Aristotelian system and rendering this new account and the problem of the use of empty terms using modern formal tools. With this in mind, details of each of the conception of the syllogism just covered are not of a main interest to us, and we do not need to describe them. From those mentioned above, the interpretation able to grant the greatest degree of theoretical clarity to our investigations is the deductivist option advocated by Corcoran and Smiley, as it describes the syllogism both as a premise-conclusion scheme and the deductive operations which are needed to account for this scheme's validity. This being said, we will mostly stick to the interpretations proposed by Boger (1998, p. 197), which highlights the operational aspect to an even greater degree and treats syllogisms not as deductions, but as *rules* used in the *deduction processes*. Thus, speaking of a syllogism (or of a syllogistic mood, which, in this case, means the same) we will have in mind an *argument pattern*, and under the name of syllogistic reasoning we will mean a *reasoning employing those patterns*. This position, needing no distinction between syllogistic mood and its particular instance, will prove to bring greater clarity to our later arguments. But first, we will start with a short description of the structure of the Aristotelian system.

1.1 The structure

1.1.1 Categorical sentences

The structure of the syllogism laid down by Aristotelian definitions has two components. First, there are "things being stated", i.e., premises (gr. προτάσεις, sg. πρότασις) of the argument, second, there is "something other" which "follows of necessity", i.e., the conclusion (most often as gr. συμπέρασμα). However, both Aristotelian premises and conclusions are specific statements, always being of the form of assertions or categorical propositions. They consist of a subject (gr. ὑποκείμενον), a predicate, and either affirmation (gr. κατάφασις) or denial (gr. ἀπόφασις) of the predicate to the subject. Both subject and predicate are referred to as terms (gr. ὅροι, sg. ὅρος), but while subject can be either individual (e.g., Casias) or universal (e.g., human), predicate can only be universal (e.g., mortal). However, Aristotle does not speak about individual terms in connection with syllogistic, and thus predication is always done by predicating a universal predicate on a universal subject, either wholly (καθόλου, i.e., of all members) or partially (εν μέρει, i.e., on some members). From this follows that all sentences recognized by Aristotle can be grouped into four types of expressions:[7]

1. Universal affirmative: All S (subject) is P (predicate)
2. Universal negative (denial): No S is P
3. Particular affirmative: Some S is P
4. Particular negative: Some S is not P

The copula

What has gathered attention of logicians and researchers throughout the centuries is also the exact framing of these sentences and, especially, the understanding of the role of the so-called *copula*, i.e., the verb *to be* (in the Greek tradition εἶναι, in the Latin tradition *esse*), here as *is* (respectively, *is not*), together with its proper interpretation. In the *copulative* interpretation, it is to be understood as an indication of a *linking* between the subject and the predicate, whereas in the *existential* interpretation it is said to signify their existence. I will not go into the details

[7] Aristotle recognizes also the so-called indefinite sentences, bearing no information on the scope of the subject of which the predicate is affirmed or denied – an indefinite affirmation reads "S is P", whereas an indefinite negation reads "S is not P". These types of statements are not, however, used by Aristotle later and are not very important when it comes to understanding the syllogistic.

and various controversies which understanding of the function of εἶναι has sparked (see (Sisson, 1939) and (van Bennekom, 1986)). The most important of them is most likely the issue of the implications of adding negation to the copula: on the one hand, one may say that it negates the *linking*, on the other, that it negates the *existence*. It is worth to have in mind the eventual existential meaning of the copula while discussing the problem of empty sets, existential import, and existential presuppositions of syllogistic (as it shall be done in Chapter 3).

1.1.2 The Square of Opposition

The relationships between the four categorical statements can be next traced and were historically codified as the "Logical Square" (or the "Square of Opposition"), the structure of which looks like:

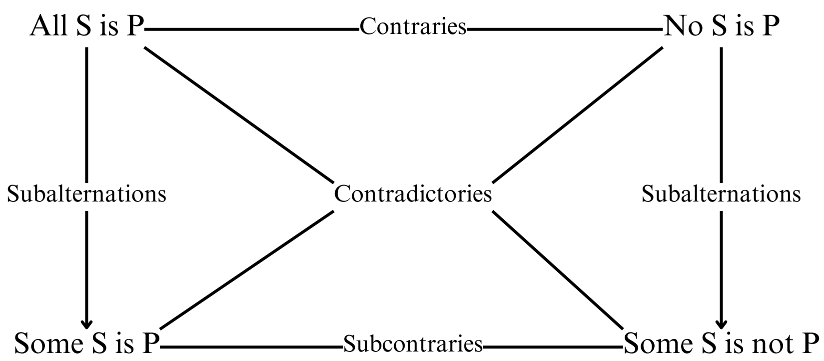

Fig. 1.1 The Square of Opposition

Where contraries are two propositions which cannot both be true at the same time, but can both be false;[8] subcontraries cannot both be false but can both be true; contradictories cannot both be true and cannot both be false, that is, cannot share the same logical value; and subalternations are interconnected such that the subalternated proposition must be true when its superalternation is true, and the superalternation must be false when the subalternation is false (but not conversely:

[8] Here it must be noted that Aristotelian syllogistic (and the traditional logic in general) is a two-valued logic.

the superalternation being false or the subalternation being true cannot tell anything about the truth values of their counterparts).

Although the diagrammatic form is said to have first appeared in Apuleius (Londey and Johanson, 1984) (although this claim is also being disputed (Christensen, 2023)) and been transmitted to the Middle Ages by Boethius (Parsons, 2021),[9] traces of the Square of Opposition doctrine are to be found already in Aristotle, though not in his *Prior Analytics*, but in *De Interpretatione*:

> I call an affirmation and a negation contradictory opposites when what one signifies universally the other signifies not universally, e.g., every man is white – not every man is white, no man is white – some man is white. But I call the universal affirmation and the universal negation contrary opposites, e.g., every man is just – no man is just. So these cannot be true together, but their opposites may both be true with respect to the same thing, e.g., not every man is white – some man is white.[10] (17b17–26)

From which the definitions of contrary and contradictory relationships can be drawn directly, and those of subcontrary and subalternation indirectly, as when ‘Some S is not P’ is false, ‘Every S is P’ must be true, ‘No S is P’ must be false, and thus ‘Some S is P’ must be true, which rules out a possibility that ‘Some S is not P’ and ‘Some S is P’ are both false at the same time. Subalternation, then, follows from the fact that if ‘No S is P’ is true, then ‘Every S is P’

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[9] For an overview of the development of the Square of Opposition present in Apuleius, Boethius, and later authors, see (Correia, 2012), (Sheikh, 2024). For an overview of Boethius’ contribution to logic see (Marenbon, 2008), and for the translation of *Peri Hermeneias* attributed to Apuleius of Madaura see (Londey and Johanson, 1987).

[10] Ἀντικεῖσθαι μὲν οὖν κατάφασιν ἀπόφασιν λέγω ἀντιφατικῶς τὴν τὸ καθόλου σημαινούσαν τῷ αὐτῷ ὅτι οὐ καθόλου, οἷον πᾶς ἄνθρωπος λευκός – οὐ πᾶς ἄνθρωπος λευκός, οὐδεὶς ἄνθρωπος λευκός – ἔστι τις ἄνθρωπος λευκός· ἐναντίως δὲ τὴν τοῦ καθόλου κατάφασιν καὶ τὴν τοῦ καθόλου ἀπόφασιν, οἷον πᾶς ἄνθρωπος λευκός—οὐδεὶς ἄνθρωπος λευκός, πᾶς ἄνθρωπος δίκαιος—οὐδεὶς ἄνθρωπος δίκαιος. Διὸ ταύτας μὲν οὐχ οἷόν τε ἅμα ἀληθεῖς εἶναι, τὰς δὲ ἀντικειμένας αὐταῖς ἐνδέχεται ποτὲ ἐπὶ τοῦ αὐτοῦ ἅμα ἀληθεῖς εἶναι, οἷον οὐ πᾶς ἄνθρωπος λευκός καὶ ἔστι τις ἄνθρωπος λευκός. For the Greek text see (Aristotle, 1938, p. 126).

must be false, and thus ‘Some S is not P’ must be true (an analogous scheme is valid for the subalternation from ‘Every S is P’ to ‘Some S is P’).

1.1.3 Simple inferences

Although many controversies have arisen concerning the precise definition and use of the Square (consult (Parsons, 2021)), for us, the most important relationships are those of contradiction and subalternation, for the first will be treated by us as a concrete application of the rule of contradiction needed to prove *per impossibile*, and the second one will be of a great importance when dealing with the six subalternated moods (see the Section 1.2.3).

Apart from those contained in the Square of Opposition, Aristotle admits some more operations that will be of great use to us. In *Prior Analytics*, he gives an account both of a simple conversion (s-conversion) and of a conversion with limitation (*per accidens*, p-conversion):

> It is necessary then that in universal attribution the terms of the negative proposition should be convertible, e.g., if no pleasure is good, then no good will be pleasure; the terms of the affirmative must be convertible, not however universally, but in part, e.g., if every pleasure is good, some good must be pleasure; the particular affirmative must convert in part (for if some pleasure is good, then some good will be pleasure); but the particular negative need not convert, for if some animal is not man, it does not follow that some man is not animal.[11] (25a2–13)

Thus, we have:

- S-conversion: from ‘No S is P’ to ‘No P is S’ and from ‘Some S is P’ to ‘Some P is S’.

[11] [...] τὴν μὲν ἐν τῷ ὑπάρχειν καθόλου στερητικὴν ἀνάγκη τοῖς ὅροις ἀντιστρέφειν, οἷον εἰ μηδεμία ἡδονὴ ἀγαθόν, οὐδ’ ἀγαθὸν οὐδὲν ἔσται ἡδονή· τὴν δὲ κατηγορηκὴν ἀντιστρέφειν μὲν ἀναγκαῖον, οὐ μὴν καθόλου ἀλλ’ ἐν μέρει, οἷον εἰ πᾶσα ἡδονὴ ἀγαθόν, καὶ ἀγαθὸν τι εἶναι ἡδονήν· τῶν δὲ ἐν μέρει τὴν μὲν καταφατικὴν ἀντιστρέφειν ἀνάγκη κατὰ μέρος (εἰ γὰρ ἡδονή τις ἀγαθόν, καὶ ἀγαθὸν τι ἔσται ἡδονή), τὴν δὲ στερητικὴν οὐκ ἀναγκαῖον· οὐ γὰρ εἰ ἄνθρωπος μὴ ὑπάρχει τινὶ ζῴῳ, καὶ ζῷον οὐκ ὑπάρχει τινὶ ἀνθρώπῳ. For the Greek text see (Aristotle, 1938, p. 202, 204).

- P-conversion: from ‘Every S is P’ to ‘Some P is S’.

And for the particular negative there is no conversion applicable. What is to note is that Aristotle nowhere considers conversion *per accidens* for a universal negative proposition (i.e., from ‘No S is P’ to ‘Some P is not S’) (Parsons, 2014, p. 29), although its validity follows directly from applying subalternation to result of a simple conversion.[12]

Thus far, we have seen the system of syllogistic consisting of three parts: first, we have four types of categorical statements: universal affirmative, particular affirmative, universal negative, and particular negative; second, we have the Square of Opposition and relations between those statements: contradiction, contrarity, subcontrarity, subalternation, and two types of conversion;[13] finally, we have the syllogisms themselves, consisting of two categorical statements as premises and a categorical statement as a conclusion.

1.1.4 The syllogisms

The premises of a syllogism are usually named major and minor. The terms, similarly: major, minor, and middle. The major term is the predicate of the conclusion, the minor term is the subject of the conclusion, and the middle term appears in both premises but does not appear in the conclusion. The major premise is then the premise containing the major term, and the minor premise – the one containing the minor term. Usually, they are also written out accordingly, so that the first premise is the major one and the second premise is the minor one. An example syllogism having two universal affirmatives as premises and one universal affirmative as a

[12] Parsons (2014, p. 29) states that it was later developed by authors such as Paul of Venice.

[13] There are also relations of obversion and contraposition (see e.g., (Santos, 2023)). The former consists of replacing the predicate term by its contradictory and changing the proposition’s quality to the contradictory one; the latter of negating both terms of the sentence and reversing their order. Thus, e.g., from SaP we may get SeP’ (obversion; with P’ indicating not-P), and from SoP – P’oS’ (contraposition; note that a contraposition of an affirmative particular statement is not a valid inference). These single-premise (or *immediate*) inferences are not of a primary concern to us as we will not use them ourselves in further Chapters. They were, however, debated at some points, most notably by the Arabic logicians (see e.g., (Fallahi, 2018)).

conclusion may look like:

1. All M is P (MaP) (major premise)
2. All S is M (SaM) (minor premise)
3. All S is P (SaP) (conclusion)

Where the major term is noted by ‘P’, the minor by ‘S’, and the middle by ‘M’, which is the usual convention.

1.1.5 The figures

Then, the syllogisms are classified into figures according to the placement of the middle term, an account of which is given by Aristotle in *Prior Analytics* 23-41b1. In the first figure, middle term is a subject in the major premise and a predicate in the minor (thus 1. M–P, 2. S–M), in the second, a predicate in both major and minor (1. P–M, 2. S–M), in the third – a subject in both major and minor (1. M–P, 2. M–S), as is shown in the Table below.

	First figure	**Second figure**	**Third figure**
Major premise	M–P	P–M	M–P
Minor premise	S–M	S–M	M–S

Table 1.1 Three syllogistic figures schema

In total, Aristotle in *Prior Analytics* discusses 14 valid syllogisms, four from the first figure (in Chapter 4), four from the second (in Chapter 5), and six from the third (in Chapter 6). To these fourteen moods, additional five first-figure moods, this time concluding indirectly, may be added[14] (Chapter 7), giving a total of nineteen valid syllogistic moods in total. The Table below shows all the valid syllogisms classified in respective figures.[15]

[14] From which later the fourth figure was formed, see Section 1.2.2 below.

[15] For this purpose of maintaining the clarity of the system, the Table is supplemented with medieval mnemonic names which will be explained in Chapter 2.

First figure	First-figure indirectly	Second figure	Third figure
Barbara	**Baralipton**	**Cesare**	**Darapti**
MaP	MaS	PeM	MaP
SaM	PaM	SaM	MaS
SaP	SiP	SeP	SiP
Celarent	**Celantes**	**Camestres**	**Disamis**
MeP	MeS	PaM	MiP
SaM	PaM	SeM	MaS
SeP	SeP	SeP	SiP
Darii	**Dabitis**	**Festino**	**Datisi**
MaP	MaS	PeM	MaP
SiM	PiM	SiM	MiS
SiP	SiP	SoP	SiP
Ferio	**Fapesmo**	**Baroco**	**Felapton**
MeP	MaS	PaM	MeP
SiM	PeM	SoM	MaS
SoP	SoP	SoP	SoP
	Frisesomorum		**Bocardo**
	MiS		MoP
	PeM		MaS
	SoP		SoP
			Ferison
			MeP
			MiS
			SoP

Table 1.2 All valid categorical syllogisms: Aristotle

1.1.6 The basis of syllogistic and the Aristotelian metatheorem on reducing moods

Once we know what figures are, we can briefly sketch what will be dealt with in detail in Chapter 5, namely the Aristotelian doctrine of reducing syllogistic moods. The idea that some syllogistic moods can be *reduced* to others, or, alternatively, that the validity of some syllogistic moods can be *proved* by others (used as rules of inference, we may say on modern terms), was deemed to be the core of syllogistic in the Middle Ages, but was already present in the work of Aristotle.

In Chapter 7 of *Prior Analytics*, Aristotle proves that the whole body of syllogistic can be ultimately reduced to the first two syllogisms in the first figure (in the medieval mnemonics framework termed *Barbara* and *Celarent*). A less strict stance, commonly held by commentators and thereafter codified in the medieval mnemonics (see Chapter 2), is that the first figure should be regarded as the "basis" of all syllogistic, and therefore moods from other figures, being ultimately reducible to (or: provable by) the first-figure moods are in a sense their derivatives.

Barbara and *Celarent* are then regarded as special and deemed the basis of the system in another sense as well, and for other reasons, which are epistemological. Namely, Aristotle regards them as the most "obviously" valid syllogistic moods whose validity can be spotted immediately. This noted "obviousness" stems most likely from the fact that, as we may say in modern terms, the first-figure moods and especially *Barbara* exemplify the schema of the relation of transitivity between three terms (and *Celarent* exemplifies lack thereof). Notwithstanding this fact, in Chapter 5 we will see that from the formal point of view, it is not necessarily the first figure that should be regarded as the basis of the whole system, and other sets of moods which serve for this purpose can be identified as well.

1.2 Historical extensions and controversies

1.2.1 *Theophrastian* moods

The moods in first three figures were clearly recognized by Aristotle and explained in detail. These included, as noted above, fourteen moods: four in the first figure, four in the second, and six in the third. In a later portion of *Prior Analytics*, Aristotle also indicates the existence of five further moods in the first figure. Those moods, canonically called indirect first-figure moods or *Theophrastian moods*, were fully recognized by Theophrastus, who systematized the system of Aristotle.[16] The five "Theophrastian" moods are equal to the four basic first-figure ones with their premises transposed (i.e., with the major premise treated as the minor and the minor treated as the major). The effect is that in order to reduce them to the first-figure moods, one needs simply to either transpose the premises back to the usual order or to convert the conclusion.[17] With the Theophrastian moods added, the system includes nineteen valid moods in total: nine in the (extended) first figure, four in the second figure, and six in the third figure.

1.2.2 The fourth figure

The status of the Theophrastian moods was then debated throughout the history, and in effect, they have become regarded not just as an extension of the first figure, but as a separate, fourth syllogistic figure. The usual account has it that Aristotle does not speak about the fourth figure *as about fourth figure* at all, and that it was introduced as such only in the Middle Ages for

[16] For the overview of the logic of Theophrastus see (Mignucci, 1997) Apart from a systematization of moods and some minor adjusting of the order of them, however, Theophrastus does not make any creative changes to the Aristotelian system (Mignucci, 1997, p. 40) and his most significant contribution remains the clear introduction of the next five first-figure moods.

[17] Some logicians have even maintained that these moods are the same accurate to the converse position of terms. Buridan, for example, writes (2001, p. 312): "[...] for *Celarent* and *Celantes* do not differ either in their premises or in their conclusions, but they differ only with respect to the converse position of the terms in the conclusions; and the same goes for *Darii* and *Dabitis*".

the purpose of maintaining the symmetry of the system, as it represents the fourth possible premise-terms arrangement, i.e., the P – M, M – S arrangement – see the Table below.

	First figure	**Second figure**	**Third figure**	**Fourth figure**
Major premise	M–P	P–M	M–P	P–M
Minor premise	S–M	S–M	M–S	M–S

Table 1.3 Four syllogistic figures schema

The person most commonly attributed with the "invention" of the fourth figure as a separate one is Galen, and the figure itself is thus sometimes referred to as "Galenian". A more detailed historical account is congruent with this narration, although a bit more nuanced. Aristotle, as stated, have not considered the fourth figure as a separate one at all, but it was "discovered" short after him, and referred to as an indirect first-figure by his first commentators, Theophrastus and Eudemus. The custom was then to treat these indirectly concluding moods not as a separate figure, but as a specific (and often considered strange) instances of first-figure moods. Throughout the Middle Ages, they were treated accordingly, which will become apparent in Chapter 2, as we consider the mnemonic devices developed by William of Sherwood, Lambert of Auxerre, Roger Bacon, and other logicians. There, what was later to be termed the fourth figure and be put at the end of the "final" version of the mnemonic, was present as the second verse, with the employment of different mnemonic names, which is also indicative of the unstable ontological status of these moods. The "Galen tale" originated in the XII century by the time of Latin Averroism and was widespread for some time – the precise reference to Galen being the "founding father" of the fourth figure stems likely from Averroes and his *Expositio media*, where he, although himself rejecting the fourth figure, talks about fourth-figure moods *as* fourth-figure moods and attributes this view to Galen.[18] Discussions concerning the view of

[18] The thing worth mentioning is that serious discussions concerning the status of the fourth figure have originated in the Middle East much earlier than in Europe. Although the "Galen tale" is being questioned by the scholars (Stakelum, 1942), (Rescher, 1966a, p. 1), there are still voices which deem it to be true and analyze the influence Galen might have had on Arabic thinkers up until the XII century (Sabra, 1965) (for the reconstruction of

Averroes can be found already in the work of Gersonides (Manekin, 1992). The ontological status of the fourth figure has started to be a serious topic around the end of the Middle Ages[19] and returned at the beginning of the Enlightenment, with, most notably, Hobbes (Hobbes' views on logic can be found mostly in (Hobbes, 1981)) and Leibniz (to be found even in his *De Arte Combinatoria* (1666), written at the age of 19) being strong advocates for treating fourth figure as a separate one. Even Kant (1762) has commented on the topic, maintaining that the fourth is to be ontologically equated with the second and the third, but only insofar as they are all reducible to the first (that is, that all other figures are valid only in the sense that they are "fueled" by the validity of the first).[20] In modern literature and interpretations of Aristotle it is also argued that the introduction of the fourth figure, being motivated by the symmetry of the premise-term arrangement, was actually a result of a misinterpretation of Aristotle's original views on predication (see e.g., (Wesoły, 2012)). According to this account, first three syllogistic figures essentially represent three different possible usage-scenarios of the relation of predication, that is: 1) the major term being predicated on the middle term, and the middle on the minor; 2) the middle term being predicated on both the major and the middle; 3) both the major and the minor terms being predicated on the middle. In this sense, adding the fourth figure, apart from being a nonsense with respect to preserving any kind of symmetry (contrary to the medieval view), would be just adding a slightly different way of looking at the first scenario, and not a whole new scenario. The peculiarity of the fourth figure and fourth-figure moods will become clear in Chapter 5, as we shall see that it indeed behaves differently from the others.

Galen's *On demonstration* see (Havrda and Koetschet, 2025)). When it comes to the Arabic world, we know for sure that even before Avicenna who i stalking about it in detail, fourth figure was already proclaimed by some Arabic logicians to be a separate figure and not just a variation of the first ((Rescher, 1965), for a more detailed account see (Rescher, 1966a)).

[19] Some authors propose to view Buridan as the first one independently identifying the fourth figure in the West (Hubien, 1975).

[20] See also (Vanzo, 2014b).

1.2.3 Subalternated moods

The last major extension of the system came not very long after Theophrastus, although its assimilation took much longer and has not been finished until, when looking at the whole life-span of syllogistic, very recently, for between the XVII and XIX century. The five "subalternated" moods were first spotted by Apuleius, as scholars claim, who, however, disregarded them as not worthwhile, for they "conclude less" when one could conclude more (Gombocz, 1990, p. 127). I was not able to find any references to the subalternated moods in the medieval treatises from the XI to the XIV century. The allegedly first place where they reappear again (and are integrated into the medieval mnemonic system, see Chapter 2) is the treatise of Peter of Mantua (1492; see also (Korcik, 1948, p. 29)), after which they are being considered in detail again by such authors as Hospinianus and Leibniz (see the discussion on Leibniz in Chapter 5).

The essence of subalternated moods consists of subalternating the conclusion of a normal mood whose two premises are general (and hence their usual conclusion is general as well), i.e., – following the phrasing of Apuleius – concluding less (particular sentence) when one can conclude more (general sentence). In practice, the only moods that can be subjected to such a procedure are the first two moods of the first figure, first two mood of the second figure, and the first mood of the fourth figure. The result of adding the subalternated moods is a sudden symmetry of a system, with each figure having exactly six moods. With the fourth-figure moods employed instead of the first-figure indirect ones and the subalternated moods considered, the list of valid syllogistic moods spans exactly twenty four ones, as is showed in the Table 1.4 below (note that the names of subalternated moods are printed in italics). These account will serve as us the basic one for further considerations (e.g., in Chapter 5).

First figure	Second figure	Third figure	Fourth figure
Barbara	**Cesare**	**Darapti**	**Bramantip**
MaP	PeM	MaP	PaM
SaM	SaM	MaS	MaS
SaP	SeP	SiP	SiP
Celarent	**Camestres**	**Disamis**	**Camenes**
MeP	PaM	MiP	PaM
SaM	SeM	MaS	MeS
SeP	SeP	SiP	SeP
Darii	**Festino**	**Datisi**	**Dimaris**
MaP	PeM	MaP	PiM
SiM	SiM	MiS	MaS
SiP	SoP	SiP	SiP
Ferio	**Baroco**	**Felapton**	**Fesapo**
MeP	PaM	MeP	PeM
SiM	SoM	MaS	MaS
SoP	SoP	SoP	SoP
Barbari	***Cesaro***	**Bocardo**	**Fresison**
MaP	PeM	MoP	PeM
SaM	SaM	MaS	MiS
SiP	SoP	SoP	SoP
Celaront	***Camestros***	**Ferison**	***Camenos***
MeP	PaM	MeP	PaM
SaM	SeM	MiS	MeS
SoP	SoP	SoP	SoP

Table 1.4 All valid categorical syllogisms: Extended

1.3 Ecthesis

Although it is not that important with respect to our later considerations, it is still worth to note that apart from the syllogistic moods, Logical Square relations and immediate inferences, Aristotle does use one more rule in his proofs, namely *ecthesis*, by which he proves the conversion of the universal negative statement (*Prior Analytics* 25a15-17).[21] Łukasiewicz (1951, p. 45) then argues that this move requires a procedure that lies "outside the limits of the syllogistic" and thus needs to be considered as an additional axiom of the system, since the conversion of the universal denial is not provable in any other way.

Ecthesis itself is essentially "setting out" a case (Smith, 1982), that is, employing an additional term, mostly although not conclusively (Joray, 2014, p. 229) regarded to be an individual term, which is then used in another syllogistic mood. For us, detailed considerations of *ecthesis* will not be of a great use, as we shall explicitly claim the s-conversion of an *e* statement to be (or not to be) part of our system. It is, however, good to be aware that every time we will be speaking of this rule, we could further reduce it to the process of *ecthesis* and that "Aristotle may have intended to base the convertibility of universal negatives on something more fundamental" (Thom, 1976, p. 299).

1.4 Preliminary notes on existential import

To all listed above, we are to add few remarks on existential assumptions of syllogistic. Although the issue does not seem to be present in *Prior Analytics* in any significant way (Malink, 2013, p. 81), by the work of Łukasiewicz (1951) it was established that Aristotelian syllogistic did not make use of any empty terms, that is, considered only the non-empty ones (as the first order predicate calculus considers only non-empty universes of discourse). For

[21] Note that he does not do this explicitly. The only place where he actually uses the Greek noun for *ecthesis* is the place where he is discussing the completion of *Disamis* and *Datisi* (29b14).

us, it is important to note that the question of existential import becomes relevant only when considering the relation of subalternation. Many various views on the existential assumptions of this relation can be found in literature (Read, 2015), but the most usual maintains that to make the inference from a universal (affirmative) statement to a particular (affirmative) one, one need to add the assumption of the existence of the subject (Parsons, 2021). Thus, a full set of premise for an example mood *Felapton* reads:

1. No M is P (PeM) (major premise)
2. All M are S (SaM) (minor premise)
3. **Some M exists** (existential premise)
4. Some S are not P (SoP) (conclusion)

Thus, particularly when considering the subalternated moods, it is to be noted that the existential assumptions are to be accounted for in the background. The issue will be described in full in Chapter 5.

Chapter 2

Mnemonic devices: encoding syllogistic moods and proofs

Mnemonic devices have been developed throughout the Middle Ages and were widely used to aid remembering of both syllogistic figures and various systematic regularities of the syllogistic system. The names for syllogisms created for mnemonic purposes were so intimately connected with the study of syllogistic itself that it is hard to imagine medieval logic without thinking about them, and the proof procedures encoded in these names played so central a role in the system that understanding them is fundamental to understanding how logicians in the Middle Ages thought about logic. In a word: to get rid of them would mean to get rid of the essence of this logic.

Although the mnemonic devices were commonly used throughout the Middle Ages, taught at the European universities up to at least the XVII century, and discussed by logicians as late as in the second half of the XIX century (see e.g., Peirce (1932)), the paradigm which views syllogistic through the lenses of modern formal logic, exemplified by Jan Łukasiewicz in his monograph (1951) (see (Rybaříková, 2024) for the discussion about Łukasiewicz's influence on the study of the history of logic; see (McConaughey, 2025a) for a discussion about how the modern formal perspective influences interpretation of Aristotelian logic), essentially dispensed with them. Neither Łukasiewicz's *Aristotle's syllogisitc* nor any of the later major works of Patzig, Corcoran, Smiley or Smith explains the mnemonic. In fact, while trying to render Aristotelian syllogistic with modern formal systems, all of these works fail to even mention the existence of the mnemonic, not to say anything about its importance for syllogistic.

This paradigm, however, starts to fall apart. This is partly due to the greater recognition of and knowledge about medieval logical texts and thus the history of medieval logic, but also to the recognition of the influence Aristotelian syllogistic have had on the "founding fathers" of the modern formal logic. We now know that the transition from traditional to modern logic was not a one, discrete moment in time, but a continual process of conceptual changes which ultimately led to the creation of a new framework.[22] In this light, Aristotelian syllogistic, although from the viewpoint of modern formal logic only a small part of the first order predicate calculus, takes on a great historical significance which must be understood on its own. It is impossible, however, to fully understand the development of Aristotelian logic without a thorough knowledge about the medieval mnemonic, the discussions and developments it has provoked, and the style of thinking about logic it was imposing for almost six hundred years. This is why the study of the medieval mnemonic, a part of medieval logic so important that it has soon became the identification mark of Aristotelian syllogistic, is crucial to properly understand the nature of syllogistic as a whole and help us again grasp the syllogistic not in relation to modern formal logic, but as it was understood in the past – as a system by and in itself.

According to the above, what follows is a detailed sketch of the history of mnemonic devices used in syllogistic. It covers its development from the utmost beginnings in the XII century to the influence it has had on later thinkers, such as G. W. Leibniz in the XVII century. It also presents instructions on how to decode the main, *Barbara, Celarent...* mnemonic, which will prove fundamental to understanding discussions around the *Dictum de omni et nullo*,

[22] See primarily (King and Raspa, 2025) and (Verburgt and Cosci, 2023). For works noting continuity between Aristotle and Boole, for example, see (Corcoran, 2003) and (Dunning, 2023).

foundations of syllogistic, methods of proof and understanding the syllogistic proof-procedure that will be important in Chapter 5.

2.1 The beginnings

The germs of using mnemonic devices to aid memorization of the intricacies of syllogistic reasoning start to appear in the logical treatises since the late XII century. The manuscripts which shall be of interest to us include the *Dialectica monacensis*,[23] dated by De Rijk (1967, p. 411) to the sixties or seventies of the XII century; the *Introductiones parisienses*,[24] dated to about 1170 (De Rijk, 1967, p. 447); and the *Logica 'Cum sit nostra'*[25] together with the *Logica 'Ut dicit'*,[26] both dated to the last years of the XII century (De Rijk, 1967, p. 443, 446).

Two forms of memory-aids are present in these treatises, both of which extend to later (i.e., XIII-century texts as well). First of them are **rules** (*regulae* or *maximae*, as they are called in the treatises), constructed so that a maximal portion of information can be fitted into a minimal amount of words. Rules appear in all four treatises and are used to describe features of both assertoric syllogistic as such and of Aristotelian logic in general (including the properties of modal syllogistic, of simple predication, of supposition, etc.). Interestingly, rules pertaining to assertoric syllogistic do appear only in the *Dialectica monacensis*. Among them, we may list such as:

> Si ex opposito conclusionis sequitur oppositum premisse, tunc ex premissis sequitur conclusio (De Rijk, 1962, p. 496).
>
> If from the opposite of conclusion follows the opposite of the premises, then from the premises follows the conclusion.

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[23] For the text, see (De Rijk, 1962, p. 453–638).

[24] (De Rijk, 1962, 353–373).

[25] (De Rijk, 1962, 413–451).

[26] (De Rijk, 1962, p. 375–411).

> Ex premissis particularibus vel indefinitis nichil[27] sequitur sillogistice, sed oportet quod altera vel utraque sit universalis (De Rijk, 1962, p. 499).
>
> From particular or indefinite premises nothing follows syllogistically, but it is needed that either one of them or both be universal.

> In tertia figura minore existente negativa vel utraque particulari, nichil sequitur (De Rijk, 1962, p. 499).
>
> In the third figure, if a minor premise is negative or either of the premises is particular, nothing follows.

Which are relatively short (usually one-sentence length) and describe certain properties of the system of syllogistic (e.g., the rule of indirect proof, as in the first example, or general observations regarding the qualitative aspects of some combinations of premises used, as in the other two) in a concise way. The rules are, in general, written in prose and do not contain any proper names or abbreviations yet; they do, however, manifest certain linguistic features that can aid memorization, such as repetitions and structural analogies,[28] various forms of parallelisms,[29] etc. Such rules are by far a most frequent study- or memory-aid present in the treatises, with their number present in *Dialectica monacensis* amounting to 96.

The second form of short mnemonic aids are **verses**, generally short (two-line) ones, written mostly in the form of Latin elegiac couplet. The verses are different from the rules in that they tend to be more concise, contain more information, and use abbreviations (e.g., "ne" for *negatio*, "aff" for *affirmatio*, "sin" for *singularis*, etc.) or made-up mnemonic words (e.g., "ave", "cato", "feci") which meaning must be separately memorized for the whole to be properly decoded and understood. Moreover, they are generally devoted to fully describe one type of properties of

[27] Such non-standard variants of the Latin words are present in the manuscript and are provided by De Rijk. I do not classicize the forms neither here nor in the rest of the quotes.

[28] Compare e.g., the two last examples above and their counterparts: "Ex premissis negativis nichil sequitur, sed ad hoc ut sit sillogismus, oportet quod utraque premissarum, vel altera, sit affirmativa", "in secunda figura maiore existente particulari vel utrisque similis qualitatis, nichil sequitur" – for both see (De Rijk, 1962, p. 499).

[29] E.g., "Plus tollit negatio quam ponat affirmatio" (De Rijk, 1962, p. 513), "De quocumque predicatur species, et genus; a quocumque removetur genus, et species"(De Rijk, 1962, p. 528).

syllogistic, for example the Logical Square relations, the figures, the conversions, and so on. In the oldest treatise under our concern, the *Dialectica monacensis*, we find a four-verse elegiac couplet describing the Logical Square:

> Sit tibi linea subcontraria prima secunde
> tertius cum quarto contrarius ordo
> quarta subalternat sibi primam, terna secundam
> contradicit tertia prime, quarta secunde (De Rijk, 1962, p. 484).
>
> Let the line from first to second be that of subcontrariety
> the one binding third with fourth that of contrariety
> the fourth subalternates the first, the third the second
> the third contradicts the first, the fourth the second.

As well as a two-verse elegiac couplet describing the subject-predicate relations within the figures:[30]

> prima: prius medium subicit, post predicat ipsum
> altera: bis dicit, tertia: bis subicit (De Rijk, 1962, p. 494).
>
> The first [figure] first subjects the middle [term], then predicates it
> The second [figure] predicates [the middle term] two times, the third [figure] subjects [it] two times.

---

[30] *Dialectica monacensis* also contains two additional short-verse-mnemonics not directly connected to our investigations. One of them is describing the equivalences between terms (here I write together four verses which are otherwise scattered throughout two pages. This is also the case for this mnemonic as it appears later in William of Sherwood's *Introductiones* (1937)):

> equivalent 'omnis', 'nullus non', 'non aliquis non'.
> 'nullus', 'non aliquis', 'omnis non' equivalent hae.
> 'non nullus', 'non omnis non', 'aliquis' faciunt hae.
> 'non omnis', 'non nullus non', faciunt 'aliquis non' (De Rijk, 1962, p. 474–475).

The other is summarizing what are the four Aristotelian "causes":

> efficiens cause Deus est, formalis ydea,
> finalis bonitas, materialis yle (De Rijk, 1962, p. 541).

The *Introductiones Parisienses*, supposed to be written a few years later than the *Dialectica monacensis*, also contains two short-verse-mnemonics, but different from the ones in *Dialectica...*. Those are:

> que?: cat. vel y.; qualis?: ne. vel aff.
> quanta?: uni. par. in. sin.[31] (De Rijk, 1962, p. 360)
> what [type]?: categorical or hypothetical; what [quality]?: negative or affirmative
> what [quantity]?: universal particular indefinite singular.

And:

> sub pre prima secunda bis pre, tertia sub bis.[32] (De Rijk, 1962, p. 363)
> subjects [and] predicates the first [figure] the second [figure] predicates two times, the third [figure] subjects two times.

The second one is a more succinct variation of the one contained in the *Dialectica...*, with abbreviations instead of full verbs (subicit, predicat, dicit), and everything condensed into one hexameter. As the one in the *Dialectica...*, it describes the subject-predicate relations in respective three figures.

The first one is slightly more perplexing, as it does not fit into a full meter. Instead, it is an unfinished elegiac couplet, consisting of an unfinished hexameter and unfinished pentameter. As such, it recounts all possible qualities of a proposition in syllogistic. A proposition can be either categorical (cat.) or hypothetical (y.), qualitatively either negative (ne.) or affirmative (aff.), quantitatively either universal (uni.), particular (par.), indefinite (in.) or singular (sin.). A slightly better version of these mnemonic verses is contained in the later *Logica 'Ut dicit'*, where it takes the form of:

---

[31] [que?: cat.[egorica] vel y.[potetica]; qualis?: ne.[gatio] vel aff.[irmatio]
quanta?: uni.[versalis] par.[ticularis] in.[definita] sin.[gularis].

[32] sub[icit] pre[dicat] prima secunda bis [predicat], tertia sub[icit] bis.

que? ca vel y; qualis? ne vel aff
u quanta par in sin.[33] (De Rijk, 1962, p. 383)

With ‘uni.’ shortened to ‘u’ and moved to the initial place of the second line. In this form, shall it be written in one line instead of two (as it is the case in *Logica ‘Cum sit nostra’*, see below), it would form a full single line of hexameter. The *Logica ‘Ut dicit’* contains also three other short-verse-mnemonics. The first of them is:

simpliciter feci convertitur aut per acci
per contra cato; sic est conversio tota (De Rijk, 1962, p. 386).
feci converts simply, aut [converts] per acci[dens] (with limitation)
per contrariety [converts] cato; this is the whole conversion.

In elegiac couplet, describing different types of conversions. Here, the words ‘feci’, ‘aut’, and ‘cato’ have mnemonic and not (if they have one) usual meaning: ‘feci’ stands for **E** and **I** types of categorical sentences (universal negation and particular affirmation), ‘aut’ is probably a misspelling of ‘eva’ (see below), signifying universal negation (**E**) and universal affirmation (**A**), and ‘cato’ refers to universal affirmations (**A**) and particular negations (**O**). The second mnemonic is:

pre contradic, post contra, pre postque subalter.[34] (De Rijk, 1962, p. 386)

In single-line hexameter, being a shorter version of the first mnemonic appearing in the *Dialectica monacensis* and describing the Logical Square relations. The last verse-mnemonic is:

sub pre prima, bis pre secunda, tertia bis sub (De Rijk, 1962, p. 395).

Being a variation of the hexameter already known from the *Introductiones Parisienses*, describing the subject-predicate relations in respective figures.

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[33] For extensions of abbreviations see the preceding footnote.

[34] pre contradic[it], post contra[riatur], pre postque subalter[nantur].

The last treatise, containing the largest number of short-verse-mnemonics (six in total), is the *Logica 'Cum sit nostra'*. It contains verses such as:

> que ca. vel ip., Qualis ne. vel aff., U. quanta par.in.sin (De Rijk, 1962, p. 421).

Which can be regarded as the final phase of the evolution of the mnemonic present already in the *Introductiones Parisienses*, now rendered as a single line of hexameter; Thereafter:

> simpliciter feci convertitur eva per acci cato per contra; sic fit conversio tota (De Rijk, 1962, p. 427).

Which is a slightly different version of the one present in *Logica 'Ut dicit'* ('cato per contra' instead of 'per contra cato'; 'eva' (correctly) instead of 'aut'); then:

> pre contradic. post contrar.
> pre postque subalter (De Rijk, 1962, p. 427).

Being a two-lines variation of a mnemonic present also in *Logica 'Ut dicit'*; next:

> sub pre prima, bis pre secunda, tertia bis sub (De Rijk, 1962, p. 435).

Being a variation of verses present in the *Introductiones...* as well as in the *Logica 'Ut dicit'*. The last two short-verse-mnemonics occurring in the *Logica 'Cum sit nostra'* are of a different kind. The first one concerns with modal syllogistic and is hence not of a primary interest to us,[35] the second is a two-verse "meta"-mnemonic, describing itself the mnemonic significance of consonants employed in the *Barbara, Celarent...* mnemonic which we will cover in the section below. It runs as follows:

[35] It runs:
a quo possibile, contingens tollitur inde
hinc inpossibile, datur opposito necesse (De Rijk, 1962, p. 430).

S simplex, P per acc.

M transpos., C notat impossibile (De Rijk, 1962, p. 436).

S simple, P with limitation

M transposition, C notes the [proof by] impossibility.

And its meaning shall become clearer when the *Barbara, Celarent...* mnemonic will be explicated.

Looking at the occurrences of short-verse-mnemonics in the aforementioned treatises, one can draw several conclusions. First of all, the number of used short-verse-mnemonics is steadily increasing. The numbers of them in respective treatises (displayed chronologically according to the dating provided at the beginning) is illustrated in the table below:

Work	**Number of short-verse-mnemonics**
Dialectica monacensis	2
Introductiones Parisienses	2
Logica 'Ut dicit'	4
Logica 'Cum sit nostra'	6

This suggest an ongoing trend of coming up with and employing mnemonic verses in the process of both encoding and teaching syllogistic. If we treat the use of rules as a first stage in this process of the "synthetization" of knowledge (be it either for pedagogical purposes or for making the fundamentals of the system clearer), then the appearance of verses and their steadily increasing usage suggest a transition to a more concise and memorable form of learning. This process is also evidenced by the development of the short-verse-mnemonics themselves, most notably by the evolution of verses such as the "prima: prius medium subicit, post..." to a more and more concise form.[36] Moreover, if we agree that the greater number of occurrences of

[36] Note here that this argument cannot hold decidedly when it comes to verses being displayed so as to fit line-to-line to a hexa- or pentameter. The reason for this is that a particular display of verses is often a fault not of an author(s) of the treatise but of a particular edition of it. This is true also when it comes to modern critical

the verse-mnemonics in the treatises corresponds with a general tendency of using more and more mnemonic devices to aid the process of learning syllogistic, it can help us also better date the works, for we may speculate that the *Logica 'Cum sit nostra'*, containing by far the most verse-mnemonic, was written a few years later than the *Logica 'Ut dicit'*, dated by De Rijk for roughly the same time span.

Short-verse-mnemonics in later texts

Although by the beginning of the XIII century, short-verse-mnemonics were overshadowed by the *Barbara, Celarent...* mnemonic on which we will speak in the next Section, they fate was not to go extinct, quite the contrary. All four major XIII-century treatises on logic which we will cover in the next Section also do contain short-verse-mnemonics, which are for most part variations on those present already in the XII-century works. Among the treatises, William of Sherwood's *Introductiones in Logicam* contain four such short-verse-mnemonics; Lambert of Auxerre's *Logica* six, including two not seen in other works;[37] Roger Bacon's *Summulae Dialectices* three, and Peter of Spain's *Summulae logicales* two. With the exemption of Peter's *Summulae...*, a trend of limiting use of **rules** is also seen, with William, Lambert, and Roger almost not using them. It looks like while the *Barbara, Celarent...* mnemonic has occupied a central place, the "creative powers" of logicians started to be put rather into the development of it than to a development of new short-verse-mnemonic devices. In this sense, the short-verse-mnemonics served the purpose of making clearer and cataloging the technical foundations of the system (e.g., single-premise inference rules, categorical square relations,

editions of treatises and their medieval or early modern copies – there are cases when the modern editions give improper verse spelling where the historical ones do get it right.

[37] Those are:
pendo suspensus; pendo dum sublevo pondus;
pendo do penas; pendet qui sustinet illas; (Lambert of Auxerre, 2015, p. 167);
And:
nunc appellatum, nunc se, nunc significatum,
dictio supponi est triplex quilibet ergo (Lambert of Auxerre, 2015, p. 240);
I do not include here all the verses appearing in later works not to make the text overdetailed.

inter-figure relations, etc.), while the *Barbara, Celarent...* mnemonic served a further purpose of cataloging existing and valid syllogistic moods. It is therefore natural that, on the one hand, it appeared some years after the short-verse-mnemonics were developed, on the other hand, that after its emergence, the short-verse-mnemonics were no longer the main point of interest.

It is worth to note, however, that making up short-mnemonic verses was not altogether abandoned. A few centuries later, further propositions of short-verse-mnemonics were being proposed: of them, I have been able to find two, that of John Wallis (1616-1703, a great British mathematician credited for a partial development of calculus) and that of G. W. Leibniz. Wallis' *Intitutio Logicae* (Wallis, 1687), immensely popular in his times (Beeley and Wardhaugh, 2022, p. 32), contains several verses which resemble a versified form of what was in the XII-century treatises a rule;[38] Leibniz, on the other hand, in his *De formis syllogismorum mathematice definiendis* (1996, p. 458–479), proposes a distich codifying how the second- and third-figure moods are to be reduced to the first-figure ones.[39]

2.2 *Barbara, Celarent...*

The pinnacle of the development of mnemonic verse-devices is a poetical fragment in Latin starting with *Barbara, Celarent...*, which is a complete guide to syllogistic reasoning for one initiated enough to decode the information it contains. Written in a Latin hexameter as to be more memorable, the construction is packed with information to the degree that de Morgan described it as "words which I take to be more full of meaning than any that ever were made"

[38] Syllogixari, non est ex particulari;
Neve negativis, recte concludere si vis.
Siqua praeit partis, sequitur conclusio partis:
Siqua negata praeit, conclusio sitque negata
Lex generalis erit, Madium concludere nescit (Wallis, 1687, p. 131);
And:
Ex falis falsum, verumque aliquando, sequetur.
Ex veris possunt nil nisi vera sequi (Wallis, 1687, p. 132).

[39] Altera majorem, sed tertia forma minorem
Ex prima servat quando regressus erit (Leibniz, 1996, p. 474).

(Morgan, 1847, p. 150). The version we shall be using (with some restrictions) in this work is the one contained in (Aldrich, 1862, p. 84–5), which runs:

> *Barbara*, *Celarent*, *Darii*, *Ferioque*, prioris:
> *Cesare*, *Camestres*, *Festino*, *Baroko*, secundae;
> Tertia, *Darapti*, *Disamis*, *Datisi*, *Felapton*,
> *Bokardo*, *Ferison*, habet: Quarta insuper addit
> *Bramantip*, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.
> *Barbara*, *Celarent*, *Darii*, and *Ferio* in the first [figure];
> *Cesare*, *Camestres*, *Festino*, *Baroko* in the second [figure];
> The third [figure] has *Darapti*, *Disamis*, *Datisi*, *Felapton*,
> *Bokardo*, *Ferison*: The fourth [figure] adds to it
> *Bramantip*, *Camenes*, *Dimaris*, *Fesapo*, *Ferison*.

and adds two more lines on the subalternated moods:

> Quinque *Subalterni*, totidem *Generalibus* orti,
> Nomen habent nullum, nec, si bene colligis, usum.
> Five subalternated [moods], all taken from the general [ones],
> Do not have any name, nor use, if you think of it.

2.2.1 Working mechanism

In the verses, nineteen direct moods of the syllogistic figures are represented by nineteen words, arranged in a proper order. Thus, *Barbara* is the name for the first syllogism of the first figure, *Celarent* for the second, *Cesare* is the first one in the second figure, and so on (for the arrangement of figures see the translation in the note). The names themselves contain information both on the structure of the syllogism and the method of proving it. The vowels *a*, *i*, *e*, *o* represent four kinds of categorical propositions: (successively) universal affirmative, particular affirmative, universal negative, particular negative. The vowels themselves stem from the Latin '**a**ff**i**rmo' ("I affirm") and 'n**e**g**o**' "I deny". The first vowel in the name of a syllogism is there to signify the major premise, the second the minor premise, and the third the conclusion. Thus, for example, *Barbara* and *Festino* are to be read as:

	Barbara	**Festino**
Major premise	All M are P [MaP]	No P is M [PeM]
Minor premise	All S are M [SaM]	Some S are M [SiM]
Conclusion	All S are P [SaP]	Some S are not P [SoP]

Table 2.1 Barbara and Festino decoded

The consonants, in turn, indicate what is to be done in order to reduce the syllogism[40] to the one in the first figure. A consonant after a vowel means that the operation it signifies should be done on the premise indicated by the preceding vowel. *s* means the s-conversion, *p* means the p-conversion, *m* stands for the Greek *metathesis*, respectively Latin *mutatio*, signifying a switch of premises, and *c* is indicating that a *reductio per impossibile* should be used as proving method (note that *c* in Aldrich is rendered as *k*. In this work, we shall proceed with the c-version, thus *Baroco* and *Bocardo*). The *B*, *C*, *D*, and *F* at the beginning of each word indicate the valid first-figure mood to be used in the proof. To see an example, take *Fresison*:

> *Fresison*:
>
> Major premise: PeM,
>
> Minor premise: MiS,
>
> Conclusion: SoP.

First-figure mood to be used: *Ferio* (since *F*).

> *Ferio*:
>
> Major premise: MeP,
>
> Minor premise: SiM,
>
> Conclusion: SoP.

[40] For what a “reduction” means see Section 1.1.6.

s after *e* indicates an s-conversion, thus from PeM we get MeP, and

s after *i* also indicates s-conversion, thus from MiS we get SiM, thus obtaining:

> Major premise: MeP,
>
> Minor premise: SiM.

From which then SoP is a validated as a conclusion by using *Ferio*.

2.2.2 The beginnings

The 5-verse "poem" thus contains a full description of all valid direct syllogistic moods together with all the information needed to prove them. The history of *Barbara, Celarent...* is, however, neither obvious nor fully understood, and the question of the original authorship remains unanswered. The first known attempt at describing syllogistic moods using mnemonics verses goes back as far as to the IX century (Turner, 1907), with the first, though much less compact (42 verses) metrical piece teaching syllogistic moods being a fragment of *Codex Galii 64*, which in metrical form enumerates all the syllogistic moods valid in the subsequent figures. However, that this pierce is of a yet different nature compared either to the *Barbara, Celarent...* or to the one we have discussed before. It does not include mnemonic words or abbreviations, and is by no means short or concise. It is a thorough descriptions on the moods in subsequent figure, with the only thing making it "mnemonic" being the metrical form in which it is written.

First traces of using abbreviations and attempts to create proper mnemonic words to refer to syllogistic moods can be found in the so-called *Ars Emmerana*[41] and *Ars Burana*,[42] dated by De Rijk to the third quarter of the XII century (1967, p. 401–3). The mnemonic code there encodes premises and conclusion of syllogistic moods, but its scope is confined just to the quantitative and qualitative aspects of them and does not give any hints for a method of

[41] For text, see (De Rijk, 1962, p. 143–174).

[42] (De Rijk, 1962, p. 175–213).

proving, an apparent innovation of the *Barbara, Celarent...* mnemonic. The first verbatim use of fragments of the *Barbara, Celarent...* mnemonic appears in the already mentioned *Dialectica monacensis*, although De Rijk argues that the use of proper syllogistic names is a later manuscript interpolation (1967, p. 412–414).

2.2.3 The Greek version

For some time, scholars had maintained that the Latin version is only a translation from a much earlier Greek source. A handbook on logic from 1843 lists Greek and Latin mnemonic names together, maintaining that the Greek ones were used by Aristotle (sic!) and the Latin by the scholastic logicians (Buchner, 1843, p. 58). Saint-Hilarie subsequently attributed the Greek version to be a creation of Byzantine monk Blemmydes (Aldrich, 1862, p. 84), but it soon turned out that only the Latin translation of Blemmydes' work has the mnemonic (added by the translator of 1607 edition, Johann Weigelin (Peter of Spain, 2014, 191)) whereas the original text has not (Blemmydes, 1885). A view formulated in (Prantl, 1861, p. 275-6) ascribed the first Greek version to a Byzantine monk Psellus from the XI century and his *Synopsis organi Aristotelici* (Latinized), but it was further contended by Thurot (1864, p. 267) (see also (Prantl, 1867)) and the Greek text is now said to be a later translation or interpretation of the Latin version (Aldrich, 1862, p. 84). A version of the Greek mnemonic present in the third book of *Institutio Logicae* by John Wallis (1687, p. 130) (who himself maintains it to be a creations of Aristotle's commentators) reads:

1st figure: Γράμματα ἔγραψε γραφίδι τεχνικός,

2nd figure: Ἔγραψε κάτεχε μέτριον ἄχολον.

3rd figure: Ἅπασι σθεναρὸς ἰσάκις ἀσπίδι ὁμαλὸς φέριστος.

The same (apart from a different location of φέριστος) is given by Aldrich (Aldrich, 1862, p. 84). Bocheński (1961, p. 214) adds as the second line:

Γράμμασιν, ἔταξε χάρισι πάρθενος ἱερόν:

Which is supposed to stand for the fourth, or the indirect first figure.

The Greek mnemonic differs from the Latin one in that the mnemonic words do carry a non-accidental usual meaning. They roughly translate as:[43]

He artfully wrote the inscription with a stylus,
He tastefully arranged the letters into a virgin temple;
He wrote: maintain harmony and calmness,
In everything strong, with a shield, the best yet humble at once.

Although there is but little evidence that the creation of Greek text exerted any influence on the further development of or on understanding of the Latin mnemonic, its existence shows that the "Latin tradition" of encoding syllogistic proofs in mnemonic devices was so widespread that it has started to be transmitted to and assimilated in the "Greek East" as well (note that the "Greek tradition" in syllogistic is usually associated with the use of the Byzantine diagrams (see Section 3.5), and not names, phrases and poems, as mnemonic aids). Its mention in later works such as the *Institutio...* of John Wallis shows, on the other hand, that the Greek text has been recognized by scholars in the West, and hence the two traditions must have coexisted.

2.2.4 Latin versions

Coming back to the Latin text of *Barbara, Celarent...*, which we know for sure to be the original one, we can note that the mnemonic was widely known and in use around the XIII century and must have first appeared around the year 1250. All three main XIII-century works on logic: William of Sherwood's *Introductiones in logicam*, Lambert of Auxerre's *Logica (Summa Lamberti)*, and Roger Bacon's *Summulae Dialectices* already contain it, as well as

[43] Note that the translations given by Bocheński and in the English translation of his work do not make much sense (1961, p. 214).

the anonymous *Introductiones Parisienses* (De Rijk, 1967, p. 364), *Logica 'Ut dicit'* (De Rijk, 1967, p. 396) and *Logica 'Cum sit nostra'* (De Rijk, 1967, p. 436). The versions given in the first three of those works run as follows:

William of Sherwood (1966, p. 66):

Barbara celarent darii ferio baralipton
Celantes dabitis fapesmo frisesomorum
Cesare camestres festino baroco darapti
Felapton disamis datisi bocardo ferison.[44]

Roger Bacon (de Libera, 1987, p. 205):

Barbara, *Celarent*, *Darii*, *Ferio*, *Baralipton*
Celantes, *Dabitis*, *Fapesmo*, *Frisesomorum*
Cesare, *Campestres*, *Festino*, *Baroco*, *Darapti*
Felapton, *Disamis*, *Datisi*, *Bocardo*, *Ferison*.

Lambert of Auxerre (2015, p. 143):

Barbara, *Celarent*, *Darii*, *Ferio*, *Baralipton*
Celantes, *Dabitis*, *Fapesmo*, *Frisesomorum*
Cesare, *Campestres*, *Festino*, *Baroco*, *Darapti*
Felapto, *Disamis*, *Datisi*, *Bocardo*, *Ferison*.

The oldest version that has survived is the one of William of Sherwood, the single available manuscript of which dates from the late XIII century (William of Sherwood, 1966, p. 13), but which is supposed to be written around 1250 (Peter of Spain, 2014, p. 191). Some scholars,

[44] Note that the version contained in (William of Sherwood, 1966) renders *Darapti* in the fourth rather than in the third verse, which apparently does not make much sense, since then the verses are no longer hexameters. The correct version must thus have been the one we propose here.

including Grabmann, the first translator and editor of William's works, have maintained that Sherwood himself may have been an inventor of it (William of Sherwood, 1937, p. 27).[45] The case is though debatable. Kretzmann (William of Sherwood, 1966, p. 67) claims that the differences in spelling Ca[m][p]estres (more on that issue below) indicate that there must have been an earlier version, being a source from which later authors drew.

The most authoritative version of the mnemonic is certainly that of Petrus Hispanus, commonly although debatably identified with the Pope John XXI (Meirinhos, 2009), which, by the end of the XIII century, has become a generally recognized standard. It runs as follows (Peter of Spain, 2014, 191):

> Barbara Celarent Darii Ferio Baralipton
> Celantes Dabitis Fapesmo Frisesomorum
> Cesare Cambestres Festino Barocho Darapti
> Felapton Disamis Datisi Bocardo Ferison

Peter's *Summulae Logicales* in which it was printed were used as a schoolbook for undergraduate courses throughout the European universities and went through more than one hundred and sixty editions (Mullally, 1945, p. 133-158).[46] The mnemonic itself must have been widely taught until the rise of empirical sciences and at least up until the late XVI century, as Hobbes in his autobiography (1994, p. 97) recounts it, though in a yet different version and omitting the second line, as being taught to him in the first year of Oxford's bachelor's degree studies:

> Barbara, celarent, darii, ferio, baralypton
> Cesare, camestres, festino, baroco, darapti
> Felapton, disamis, datisi, bocardo, ferison

[45] See also (Kneale and Kneale, 1962, p. 233). Here, it is also worth to note the influence that William had on other logicians of his time, for among his students in Paris were both Bacon and Lambert of Auxerre (and even Peter of Spain before his venture to Sienna) (William of Sherwood, 1966, p. 4). This can be an additional argument for stating that his version of the mnemonic, while maybe not the first, was the one which had the greatest influence on the later ones.

[46] Let only add that Dante, in *Canto XII* of *Paradiso*, places Peter in the second circle (Alighieri, 2013, p. 813).

What the aforementioned authors differ on when it comes to the form of the mnemonic is the spelling of Aldrich's *Camestres* and the rules for understanding consonants.[47] William gives *Camestres*, Bacon *Campestres*, Lambert *Capestres*, and Peter *Cambestres*. Since in order to reduce the mood under question to *Celarent* one needs to switch the premises, simply convert the universal negative and simply convert their conclusion, the correct version is undoubtedly *Camestres*. Thus *Campestres* wrongly tells us to convert with limitation the major premise, *Capestres* does the same without telling us to switch the premises, and *Cambestres* contains a *b* insignificant from the mnemonic point of view. It is hard to tell why this mood was rendered in so many different versions, whereas others (apart from minor differences like *Felapto*/*Felapton*) mostly stayed the same. The other discrepancy concerns the letter chosen to indicate the *reductio per impossibile*. Here, Bacon uses *c*, the same being true for Peter; Lambert is speaking of the appearance of *b* and *o* in the same word, William does the same for *b* and *r*,[48] and Aldrich, as already noted, writes *k*. Finally, Peter and Lambert precautionary tell us that everything beyond three syllables is devoid of mnemonic meaning – thus, the last syllables of *Baralipton* and *Frisesomorum* are not to be looked on.

2.3 Evolution of the mnemonic

In the years following Peter's *Summulae...*, the mnemonic must have became a common knowledge. Ockham's *Summa Logicae* (1974), although it does not in any place explicate the working of the mnemonic verses, includes few mnemonic names without previously giving any introduction to them. It also seems that by the end of the Middle Ages, the standard spelling of the mnemonic names has already been well settled (with a correct spelling of *Camestres*!). The *Logica Parva* of Paul of Venice, being on the second place (next to Peter's *Summulae*) when

[47] The succinct description given by Sherwood is available in English in (William of Sherwood, 1966, p. 67) and the more elaborated ones of Bacon in (Bacon, 2009), Auxerre in (Lambert of Auxerre, 2015, p. 144) and Peter in (Peter of Spain, 2014, p. 190-3).

[48] Note: thus leaving *Baralipton* unaccounted.

it comes to popularity and number of reprints, contains a version closed to the one of Peter, which can be taken for a standard spelling at the end of the Middle Ages (see (Venetus, 2002, p. 20) and (Venetus, 1536, p. 10)):

> Barbara, Celarent, Darii, Ferio, Baralipton
> Celantes, Dabitis, Fapesmo, Frisesomorum
> Cesare, Camestres, Festino, Baroco, Darapti
> Felapton, Disamis, Datisi, Bocardo, Ferison[49]

Thus, apart from the unification of spelling (although it is not by any means historically conclusive, see for example Section 2.4 below), the most important tendency in the development of the *Barbara, Celarent...* mnemonic pertains to the division of figures throughout the verses. We have already noted that *Darapti* in Kretzmann's version of Sherwood's *Introductiones...* is placed wrongly in terms of the hexameter, and this turns out to be the case not only there, as *Dialectica monacensis* also puts it not at the end of the third line, but at the beginning of the fourth. In *Logica 'Cum sit nostra'*, *Baralipton* is placed not in the first, but in the second line, highlighting the fact that it belongs to the other figure, but losing the meter. In works of Bacon, Auxerre, and Peter, *Darapti* is placed at the end of the third line, but consequently treated as belonging to the fourth figure. We can thus see that there is a tension between a willingness to keep the hexameter clear and that to indicate the proper figure of the mood. It also becomes apparent that all XIII-century versions of the mnemonics does not contain an information on the assignment of a specific mood to a specific figure, which must be memorized separately. This, together with the fact that the four-syllable names had parts redundant, might have led later authors to include the instructions concerning the assignment of moods to respective figures directly in the text as well. Thus, the four-syllable *Frisesomorum* became the three-syllable *Fresison*, *Baralipton* has evolved to *Bramantip*,[50] and the indications of *prioris*, *secundae*,

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[49] Exactly the same version is present in Buridan's *Summulae...*, see (Buridan, 2001, p. 26).

[50] This tendency must have started already in the Middle Ages, as Buridan is already talking about *Frisesom* and *Baralip* instead of *Frisesomorum* and *Baralipton* (Buridanus, 1976). Fresison and Bramantip are then nothing

tertia, and *quarta* have appeared (see Aldrich's version above), rendering the verses both more readable and mnemonically succinct, at the same time making them fit into the meter.

2.3.1 Alternative names for fourth-figure moods

It is important to note that with different ways of treating the fourth figure (see Section 1.2.1 and 1.2.2) names of its moods had to be changed accordingly. Thus, apart from *Frisesom(orum)/Fresison* and *Baralip(ton)/Bramantip*, we have also pairs of *Camenes/Celantes*, *Dimaris/Dabitis*, and *Fesapo/Fapesmo*.

It is hard to tell where exactly these names appeared for the first time, but they seem to be quite a late contribution. Leibniz in his youth-work *De Arte Combinatoria* (1666) uses some of the names for indirect first-figure moods, but also the new "fourth figure" ones. In his later *De formis syllogismorum mathematice definiendis* (2019) and *Schedae de novis formis syllogisticis*, however, he gives up the indirect first-figure ones and does not even mention them, advancing his reasoning solely with the use of fourth-figure ones.

The cited account of Peirce (1932), although underestimating the systematical difference between indirect first-figure moods and direct fourth-figure ones, appears then to be historically right – the names used to designate indirect first-figure moods were widely used and acknowledged at least from the time of Peter of Spain to the times of Leibniz and thereafter, while the names for direct fourth-figure moods started to be advanced around the time of Leibniz, and, by the time of Peirce, were indeed not yet "very well settled" (Peirce, 1932, p. 349).

2.3.2 Subalternated moods

Apart from the moods described above, we need to briefly refer also to the mysterious "quinque *Subalterni*" which, according to Aldrich, "do not have any use". Subalternated moods are

more than Frisesom and Baralip treated as fourth-figure moods instead of first-figure indirect ones – see also the Section below.

essentially versions of some main moods which yield general conclusions with their conclusions subalternetad so as to be particular (see Chapter 1). Thus, a subalternated counterpart of the main mood *Barbara* is *Barbari*, of *Celarent* is *Celaront*, and so on. Classically, five moods were accounted for as subalternated ones, with two being a part of the first figure (*Barbari* and *Celaront*), two of the second (*Cesaro* and *Camestros*), and one of the fourth (*Camenos*).

Aristotle himself does not speak about subalternated moods at all, which was not the custom even for many medieval authors; Buridan is including some new names for syllogisms, but he is not talking about the subalternated moods yet. Quite the contrary – while talking about *Barbara* and *Baralipton* in his *Summulae de dialectica*, he admits that there are moods which can conclude in two ways (such as *Barbara*), but has in mind only that they can do so either directly or indirectly. In this way, he is treating *Barbara* and *Baralipton* as two instances of the same mood, but he does not yet recognize the mood *Barbari* (Buridan, 2001, p. 311-312). The work of Hieronymus Savonarola (1497) from the end of the XV century does discuss the subalternated moods (although briefly), but does not include their names yet.

The main historical figure to comment on the names for subalternated moods was Leibniz in his *De formis syllogismorum mathematice definiendis* (2019) and *Schedae de novis formis syllogisticis* (2019). Leibniz himself ascribes the names for subalternated moods to Hospinianus (for a comparison between Leibniz and Hospinianus see (Korcik, 1954)) – Hospinianus was the first one to systematically analyze the numbers of possible and valid syllogisms, and he himself believed that he had "invented" the subalternated moods together with their names (Dürr, 1955). However, that does not prove to be right, and the names for subalternated moods do appear as early as in the work of Peter of Mantua (consult e.g., (Korcik, 1948, p. 29), for life and logic of Peter of Mantua see (James, 1974)). In his *Logica* (Mantuanus, 1492), Peter talks explicitly about subalternated moods and names them by the known names of *Barbari*, *Celaront*, etc. However, he does not present them in a way that would suggest he is the one to construct and propose them. On the contrary, he does not treat them as anything novel and uses the verb *solere* ("to be used to") while talking about their use (see (Mantuanus, 1492, p. 100 of the .pdf file, the manuscript itself is unnumbered)), which suggest that they were at least relatively known to his readers and must have been proposed by some scholars earlier. The precise origin of this proposal, however, remains unknown as for now, but the dating of the invention of the names should be placed at the end of the XV century, that is, around the time of the work of Savonarola and Mantuanus.

As a counterpart to the work of Leibniz and as a proof of a somewhat "uneven distribution" of syllogistic novelties, however, we may view the already-mentioned *Institutio Logicae* of John Wallis, which, being an oeuvre contemporary to the one of Leibniz, does not mention neither the names for subalternated moods, nor the previously described names for fourth-figure moods. Instead, while talking about the fourth figure as an autonomous one, John advances several alternative names different from the ones mentioned by Leibniz, which must have not been widely accepted.

Throughout the history, there was also some discussion on whether some other moods with a particular conclusion (for example *Felapton* or *Darapti*) should be accounted for as subalternated moods. Bocheński (1961, p. 140), for example, maintains that Galen had advocated for *Darapti(s)* to be considered as a third figure subalternated mood. The most important fact is that only after the introduction of the subalterns we obtain a complete system of 24 valid moods, with six moods per each figure. In Chapter 5, however, we shall see that the status of some of the moods is somewhat in-between when it comes to being subalternated also in our own schema.

2.4 Other expansions of the mnemonic

Apart from the development of names forced by the historical discussions happening on the fourth-figure and subalternated moods, there were also other developments taking place, which were themselves stretching the boundaries of syllogistic logic. Two most notable of such developments are those of Jean Buridan and G. W. Leibniz.

Jean Buridan proposed new names for syllogistic moods twice, and both of these times they were supposed to stand for new moods invented by him. His first proposition appears in *Tractatus de consequentiis* (1976), the second one in his opus magnum, *Summulae de dialectica* (2001, 2015). Apart from being among the first European logicians to admit the fourth figure as an autonomous one instead of treating it as an indirect first-figure (Hubien, 1975), he was also eager to consider "indirect moods" not only of the first figure, but also of the second and third one. Buridan then proposes encoding indirect moods not only for the first figure, but for the second and third figure as well. In his *Tractatus de consequentiis* (see (Buridan, 2015) for an English translation and (Buridanus, 1976, p. 93) for critical edition), he proposes alternative names *Tifesno* and *Robaco* for second-figure moods *Festino* and *Baroco*, as well as *Lapfeton*, *Carbodo*, and *Rifeson* for third-figure *Felapton*, *Bocardo*, and *Ferison*, respectively. And while those names are just variants of the "classical" ones with part of the words being switched, in his major work, the *Summulae de dialectica* (2001, p. 331–334; 1976, p. 44–48), he advances his proposition and poses names *Fitesno*, *Boraco*, *Fapemton*, *Bacordo*, and *Fisemon*, respectively, which are of his own invention.

While Buridan is in general not very interested in the Aristotelian doctrine of reduction (and in the assertoric part of syllogistic in general (Lagerlund, 2022)) and the mnemonic encoding of proofs is not of much interest to him, it is worth to note that the names proposed later in the *Summulae de dialectica* are built in accordance with the rule that the initial consonant denotes the first-figure mood to which a particular mood is to be reduced, whereas the names proposed in the *Tractatus de consequentiis* are not. This, in turn, can hint that Buridan must have acquired a greater understanding of the functioning of the mnemonic in the meantime. Buridan also takes into consideration the remaining second- and third-figure moods and builds their indirect counterparts. The whole of his proposition looks like:

First figure		Second figure		Third figure	
Direct	**Indirect**	**Direct**	**Indirect**	**Direct**	**Indirect**
Barbara	Baralipton	Cesare	(Cesare)	Darapti	(Darapti)
Celarent	Celantes	Camestres	(Camestres)	Disamis	(Disamis)
Darii	Dabitis	Festino	Firesmo\Tifesno[51]	Datisi	(Datisi)
Ferio	Fapesmo	Baroco	Boraco\Robaco	Felapton	Falepton\Lapfeton
	Frisesomorum			Bocardo	Bacordo\Carbodo
				Ferison	Fiseron\Rifeson

Table 2.2 Buridan's mnemonic names

The last significant extension of the mnemonic is due to Leibniz. In his *Schedae de novis formis syllogisticis*, he introduces several new syllogistic names, mainly for the subalternated moods. Those include *Gabali*, *Legano*, *Lesaro*, *Gaceno*, *Digamis*, *Cademop*, *Calmentes*, and *Baralimp*.[52] *Gabali* and *Legano* are meant to stand for *Barbari* and *Celaront*, and *Gaceno* with *Lesaro* for *Camestros* and *Cesaro*. Unlike Buridan, Leibniz is very careful about the initial letters of the mnemonic names – his reasoning for using **G** and **L** as initials stems from a rigid sticking to the rule which was partly neglected by Buridan: that every mood from figures II-IV ought to begin with the same letter as its "mother" mood from figure I; and as he shows how *Camestros* is reduced to *Barbari* and *Cesaro* to *Celaront*, he treats *Barbari* and *Celaront* as legitimate first-figure "mother" moods that ought to have their own letter. He also alters the names of *Bramantip*, *Dimaris*, *Camenos*, and *Camenes* to *Baralimp*, *Digamis*, *Cademop*, and

[51] The names after backslash are taken from *Tractatus...*, the ones before it from *Summulae...*.

[52] It is, moreover, worth to note that apart from Leibniz, there have also been much later propositions of mnemonic improvements of certain names, for example (LaFleur, 1942).

Calmentes, reflecting his views on syllogistic reduction processes, which, once again were earlier neglected by Buridan. The whole of Leibniz's proposition looks like:

First figure	**Second figure**	**Third figure**	**Fourth figure**
Barbara	Cesare	Darapti	Baralimp
Celarent	Camestres	Disamis	Digamis
Darii	Festino	Datisi	**Cademop**
Ferio	Baroco	Bocardo	Calmentes
Gabali	**Lesaro**	Felapton	Fesapo
Legano	**Gaceno**	Ferison	Fresiso

Table 2.3 Leibniz's mnemonic names

2.5 Conclusions

As we finish the overview of the history of the development and use of the mnemonic devices in syllogistic, it is worth to emphasize that their historical role was not to merely aid the memorization of syllogistic moods; although developed primarily for this purpose, significance of these devices encompasses a much bigger range of phenomena, including structuring the system of syllogistic, providing theoretical lenses from which the system could be thought of, and guiding further formal explorations within the boundaries of syllogistic.

The last statement is evidences primarily by the developments of the mnemonic system done by Buridan and Leibniz. After commenting on them in detail, we can see, for example, how the developments of names done by Leibniz can be viewed as a next step forward relative to those of Buridan. Namely, whereas Buridan is advancing the doctrine while taking into consideration only the division between *direct* and *indirect* moods, Leibniz is talking about the nature of the *subalternated* moods. This, in turn, aligns with a general tendency in the history of syllogistic, where the discussion on the subalternated moods has taken place after the questions concerning the direct and indirect moods had been settled (as was briefly mentioned in Chapter 1).

What the history of the mnemonic devices shows is that, contrary to the popular belief, Aristotelian syllogistic was not a mortified system, but was alive and ardently debated even in the time of Leibniz, with various extensions of it being proposed. The nature of these extensions, as dictated almost solely by the nature of the *Barbara, Celarent...* mnemonic, should, in turn, remind us of the crucial role the mnemonic system have played in the history of syllogistic and induce us to study syllogistic logic not by reducing it to modern formal notation, but by engaging with its rich historical tradition.

Moreover, using the mnemonic can prove beneficial even now, with the theoretical framework of new mnemonic names provided by Leibniz, for example, being useful for spotting several regularities at the level of the numbers of proofs for each syllogistic moods, which will be done in Chapter 5. On the same occasion, we will also speculate how the recognition of empty terms and subalternation by Leibniz may have prompted him to both rename certain syllogistic moods (as we have seen above) and organize the whole body of syllogistic in a slightly different fashion that the usual, which will be again exploited in Chapter 5.

Chapter 3

Existential import, or the history of ontological commitments

Introduction: the problem of existential import

In the first Chapter, we have already pointed out that the issue of assuming existence of term-subjects becomes relevant when performing certain syllogistic inferences, most notably subalternation and conversion *per accidens*. We have also mentioned that reconsidering the issue of empty terms is important for psychology of reasoning and may help optimize future empirical studies. The purpose of this Chapter is to provide an account of the issue of empty terms and ontological assumptions as they play throughout the history of traditional logic, which will prove important to correctly understand the following Chapters.

In modern formal logic, first order predicate calculus, using the notion of the universe of discourse, does not allow for constructing interpretations of formal languages for empty universes, and in contemporary discussions concerning the history of logic, there is a widespread assumption that the Aristotelian syllogistic did not allow for the use of empty terms as well. The issue of empty terms is, however, more complex, and has been discussed extensively throughout history. Most importantly, it has been discussed in connection with the so-called "existential import".

The problem of existential import can be defined as the question of whether and in what sense a proposition presupposes the existence of its subject, and, with respect to syllogistic, it is also inherently connected to issues such as the validity of the relation of subalternation and the question of whether particular propositions imply existential ones (see e.g., (Leibniz, 2019, p. 151)). More generally, it is the question of whether the categorical statements preserve their truth-value if the entities that appear as denotations of their terms are nonexistent, and, with respect to syllogism themselves – whether the entities appearing as denotations of their terms need to exist for the syllogisms to be logically valid. Thus, a proposition is said to have existential import when "it entails the corresponding existential proposition based on its subject term" (Horn, 2001, p. 24), or, alternatively speaking, "it cannot be true unless its subject refers to some existing object(s)" (Chatti and Schang, 2013, p. 102). If the categorical sentence SxP (we are not defining the quality of this sentence yet) has existential import, then it is true if and only if its subject term S refers to something that is existent, i.e., if there is something which is S (again, without defining the precise *mode* of existence of this denotation).

The problem of existential import is especially important when it comes to understanding the relation between syllogistic and modern formal systems. In terms of modern first order predicate calculus, the definition of existential import for quantified proposition needs to be rewritten like, for example: "A quantified proposition has existential import if and only if *Sx* is true of some (real) value of the variable *x*"[53] (Chatti and Schang, 2013, p. 102), which means that if "*Sx* is true of some of the variable *x*, then the [quantified] proposition implies [that] *(∃x)Sx*" (Chatti and Schang, 2013, p. 102).

The main discrepancy between traditional (syllogistic) logic and first order predicate calculus in modern logic concerns the universal affirmative statement. It stems from the fact

[53] Following the famous account of Quine that "To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable" (Quine, 1953).

that in the latter, this type of statement **does not have existential import**, whereas in the former, it seems like the existential import of it **has to be assumed** in order to preserve the validity of subalternation and, consequently, the Logical Square.

If we were to rewrite the traditional Square of Oppositions using modern notation, the result would be as follows:

1. Universal affirmation: $\forall$x(S(x) $\rightarrow$ P(x))
2. Particular affirmation: $\exists$x(S(x) $\wedge$ P(x))
3. Universal negation: $\forall$x(S(x) $\rightarrow$ $\neg$P(x))
4. Particular negation: $\exists$x(S(x) $\wedge$ $\neg$P(x))

With the inference

$$\forall x(S(x) \rightarrow P(x)) \rightarrow \exists x(S(x) \wedge P(x))$$

being true, as the left side of it (the universal affirmation) is formulated as a conditional and is thus true either if no x exists.

The issue of how the relation between the universal and particular statements is to be interpreted on the ground of traditional logic is, however, more complicated, and constitutes only a small part of debates on **existential (or ontological) assumptions** of traditional logic which have been taking place throughout the history. These debates, often more general, extend to other types of sentences (e.g., negative statements), different types of negation, different modes of existence, and sometimes also from categorical statements to syllogisms themselves. What follows, is a historical review of the issue of existential import of terms in traditional logic from Aristotle to the times of the development of Boolean algebra and modern logical systems. As a starting point, I take the influential statements of Łukasiewicz, saying that Aristotelian syllogistic, like modern formal logic, presupposed all its terms to be non-empty. I contest this view by relating extensive historical discussions on the existential import problem and the evolution of understanding the use of empty terms in traditional logic. In addition, I sketch the parallel development of the modern notion of universe of discourse for first order predicate calculus adopted by modern logicians. To this end, I cover four areas: the logic of Aristotle, Arabic logic, medieval European logic, and later discussion up until the emergence of modern formal logic, focusing on Leibniz. In addition, I discuss the influence which various forms of diagrammatic representations of categorical statements and syllogistic inferences may have had in the processes under description.

I will not cover the many amendments which have been proposed for the Aristotelian Logical Square in the last few decades to account for its shortcomings related to admitting or denying empty terms (see (Jacquette, 2016)), neither I will cover the recently developing field of the so-called "free logics" (Nolt, 2007), which dispense with the necessity of assuming existence present in modern formal logic. What is of primary interest to us here is when and to what extent empty terms were discussed and/or allowed by logicians, what has provoked these discussions, and what implications they had on the authors' theories of syllogistic. Along relating the history, I also make some claims, main of them being that the dominant modern interpretation of Aristotle stems from the attempts to render syllogistic using modern notation (a trend which began with Łukasiewicz), and that in the conditional treatment of the existential assumptions proposed by Ockham, we can trace the origins of the first order universe of discourse idea. By a closer look into the history and by answering the posed questions, we will be able to better grasp the issues discussed in the next Chapters, particularly the not-so-simple relation between traditional and modern logic which will be explicated in Chapter 4.

3.1 Interpreting Aristotle: at a crossroad of modern and traditional logic

The interpretation of Aristotle's logic which seems to have been the most influential for the second half of the XX century is that of Jan Łukasiewicz, who in his 1951 *Aristotle's Syllogistic* claimed that “In building up his logic, Aristotle did not take notice either of singular or of empty terms” (1951, p. 4). The claim that Aristotle supposes all terms used in his syllogisms to be non-empty was held by Łukasiewicz without a further justification, and he talks about it on more than one occasion in the 1951 work. Note, for example: “Singular, empty, and also negative terms are excluded as values” (1951, p. 72), “Aristotle does not introduce into his logic singular or empty terms or quantifiers” (1951, p. 130). The same is true for his earlier *Elements of Mathematical Logic* (*Elementy logiki matematycznej*): “As it is the case in the logic of Aristotle, values of our name variables *may not be empty* [original emphasis], such as, for example, is the ‘square circle’” (1958, p. 86).

The view of Łukasiewicz was consequently held by many scholars. Kneales in their *Development of Logic* (Kneale and Kneale, 1962) state that “In order to justify Aristotle’s doctrine as a whole it is necessary, then, to presuppose that he assumed application for *all* [original emphasis] the general terms with which he dealt” (1962, p. 60). The same is true for Patzig, who, while analyzing Aristotle’s *Prior Analytics*, writes that “The expression ‘one must examine the set of subject (predicate, contrary) terms of S (P)’ clearly presupposes that in each case these sets have at least one member” (1968, p. 6). Corcoran, in his formalization of Aristotle’s syllogistic, although proposing to read it as a natural deduction system rather than an axiomatic one (as it is the case for Łukasiewicz or Bocheński (1961, p. 63)), maintains the assumption of the non-emptiness of sets as well (Corcoran, 1972a, p. 696).

This rendering seems to be an orthodox way of interpreting up until now. In his *Stanford Encyclopedia of Philosophy* article, Smith states rather conclusively that “(...) Aristotle in effect supposes that *all* [original emphasis] terms in syllogisms are non-empty” (2022).

Unfortunately, this viewpoint was rather never held by Aristotle himself. If modern scholars ascribe it to him, they usually do it either by an analysis of his description of the Square of Opposition (see Section 1.1.2), maintaining that drawing all the conclusions from his formulation leads to admitting existential import of affirmative sentences (see e.g., (Corkum, 2018)), or by interpreting parts of his work not directly connected to syllogistic, as it is the case with the work of Patzig mentioned above. Empty terms and precise consequence of their admission were not, however, explicitly addressed in any of Aristotle work, including those concerned with syllogistic reasoning. Malink (2013, p. 82), for example, states that "The question of whether or not an individual falls under a term seems to be irrelevant in Prior Analytics 1.1–22". Aligned with this, a view that, contrary to Łukasiewicz, admits the emptiness of terms in the Aristotelian syllogistic is gaining more and more advocates, and the discussion is ongoing.

Scholars who hold that Aristotle was not restricting terms in his syllogistic to non-empty ones refer to fragments from *Prior* and *Posterior Analytics* in which Aristotle speaks about a "goat-stag" as a syllogistic term (Bäck, 2000, p. 243), (Malink, 2013, p. 81), (Parsons, 2014, p. 12):

> (...) Similarly if it should be proved that the healthy is an object of knowledge quâ good, or goat-stag an object of knowledge quâ not existing, or man perishable quâ an object of sense (...). (*Prior Analytics*, 49a24).

And:

> (...) for of that which is not, no one knows what it is. You may know what the account or the name signifies when I say goat-stag, but it is impossible to know what a goat-stag is (...). (*Posterior Analytics*, 92b6–8).

As a goat-stag is a nonexistent, it is further argued that Aristotle is aware of such a possibility and wants his theory to account for empty terms as well. It is a fact that Aristotle does not refrain from speaking about complex terms that *can* be empty and considers such entities as "sleeping horses", which can be hypothetically empty (i.e., when no horse is actually sleeping). For example, in *Prior Analytics*, 40a38: "To illustrate the former take the terms sleep, sleeping horse, man; to illustrate the latter take the terms sleep, waking horse, man". Wedin (1978,

p. 179) quotes *Categories*, 13b12–36, where Aristotle states that both “Socrates is sick” and “Socrates is healthy” are false in case Socrates does not exist, but “Socrates is sick” and “Socrates is not sick” become opposites in that case:

> For if Socrates exists one will be true and one false, but if he does not both will be false; neither “Socrates is sick” nor “Socrates is well” will be true if Socrates himself does not exist at all. (...) For take “Socrates is sick” and “Socrates is not sick”: if he exists it is clear that one or the other of them will be true or false, and equally if he does not; for if he does not exist “he is sick” is false but “he is not sick” true. (*Categories*, 13b12–36).

Thus, Aristotle is claiming the existence of a subject as a precondition for having a truth value. This means that he was at least aware of the issue of additional existential premises required for some statements to be true. Nevertheless, the interpretations of such passages vary greatly. For example, Crivelli (2004, p. 161–162) states that “[Such passages] only commit [Aristotle] to the claim that the subject of some true singular predicative assertion refers to an object that does not exist at the time with respect to which the assertion is evaluated as true or false” and thus Aristotle is not speaking about nonexistence as such, but, speaking in medieval terms, only about the nonexistence of the object’s actual accidental supposition. Whatever the interpretation, the issue taken in relation to simple statements cannot be easily generalized to the existential assumptions of syllogistic as such.

Corcoran (1972a, p. 702), advocating for the non-emptiness assumption, refers to Aristotle’s metaphysical requirement of every universal term holding at least one particular as a motivation for the assumption of the non-emptiness of terms:

> The fact that Aristotle’s metaphysics required that each universal term hold of at least one particular provides the motivation for assigning non-empty sets to “terms” in addition to providing the key to a theoretical account of why Aristotle’s logic had “existential import”.

Bäck (2000, p. 244) proposes to read Aristotle in yet another way. What he says is that as Aristotle wants his theory to engage with sophistical arguments, he must be willing to abandon his sense of reality in order to take part in the discussions. Only after accepting the universe held by the sophists (i.e., presupposing the existence of the designata of empty terms, if needed) is one able to defend his own. Thus, even the non-existing objects are assumed to exist for the sake of the logical argument. When it comes to the validity of the syllogism, it becomes independent from the truth of its premises, which are assumed to be true regardless of the existential status of the terms they use. The syllogism containing such terms is then valid formally during the discussion but excluded from scientific demonstration since its premises are not true. This view can be considered as a proposition that presents Aristotle as a precursor to the modern notion of the universe of discourse in first order predicate calculus, which will be discussed in more detail in the next Sections.

The discussion summarized above is far from being conclusive. The tendency one may observe is that leading from the one-sidedness of first interpretations to a more and more nuanced view. While Łukasiewicz (1951, 1958) simply states that Aristotle himself held the non-emptiness assumption, the Kneales (1962) are more cautious, saying only that the assumption is needed to justify the Aristotelian system (whether or not Aristotle was aware of it), and Patzig (1968) talks not about the views of Aristotle, but about his presuppositions. The question of how Aristotle originally intended his syllogistic to treat empty terms remains open. The safest option is to say that he does not make any explicit statements about it and that the (non)emptiness does not yet emerge as an issue. We will also not be the first to claim that the existential assumptions are forced by intuitions associated with Boolean algebra and attempts to render Aristotle using modern notation (Martin, 2016, p. 74).

3.2 The Arabic world and the recognition of existence in logic

Whatever might be said about Aristotle, empty terms and existential presuppositions started to be discussed in the logic of the Arabic world, and were discussed there in detail. According to Hodges and Chatti (2020), the first one to explicitly talk about the existential import is Al-Fārābī, although his statements about it are only occasional and still indirect. In his *Categories*, 120,14-126,19, he writes that two quantified affirmative propositions with contrary predicates (e.g., every fire is cold – every fire is hot) cannot share a truth value when their subject exists and are both false when it does not exist (Chatti, 2019, 34–35) – thus, every true affirmative statement presupposes the existence of its subject. It can be also said that he is the first one to generalize to *quantified propositions* what Aristotle has only mentioned with regard to *singular* ones, as he is considering a general syllogistic form of "Some C is not a B" (Hodges and Chatti, 2020, 52). Nevertheless, in his *Syllogism*, he does not talk about empty terms at all, and so the discussion is confined to categorical statements (Chatti, 2019, p. 39). For a detailed discussion of Al-Fārābī's views on existential import consult (ku Yi, 2023) and (Chatti, 2024).

Avicenna continues to make existential assumptions more and more explicit. He holds that we cannot talk about a nonexistent and maintains that a negative categorical statement with an empty subject is true and an affirmative statement with an empty subject is false (Mousavian, 2022, p. 142), (Hodges, 2012, p. 120). What follows is that every true affirmative categorical statement must have an existing subject (Hodges, 2012, p. 132), (Bäck, 1987, p. 356). He makes statements about existential import in syllogistic, where he requires that negative propositions have an existential import as well (Bäck, 2000, 293).

Moreover, with Avicenna, the issue of *modes of existence* starts to be considered. Although Rescher (1966b, p. 72–73) holds that Avicenna continues to admit only existence *in re*, Bäck (1987) states that the existence Avicenna is talking about is not restricted to real existence as it is the case with Al-Fārābī, but apart from existence *in re*, existence *in intellectu* is being considered as well (Bäck, 1987, p. 356, 360), (Daşdemir, 2019, p. 90). Hodges (2012) refers to *al-Ibara*, 79.13f, where Avicenna says that "the subject term of an affirmative predicative sentence must be satisfied 'either in the world or in the mind'", and gives an example of a regular icosahedron which cannot be said with certainty to exist in the world and is nonetheless justified as an intension on which we can perform mathematical reasoning. Even firmer statements are to be found in Avicenna's *Al-mantiq* (a part of *Al-Sifa*, the major work of Avicenna, concerning syllogistic logic) (Daşdemir, 2024). For an overview of how Avicenna's distinction of two modes of existence was taken up and modified by later philosophers consult (Ibrahim, 2025).

There is some discussion in the literature concerning the specific nature of things existing *in intellectu* (Mousavian, 2022, p. 145–177), (Daşdemir, 2019, p. 90–96), (Bäck, 2000, p. 293–294). Here, it is important that Avicenna requires the subject of an affirmative sentence to exist *somehow*. The question of existential assumptions at this stage became a fundamental issue and had to be treated accordingly. The view which allows for existence *in intellectu* was also endorsed by Averroes (Bäck, 1987, p. 361). As Rescher (1966b, p. 74–75) refers, Averroes considers both actual and possible existence, where the latter encompasses both existents and nonexistents. In the later Arabic logic, this problem gets more pronounced and different views are considered as well. Al-Razi, for example, holds that we cannot make any judgments about anything unless it exists *in intellectu*, which means he makes a claim for the necessity of existential import on all type of statements, including the negative ones (Daşdemir, 2019, p. 96–115). In later Arabic logic, there have also been discussions on the existential import of propositions with negative predicates (Daşdemir, 2024), as well as a (much later) separate discussion on the existential import of negative propositions (Daşdemir, 2024).

The most important issue here is that whereas Aristotle does not make any open claims about existential presuppositions of his logic, especially with respect to categorical statements and syllogistic, Arabic logicians consider this issue explicitly. Starting with Al-Fārābī, all major philosophers have to take into consideration the question of existential import which from then on starts to be more and more clearly pronounced. And although the authors are mostly concerned with categorical statements, occasional comments pertaining to syllogistic inferences as such are made as well.

3.3 Medieval Europe and the origins of the universe of discourse idea

A logician to which the "discovery" of the problem of existential import in the west is ascribed is said to be Peter Abelard (King, 2023), or, by some scholars, Garland the Computist (Henry, 1984) (XI century). These claims are though based on passages which diagnose the problem of existential import only indirectly, and looking for an explicit systematic treatment of the issues associated with existential import with respect to categorical sentences and/or syllogistic inferences in these authors is looking in vain.

Later medieval logicians have developed the so-called *theories of supposition*, roughly corresponding to modern theories of reference, which provided various accounts on relation between a term and its denotations (for an overview see (Klima, 2001)). When a term is to be taken in its *natural supposition*, for example, it is to be treated as referring not only to the actually existing subjects, but also to those existing absolutely, i.e., either in the future, or in the past. The precise wording of the definition of and the precise understanding of different suppositions varies between authors (Rijk, 1971), such discussion, however, indicate that medieval logicians were aware of the possibility of considering empty terms from early on, and the doctrines of supposition can be distilled so as to provide their hypothetical views on empty terms. However, doing so obscures the discussion in the sense that it quickly becomes less about syllogistic logic and more about general problems of semantics, ontology, and even metaphysics.[54] I will therefore confine myself to note that applying, e.g., the natural supposition allows to overcome the problem of existential import, but only seemingly, as it does not provide the answer of what to do with empty terms, but only of how to understand the terms so as they are non-empty.[55]

Notwithstanding the development of the theories of supposition, in Europe, unlike in the Arabic world, the question of the implications of allowing empty terms in syllogistic was virtually nonexistent on a larger scale before the modern formal logic and discussions addressed in Section 3.1. The majority of earlier comments on the topic are from the XIV and XV centuries. Early scholastics did not see the issue at all. Peter of Spain in his *Summulae Logicales* does not say a single word concerning existential presuppositions, both of categorical statements and syllogistic reasoning (Church, 1964, p. 417). The issue was not addressed explicitly until the rise of nominalism, and the first one to pronounce it was William of Ockham (Church, 1964, p. 420). Thereafter, it was considered also by another nominalist, Jean Buridan. As we shall argue, Ockham and Buridan developed explicit theories about empty terms that can even be said to anticipate modern solutions to the problem of existential import.[56]

Ockham holds a doctrine almost identical to that of Avicenna. He says that the truth conditions of both affirmation and negation are disjunctive. He states clearly that for a negative categorical statement to be false, it is needed that its "subject and predicate does not refer to the same, and moreover, that the subject *does not refer to anything* [my emphasis] or that it refers to something to which the predicate does not" (*Summa Logicae*, 255, 13–17). Similarly, for the

[54] Or, in fact, theology. See for example the alleged role existential import has played in theological discussions concerning Anselm's argument for the existence of God (Bäck, 1981) and Acquinas' argument for God's will mutability (Villegas, 2024); note also that the medieval semantics is argued to be inherently intertwined with metaphysics (Strunk, 2013).

[55] The difference is that unlike the later discussion around modes of existence, which, as we shall argue, has led to the emergence of the notion of universe of discourse, this one has no direct impact on the formal properties of inferences, i.e., what is under question is the scope of entities which can be considered by logicians, and not the implications of using terms which do not have denotations falling under this scope.

[56] How exactly the theories of existential import developed by Ockham and Buridan correspond to modern formalizations shall be described in (Urbański et al., 2026).

affirmation to be true, we must claim both the existence of the subject and of the predication (Bäck, 2000, p. 302–303). He is thus perfectly aware of the consequences of existential presuppositions and treats the issue openly. He is not restricting logic to non-empty terms only, but he is making the truth of statements dependent on the existence of terms' denotata. The difference between Ockham and Avicenna or Averroes is that Ockham requires every subject to exist *in re* and is not considering existence *in intellectu* (Bäck, 2000, p. 304), a claim that can be ascribed to his nominalism.

The thing most interesting about Ockham is that his treatment of existential assumptions is at times conditional. When talking about the *Dictum de omni et nullo*, he states that for an example sentence to be true, its subject need not always exist. He distinguishes between a categorical and a conditional proposition and seems to be treating some of the syllogisms as a conditional reasoning accordingly (*Summa Logicae*, II.4):

> Similarly, just as a syllogism holds for all terms, so it holds no matter how things change. But according to Aristotle this is now a good syllogism: "Every colored thing exists; every white thing is colored; therefore every white thing exists". Similarly, this syllogism is now in fact good: "Every animal is a man; every donkey is an animal; therefore every donkey is a man". For these syllogisms are regulated by the rules governing "every" (*dici de omni*).
>
> And surely, whoever denies such syllogisms is incapable of perceiving any truth whatsoever. Therefore, even if every white thing and every donkey were destroyed, these would still be good syllogisms. And they would still in that case conform to the rules governing "every" (*dici de omni*), just as they now do.
>
> Hence, distinctions such as that between the "to be" which is an operation of a being and the "to be" of condition are frivolous, and they are posited by those who do not know how to distinguish between a categorical proposition and a conditional proposition. Hence, these propositions are distinct: "A donkey is an animal" and

> "If a donkey exists, an animal exists". For the one is categorical and the other is conditional and hypothetical – and they are not interchangeable. Rather, one can be true while the other is false. In the same way, "A non-creating God is God" is now false, and yet these conditionals are true: "If a non-creating God exists, then God exists" and "If this is a non-creating God, then it is God". (Ockham, 1998, p. 99).

For the truth of the conditional statement, it is not needed that its subject exists – it suffices only that the sentence is true whenever its subject exists (Spade, 1999, p. 42). This subject, however, must exist actually. Ockham can thus be viewed as a precursor of the modern notion of the universe of discourse (quantifying over a given and presupposed domain), only that there exists only one possible interpretation for him, namely objects existing *in re*, i.e., he is considering only things existing actually as possible members of this quantification domain.

Ockham's treatment of categorical propositions thus distinguished between two types of truth conditions: categorical and conditional. A categorical proposition asserts something about actually existing things, while a conditional proposition asserts something about what would be true if certain things existed (Ockham, 1974). For Ockham, the proposition "All S are P" can be interpreted in two ways:

1. Categorical interpretation: "Everything that is S is P", which requires that at least one S actually exists for the proposition to be non-vacuously true

2. Conditional interpretation: "If anything is S, then it is P", which can be true even if no S exists (vacuous truth)

Ockham explicitly recognized that these two interpretations have different logical properties. Under the categorical interpretation, "All S are P" implies "Some S is P" (subalternation holds). Under the conditional interpretation, this implication fails when S is empty. Ockham's analysis of syllogistic validity incorporated this distinction – he recognized that certain syllogisms are valid only under the categorical interpretation, while others are valid under both interpretations. For example, *Barbara* (All M are P; All S are M; therefore All S are P) is valid under both interpretations, while *Barbari* (All M are P; All S are M; therefore Some S are P) is valid only under the categorical interpretation (Klima, 2017).

Ockham's distinction between categorical and conditional interpretations provided conceptual resources for handling empty terms systematically. Under the conditional interpretation, universal propositions can be true even when their subject terms are empty, which is a significant departure from any simple extensional reading. Under the categorical interpretation, universal propositions presuppose non-empty terms and support subalternation to particular conclusions. Whether a syllogism requires existential assumptions thus depends on which interpretation is adopted.[57]

Jean Buridan talks about existential assumptions as well and also requires the subject of every true affirmative sentence to exist *in re*. He makes this claim more explicit than Ockham and writes, for example, that the statement "chimera is a chimera" is false since chimeras do not exist actually (Bäck, 2000, p. 304). As quoted by Bäck, van der Lecq (1983, p. xxiv) says that according to Buridan, "The name 'chimera' signifies all things that have been composed in the description of the term, although it does not refer to anything". He is, however, skeptical about Ockham's conditional approach and requires the existence of a subject to be read *verbatim* (King, 1985, p. 25): the proposition "every S is P" is not to be read "if S exists, then it is a P" but rather as "consider the S's: they are each P". The approach of Ockham is also vulnerable to paradoxes taken up by Buridan. For example, we can imagine a situation when "God annihilates all negatives", and yet the proposition "no proposition is negative" is and would be false whenever formed (*Sophismata*, 8, O.1). From the points of view of both philosophers, however, talking about non-actual beings existing somehow is strictly forbidden and regarded as nonsense (Klima, 2009, p. 159).

[57] Read's analysis of medieval theories of consequence provides detailed historical context for Ockham's distinction and its role in XIV-century discussions of validity (Read, 1995).

Buridan also developed Ockham's division of syllogistic moods further, providing a more systematic treatment of existential commitments in syllogistic logic. In his *Summulae de Dialectica*, he explicitly discussed the problem of existential import and formulated rules for determining when syllogisms require existential assumptions (Buridan, 2001). He also developed the supposition theories further, making them applicable to classification of syllogistic moods – he distinguished between a simple supposition (*suppositio simplex*) and a personal supposition (*suppositio personalis*) of terms. In simple supposition, a term stands for a universal or concept; in personal supposition, it stands for actual individuals. This distinction allowed Buridan to analyze the existential commitments of different propositions systematically (Buridan, 2001).

Thus for Buridan, particular propositions always involve personal supposition and therefore require actual instances. "Some S is P" cannot be true unless at least one actual individual is both S and P. Universal propositions, however, can involve either simple or personal supposition. "All S are P" with personal supposition requires actual instances of S, while "All S are P" with simple supposition does not (Klima, 2003), (Read, 2013). Buridan applied this framework systematically to syllogistic validity, recognizing that certain syllogisms require terms to have personal supposition while others succeed with simple supposition alone. He formulated explicit rules for determining when existential assumptions are needed (Buridan, 2001), rules that addressed what we would now recognize as questions about when reasoning requires existential witnesses.

What is to take from the above is that even if we agree on the tacit existential assumption of non-emptiness in Aristotle, this view was certainly not held by the medieval logicians mentioned above. Although for a short time, empty terms were discussed fervently by the logicians associated with nominalism (Klima, 2009, p. 143), and the sole discussion between Ockham and Buridan concerning discrepancies in their views is vast. The culmination of this trend is the *Ars Logica* by John of St. Thomas. John is holding the doctrine of existential import as developed by the nominalists, and parts of his theory greatly resemble the modern notion of the universe of discourse, a tendency which starts with Ockham and is becoming more and more pronounced. John does not only take into consideration things existing *in re* and *in intellectu* but also those existing in the past or in the future (Church, 1964, p. 420). Unfortunately, he does not comment on syllogistic as such and talks only about categorical statements.

It is then seen that medieval logicians after Ockham were perfectly aware of the existential presuppositions of categorical statements. Why it was left unaddressed earlier and began only with Ockham is a subject of speculation. One can suspect that what sparked the sudden discussions was the character of the doctrine of nominalism and consent to challenge old modes of thinking among the protagonists of *via moderna*.[58] References to syllogistic reasoning are still quite rare, but they do appear at times (as seen in Ockham and Buridan). What is most interesting are the first efforts made by Ockham and later by John of St. Thomas in the direction of the modern idea of the universe of discourse.

Nevertheless, it must be admitted that the nominalist tradition thus developed a sophisticated understanding of existential import. Ockham's conditional interpretation and Buridan's theory of supposition provided them conceptual tools for handling empty terms and determining whenever existential assumptions are required for valid inference. These medieval insights were largely forgotten in the early modern period but were rediscovered, in different form, in XX-century logic.

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[58] The *via moderna* as a theological and philosophical movement developed in XIV and XV centuries and, in opposition to the realist and rationally oriented *via antiqua*, emphasized nominalism, freedom of will and God's absolute power.

3.4 *Logique de Port-Royal*, Leibniz, and the mathematization of logic

Until the early XVII century, views of Ockham and the nominalists must have become widespread, and empty terms were considered openly. Hobbes, being a philosopher from the later and empirically oriented generation, criticizes scholasticism as a "vain philosophy", for (among others) employing "barbarous" empty terms (Sgarbi and Cosci, 2018). He also talks about syllogistic in rather depreciating terms in his autobiography, which shows that he must have been contemptuous about it indeed – see (Maurer, nd) and (Hobbes, 1839).[59] We shall claim that the issue of empty terms became obsolete and hence cast aside (and almost forgotten) due to the triumph of Enlightenment and empirical sciences, which in turn allowed for one-sided interpretations of Aristotle's ontological assumptions in the first half of the XX century.

Indeed, serious and detailed investigations of existential import were abandoned around the third decade of the XVI century, and the discussions ceased again (Ashworth, 1973). What was discussed later concerning syllogistic and logical reasoning in general were the differences between extensional and intensional approaches. Through this, we can further trace how the

[59] However, logic played an important role in the philosophical system of Hobbes, his intention being not to fully dispense with it, but to reform it and free it from the vain discussions about metaphysics and semantics (Pécharman, 2015). In this context, the thing most noteworthy here is that he openly treats syllogistic inferences as **conditionals** (similar to Ockham); to quote from (Jesseph, 2018), according to Hobbes, the categorical syllogism

> Every man is a living creature,
> Every living creature is a body, *therefore*
> Every man is a body.

is equivalent to the *hypothetical syllogism*:

> If anything be a man, the same is also a living creature,
> If anything be a living creature, the same is a body, *therefore*
> If anything be a man, the same is a body. (consult *De Corpore*, 1.4.13).

We may then treat his approach as a radical counterpart to the early medieval discussions on the theories of supposition described earlier in this Chapter.

notion of the universe of discourse has been gaining more clarity. In the *Port-Royal Logic*, Nicole and Arnauld speak of signification as a relation between the intensional concepts and extensional material things, giving an intensional interpretation to extensions (Martin, 2017). The first one to clearly allow for denotata of terms in his syllogistic to be systematically interpreted as things *in intellectu* was Leibniz (Martin, 2017), who even ascribes such a view to Aristotle:

> I have previously observed that Aristotle may have had a specific reason for the common arrangement [of terms]. For instead of saying A is B, he has a custom to say B is in A. (. . .) And this manner of expression should not be despised, as it is indeed the case that the predicate resides in the subject, or better, that *the idea of the predicate is contained in the idea of the subject* [my emphasis]. (. . .) The common way of expression pertains primarily to individuals, but the one of Aristotle has *more consideration for ideas or universals* [my emphasis]. (Gerhardt, 1962, p. 468).[60]

Moreover, Leibniz does not treat extension in his logic as a set of actually existing beings that fall under a particular concept. Rather, he speaks about the set of all possible individuals (Lenzen, 2004), from which follows that even extension cannot be treated simply as existence *in re*. When it comes to empty terms, Leibniz builds his logic using non-empty sets, and his system is claimed to be sufficient for deriving the laws of Boolean algebra of sets ((Lenzen, 2004), see also (Peckhaus, 2024)).

[60] See also (Leibniz, 2019, p. 138, 140): "Aristotle himself seems to follow the [intensional] path of ideas, for he says that animal is in human, naturally the concept in the concept, wile, looking from the other side, humans would have be in animals" (Aristoteles ipse viam idealem secutus videtur, nam dicit Animal inesse homini, nempe notionem notioni, cum alias potius Homines insint animalibus).

Difficultates quaedam logicae

Of a particular interest is Leibniz's paper *Difficultates quaedam logicae*,[61] in which he develops his views on existential import in detail – in this work, as we shall argue, one can search not only for a further development of the universe of discourse idea, but also for the beginning of the processes which have resulted in the "mathematization" of logic in the XX century. Syllogistic has played a principal role in Leibniz's logical endeavors, which were ultimately aimed at creating the *calculus ratiocinator* and *characteristica universalis*. His forerunning of ideas that underlie Boolean algebra and algebraical logic (Hailperin, 2004), as well as his various efforts at both algebrizing and arithmeticizing,[62,63] syllogistic, are facts now well recognized. However, as Leibniz did not consider his results to be satisfactory, he, as "one of the greatest logicians" (Bocheński, 1961, p. 266), did not publish a single paper on logic throughout his lifetime (Castañeda, 1990, p. 15). What we now know is due to the work of Couturat (1901), (Leibniz, 1903), and later scholars, but there is most probably a great amount of Leibniz's papers concerning syllogistic which are still not discovered,[64] and many which are known but which remain understudied and which significance is not fully recognized. One of such papers

[61] The *Difficultates...* were first printed in the 1765 *Œuvres philosophiques latines et françaises de feu Mr de Leibnitz, tirées des ses Manuscrits qui se conservant dans la Bibliothèque royale à Hanovre et publiées par M. Rud. Eric Raspe* (Leibniz, 1765), and then brought back to life only with Couturat's *Opuscules et fragments inédits de Leibniz : extraits des manuscrits de la Bibliothèque royale de Hanovre* (Leibniz, 1903). For the English translation of this text see (Leibniz, 1966, p. 115–121); for the Latin text together with German translation and a brief commentary see (Leibniz, 2019, p. 122–170).

[62] The main goal of Leibniz at which his attempts were aimed was to **extend** syllogistic to a more general logical system. His ultimate failure to overcome the Aristotelian style of thinking about logic is said to be due to his excessive respect for Aristotle and his thought; but Leibniz's work is also said to be the first to *redirect* the thinking about logic which ultimately have led to the creation of more general systems in the XX century (Agrela, 2010).

[63] For a descriptions of Leibniz's *arithmetizations* of syllogistic see (Sotirov, 1999), for descriptions of his algebraizations, see (Robering, 2014), (Doull, 1991), (Leibniz, 2021), (Leibniz, 1966, 25–33), for a general account of Leibniz's logic see (Dürr, 1949) and (Kauppi, 1960).

[64] The unpublished manuscripts of Leibniz concerning *mathematics* are still being edited and translated (see, e.g., the project PHILIUMM: https://eman-archives.org/philiumm/home), and the edition of the whole body of Leibniz's oeuvre, amounting to several hundred thousands of pages (Poser, 2004), is said to take at least another fifty years to complete.

is titled *Difficultetes quaedam logicae*, in which are present all of the most significant themes of Leibniz thought concerning syllogistic reasoning: algebraization of the Square of Oppositions categorical sentences, development of an intensional approach to syllogistic terms,[65] and, most importantly for us, detailed treatment of the existential import issue.

The answer to the problem of existential import is, according to Leibniz, very simple,[66] as it is the misunderstanding that causes it. The latter is that in natural language, we tend to mix two different interpretations of syllogistic terms. As an example, Leibniz looks at the conversion *per accidens*: "Everyone laughing is a human, therefore some human is laughing",[67] and writes that its fault lies in the fact that "it can and could have happened that no human is laughing right now, or that no human has ever laughed, or even that no human has ever existed at all". The source of this confusion is, however, rooted in our preference to read the statement "some human is laughing" as pertaining to reality, whereas the statement "everyone laughing is a human" is read as a **conditional**, or, more precisely, as pertaining to *possibility* instead. In other words, the first statement ("everyone laughing is a human") treats "laughing human" as a **species** of a genre "human", whereas the second one ("some human is laughing", or better: "there is some laughing human"[68]) treats "laughing human" not as a species (therefore, "within the realm of ideas"), but as an actuality.

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[65] The intensional interpretation of syllogistic proposed by Leibniz is in itself subject of many debates. Among others, see (van Rooij, 2014), (Brassler, 1998), (O'Briant, 1967), (Swoyer, 1995), (Konkova and Legeydo, 2022). For example, the first major interpretation of Leibniz's logic, the one of Couturat (Couturat, 1901), has been criticized for having to much preference for extensional approach to syllogistic terms and therefore missing the significance of many of Leibniz's crucial ideas (Rescher, 1954), (Schmid, 2012).

[66] The *Difficultates...* are Leibniz's most detailed paper on existential import, but there are also voices stating that his views on this problem were changing, and the stance developed there is not to be regarded as the ultimate one (O'Briant, 1967). However, a detailed analysis of the evolution of Leibniz views is beyond the scope of this work.

[67] In general form: SaP $\rightarrow$ PiS.

[68] The problem with translating categorical statements from Greek and/or Latin into other languages (or, more generally, the problem with translating them from any and to any language) is not a trivial one and it is often hard to render the original construction. It is worth remembering that already in antiquity, there have been discussions regarding the ultimately correct spelling of Aristotelian sentence: see the fragment "Copula" in Chapter 1.

To resolve this, Leibniz proposes a rather obvious solution: the problem disappears, when both of the statements are given the same interpretation, i.e., when we treat both of them as pertaining to reality (extensionally) or to possibility (intensionally). What causes as to treat them differently is in turn the ambiguity inherent to natural languages. At the same time, Leibniz states that his algebraical calculus for categorical sentences[69] removes this difference in interpretation by a precise analysis which neutralizes this natural-language ambiguity – The *being* present in his analysis is always to be read either extensionally or intensionally (with the preference of Leibniz given to the latter (e.g., (Lenzen, 1983)), and hence does not allow the problem of mixing the two approaches, according to Leibniz the source of the whole confusion, to occur. For from "everyone *possibly* laughing is a human" we may safely infer "someone *possibly* laughing is a human", as we can safely infer "someone *actually* laughing is a human" from "everyone *actually* laughing is a human".[70]

Leibniz is therefore further and more explicitly expounding the two ways of interpretation of syllogistic terms which we have previously identified: *in re* – the actual existence of the subject; and *in intellectu* – the one pertaining to the possibility or to existence as an *idea*. What is more interesting and more important historically speaking, is that the intensional treatment of terms does not eliminate the problem of existential import, as Leibniz has thought, but instead "moves it one step higher". If at the *in re* "level", the existential assumptions are about the

[69] In *Difficultates...*, Leibniz refers to his "old analysis" ("*reductio mea vetus*", see (2019, p. 126)), which most likely means the one developed in *Generales Inquisitiones de Analysi Notionem et Veritatum* ((Leibniz, 2021), see also (Malink and Vasudevan, 2016), (Castañeda, 1990), (Castañeda, 1976)). Precisely, his calculus presented in *Difficultates...* looks like:

Every A is B $\Longrightarrow$ AB = A
Some A is not B $\Longrightarrow$ AB $\neq$ A
No A is B $\Longrightarrow$ AB $\neq$ AB-*being*
Some A is B $\Longrightarrow$ AB = AB-*being*.

Where *being* is to be read consequently according to only one interpretation: either extensional or intensional (see below in the main text).

[70] We cannot, however, safely infer "someone *actually* laughing is a human" from "someone *possibly* laughing is a human".

actual existence of a subject, at the *in intellectu* level they are not merely disappearing, but become about the logical "non-contradictoriness" of a concept under question instead (Leibniz, 2019).

The stance taken by Leibniz, although being perhaps a sole that detailed exposition of the empty terms problem during this historical period, gives us a right to speculate about the influence of Leibniz's on the development of logic in next centuries. For if the existential assumptions become about the logical "non-contradictoriness" of the *concepts*, then Leibniz inadvertently shifts the notion of existential presuppositions of logic from concerning specific instances of the existence of sentence subjects (i.e., specific individuals) to a general existential commitment to the existence of possible individuals *as a category* of entities over which we can quantify.

As this general (and generalized) understanding of existential commitments can be further applied to talk about entities categorically different from individuals (such as classes, sets, functions, etc., with their corresponding subfields in modern terms being first order logic for individuals as a class, second order and higher order logic for sets, sequent calculi for functions, etc.), we can speculate to view Leibniz's endeavor as a first step in the direction of the *mathematization of logic*, and Leibniz himself not only as a "father of algebraical logic" (compare (Hailperin, 2004)), but also as the father of the conceptual foundations on which modern formal systems have been built.[71]

[71] This speculation can be further advanced if we look at Leibniz's forerunning of the purely formal aspects of Boolean algebra and modern formal logic (Lenzen, 2004), (Peckhaus, 2024), level of sophistication of Leibniz's logic (Castañeda, 1974), (Lenzen, 1986) compared to the traditional formulation of syllogistic, and Leibniz's influence on the founders of analytic philosophy (O'Briant, 1979), (especially Russell, who was a great specialist on Leibniz (e.g., (Russell, 1903)), but whose precise reception of Leibniz's work remains as a topic to study). In light of the latter, we could even juxtapose the aforementioned alleged "mathematization of logic" with the analytical project of the "logicization of mathematics", stating that the latter must have had collapsed because the logical framework of it was already "infected" by the Leibnizian "mathematization", i.e., the ontology of logic used by the analytic philosophers was already "mathematical".

3.5 Side discussion: the role of diagrammatic representations

Whatever might be the case with Leibniz's views on existential import and his influence on the later thought, the detailed discussion of this topic have ceased thereafter.[72] Some authors point out that what helped to make the issue of empty terms more explicit again was the invention of the Venn diagrams in 1881 (Wu, 1969). This is at least intuitively true with respect to syllogistic, and the use of similar diagrams can be witnessed in works devoted to Aristotelian syllogistic as well – In his *Elements of Mathematical Logic*, Łukasiewicz (1951) uses Eulerian circles to interpret both affirmative and negative categorical statements and whole syllogisms, from which he then deduces his axioms for syllogistic. After him, Bednarowski (1956) interprets the logic of Aristotle as a theory of relations between two classes, which can be represented using Eulerian diagrams. It is possible that the influence of this method of representation was in part responsible for making the non-emptiness assumption a natural choice, and as the cognitive role of diagrams is being more and more recognized recently ((Mancosu, 2005), (Giardino, 2018), (Carter, 2012), (Carter, 2019a), (Carter, 2021), (Shimojima, 2015)), we may also go into some details of the history of the use of diagrams in syllogistic, which will help us better understand the shifts in attentions to the problem of non-emptiness described so far.[73]

Logical diagrams were widely used to represent syllogistic inferences throughout the centuries. Here, we want to extend the aforementioned remark on Venn diagrams and Eulerian

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[72] As always, however, some comments can be identified: Kant, for example, has a theory of existential import which is assumed to have held affirmative sentences to have it and negative sentences not to have it, but his logic has been also interpreted as being free from existential commitments whatsoever (Vanzo, 2014a). Our point here is that the issue ceased to be of a great significance to the later thinkers and was discussed on the side, if at all.

[73] This approach is also in accordance with the recent trends in philosophy of science to look at the scientific *practices* undertaken in various disciplines, and can be treated as a study which extends to logic what has been stated with respect to the role diagrams play in mathematics and in mathematical practice. For an overwiev of the Philosophy of Mathematical Practice (PMP) see (Giardino, 2017), (Schlimm and González, 2020), (Carter, 2019b); for a study in practice-based philosophy of logic see also (Novaes, 2012).

circles,[74] and give an account of how the logical diagrams historically used for diagrammatically rendering syllogistic inferences might have affected the historical evolution of the debate on existential import, empty terms, and their use in syllogistic. The proposition is then to view the evolution of syllogistic diagrams and the evolution of answers to the problem of existential import in parallel, and consequently draw lines of possible mutual influences between these two areas.

We have seen that the problem of empty terms and existential import was debated throughout history, although with different levels of engagement: It was not yet stated by Aristotle; was first explicitly addressed in the Arabic world, starting with Al-Fārābī; In Europe, it was not debated in detail until the rise of nominalistic philosophy, to be thereafter resurrected by Leibniz and receive a full attention only after the development of the Boolean algebra and modern formal systems.

To better understand these sudden historical shifts in attention, let us investigate the parallel evolution of syllogistic diagrams, noting that:

1. In the Late Antiquity[75] and, most notably, in the Byzantine Empire, a sophisticated system of diagrams (the so-called "Byzantine diagrams"[76]) was gradually developed

[74] There has been recent research on how the modern forms of visual representations have treated the concept of existence (Lemanski, 2024). What we propose is to, again, extend such considerations to encompass the whole body of diagrammatic representations used to represent syllogistic inferences throughout the history.

[75] For the alleged use of diagrams by Aristotle himself see (Englebretsen, 2020, p. 8–26), (Wesoły, 2012), and (McConaughey, 2025b).

[76] Diagrams were, of course, used by logicians from the "Latin" tradition as well. The Square of Opposition or The Logical Square is an obvious example, but one may also point to the Tree of Porphyry (Verboon, 2014) or the so-called *pons asinorum* (Hamblin, 1976) (note that the later use of the *pons asinorum*,i.e., as a linguistic expression, is a later invention). For an example of western (or "Latin") use of the Byzantine diagrams themselves, see for example (Korcik, 1938, p. 284), where he talks about a diagrammatical illustration of sorites found in the work of Julius Pacius (Pacius, 1584). A renaissance edition of Paulus Venetus *Logica Parva* (Venetus, 1536) contains a similar diagram as well, but it is not explicated, nor in any way are the rules of the diagrams explained. What follows is that the western logicians must have been aware of the existence of the Byzantine diagrams, but nevertheless did not make use of them, and their use (together with potential implications of it, which we will discuss) never played a major role in the understanding and development of logic in the West. Our main argument that the omission of the existential assumptions on the diagrams has further prompted the neglect of these assumptions in the West nevertheless holds, as we note that the "Latin" counterpart of the Byzantine diagrams, the *Barbara, Celarent...* mnemonic described in Chapter 2, does

(Agiotis, 2022), (Cacouros, 2001), (Panizza, 1999), (Safran, 2020). These diagrams were constructed so as to allow for rendering both all the moods from respective syllogistic figures as well as the reductions of third- and second-figure moods to first-figure ones (see (Triantafyllou, 2023)). However, they did not allow for indicating whether or not the terms are (not)empty.

2. In the Arabic world, Al-Barakāt has proposed a system of linear logical diagrams. Although at first he allowed for empty terms, his diagrammatic method had no way of representing them, and he later renounced them altogether (Hodges, 2023).

3. Leibniz himself and, following him, Lambert (1764), have created systems of linear diagrams for syllogistic as well. These diagrams were developed so as to explicitly display the distribution of terms in statements (Bellucci et al., 2014), and, in the case of Lambert, treated a single dot as indicating the denotation of individual term (Englebretsen, 2020, p. 31).

4. Leibniz also developed a system of spatial diagrams for representing syllogistic moods, similar to the later system of Euler circles, which allowed for representing the set of individuals constituting the extension of the term. Euler, later, introduced an asterisk to explicitly indicate that a part of diagram is non-empty (Englebretsen, 2020, p. 33).

5. Venn diagrams allowed for indicating the emptiness of a term by shading its relevant area (Englebretsen, 2020, p. 37). Later, Peirce[77] added a method for explicitly indicating the denotation of a term by placing an "x" in the area of its extension (Bellucci, 2013) (see also (Moktefi and Pietarinen, 2016)).

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not account for the (non)emptiness of terms as well, and that *status quo* has not been changed with the later influence from the "Greek" tradition either. The Greek tradition itself was in turn so vast that traces of it can be found even in Serbian manuscripts (Žunjić, 2011).

[77] For an overview of Peirce's diagrammatic systems for logic see (Roberts, 1973), (Shin, 2002), (Forster, 2013), and (Carter, 2020).

With this in mind, several mutual influences can be observed: first, as the existence of terms' denotata in syllogistic was not explicitly addressed by Aristotle, it was not explicitly addressed in the Greek diagrams either, which further prompted their neglect in Europe; second, in the Arabic world, where the issue was discussed explicitly, it was also discussed with respect to diagrams and prompted the creation of diagrams which are able to note solely the existing subjects; third, in the West, diagrams which allowed for explicitly marking the denotation of term-subjects were not developed until the time of Leibniz, who has himself discussed the problem of existential import and hence was aware of it. Moreover, the linear diagrams of Leibniz and Lambert also seem to have no way of indicating the emptiness of terms, which, in the case of Leibniz, aligns with his intensional interpretation of terms and his view on existential import as presented in the discussed *Difficultates quaedam logicae*. Lastly, the Venn and Venn-Peirce diagrams, devised to map Boole's algebraical logic and thus allowing to explicitly mark the denotation of terms, were further applied to syllogistic, making the issue of existential import in syllogistic more explicit and provoking further discussions on the topic.[78]

3.6 Towards modern formal logic

What sparked renewed interest in the question of empty terms was the development of Boolean algebra, although Boole himself refrains from making any direct comparison between his system and that of Aristotle:

[78] Another example of how the particular method of representation (in this case: either Byzantine diagrams or the Latin mnemonic) has influenced the development of syllogistic can be the fate of the fourth figure in either of the traditions. In the Byzantine tradition, the diagrams were the main method of representation of syllogistic inferences and they could not accommodate representing the fourth figure; hence, the fourth figure was not recognized as a separate one. In the Latin tradition, the mnemonic names *could* be altered so that they have represented the fourth-figure moods instead of the first-figure indirect ones, and hence, although it took some time – the fourth figure was finally recognized as a separate one. This line of thought is especially interesting as we note that the Byzantine tradition is mainly a *manuscript* tradition, i.e., the diagrams were actively used by by learners and logicians to represent the syllogistic moods as they were reading, e.g., Aristotle, and hence played a big imaginative role when it comes to understanding the nature of syllogistic and syllogistic moods.

> My object, indeed, is not to institute any direct comparison between the time-honoured system of the schools and that of the present treatise. (Boole, 1854, p. 226).

Venn, in *Symbolic Logic*, considers the issue of existential import openly:

> Now if we adopt the simple explanation that the burden of implication of existence is shifted from the affirmative to the negative form; that is, that it is not the existence of the subject or the predicate (in affirmation) which is implied, but the non-existence of any subject which does not possess the predicate, we shall find that nearly all difficulty vanishes. (Venn, 1881, p. 141).

He writes that taking up the position of symbolic logic will resolve all the perplexities of the subject (Wu, 1969). Peirce and Russell also comment on the issue. Peirce writes:

> A and E, All S is P, and No S is P, are true together when no S exists, and false together when part only of the S's are P. I and O, some S is P, some S is not P, are true and false together under precisely the opposite conditions. (Peirce, 1932).

Russell states that we will get rid of the problem by distinguishing two meanings of existence and reading existence employed in symbolic logic as:

> To say that A exists means that A is a class which has at least one member. (Russell, 1905, p. 398).

The first one acknowledged to explicitly state that Aristotle's syllogistic requires its terms to be non-empty was Śleszyński:

> We assume that these classes are non-empty, which means that each class contains at least one element (some logicians either do not see, or choose not to see, that Aristotle assumes this). (Śleszyński, 1921, p. 5).

The issue was then discussed also among logicians from the Lwów-Warsaw School, with Kazimierz Ajdukiewicz contributing to the discussion (1926). Łukasiewicz (1923) read and reviewed Śleszyński's articles, and the widespread popularity of the non-emptiness assumptions stems from his 1951 work (1951).

3.7 Conclusions and questions

The historical development of the issue of empty terms can be seen as twofold. Most importantly, we observe a rising awareness of empty terms as an issue that needs to be covered. The issue was irrelevant or nonexistent in Aristotle, present in the Middle Ages, and played a substantial role in the modern interpretations of Aristotle's work. The difference which is seen is that up until Łukasiewicz it was discussed either with respect to the validity of the Logical Square, as in the medieval and early modern period (as the problem of existential import, mostly), or with respect to categorical statements in general, as when the Boolean algebra emerged. Only with the works of Łukasiewicz and their popularity, the discussion turned to syllogistic as such and ontological assumptions of syllogistic reasoning started to be considered.

Furthermore, in parallel with the above, we can trace the origins of the universe of discourse idea, beginning in the works of Ockham and getting more and more pronounced, with *Logique de Port-Royal*, Leibniz, and finally Boole stating this idea more and more explicitly.

Apart from the above, we can raise some minor remarks. Firstly, we can speculate about why the question of existential assumptions was nonexistent at times and then raised to the awareness and all of a sudden discussed with great detail. In the case of The Arabic world, we propose to say that the sudden and open rise of interest can be explained by the language differences between Indo-European and Semitic languages; in the case of medieval Europe, we attribute it to the development of the school of nominalism. Both cases, however, need further study. Secondly, we say that the one-sidedness in interpretations of Aristotle's ontological commitments in the XX century can be explained by the fact that the issue of non-emptiness became obsolete and almost forgotten up until most recent, due to the triumph of Enlightenment in the XVII century and the critique of scholastic logic.

Chapter 4

Existential import in first order logic

4.1 Interpreting syllogistic in modern terms: first order predicate calculus

Building upon the rich historical background presented in previous Chapters, we now turn to addressing the problem of what additional existential premises should the syllogistic moods have once rewritten using first order predicate calculus. We choose first order predicate calculus for several reasons: (1) Epistemic transparency – it makes existential commitments explicit through quantifiers, illuminating what traditional systems presupposed rather than solving the problem through alternative semantics; (2) Contemporary dominance – it is the standard framework in logic courses and philosophical analysis, making the comparison broadly relevant and approachable for wide audience of non-specialist philosophers; (3) Theoretical neutrality – it is a general framework for quantification, not designed specifically for syllogistic, so it reveals genuine differences rather than engineered ones. When we say *Barbari* "requires" $\exists x S(x)$ in first order predicate calculus, we mean first order predicate calculus makes explicit a commitment traditional logic handled through categorical interpretation, presuppositions, or term restrictions.[79]

To analyze the existential requirements of Aristotelian syllogisms systematically, we adopt the following translation scheme for categorical propositions, already discussed in the previous Chapter:

A (Universal Affirmative): “All S are P” $\forall x(S(x) \to P(x))$

E (Universal Negative): “No S is P” $\forall x(S(x) \to \neg P(x))$

I (Particular Affirmative): “Some S is P” $\exists x(S(x) \land P(x))$

O (Particular Negative): “Some S is not P” $\exists x(S(x) \land \neg P(x))$

This translation scheme is standard in modern logic and makes the existential commitments of particular propositions explicit while allowing universal propositions to be vacuously true when their subject terms are empty.

With this in mind, three different answers to our initial question (what additional existential premises should the syllogistic moods have once rewritten using first order predicate calculus?) are possible, with the response varying between three different types of moods:[80]

G1 No additional premises are needed, as no special ontological commitments are required; this is the case for syllogisms in which both premises and the conclusion are general sentences.

G2 No additional premises are needed, as required ontological commitments are addressed by a particular premise.

[79] See Sommers (1982) and Englebretsen (1981, 1987) for alternative formalizations that preserve traditional inferences through different semantic mechanisms. These alternative formalizations are valuable for those seeking to preserve natural language inference patterns or develop non-classical logical systems; our different purpose is to understand what traditional logic presupposed by translating into the contemporary standard framework.

[80] A comprehensive description of these groups, together with considerations about how they map onto frameworks of historical figures discussed in Chapter 4, has been developed in (Urbański et al., 2026).

G3 Additional existential premise is needed, as required ontological commitments are not warranted by the premises of the syllogisms.

The moods can then be divided, accordingly, into three different categories:

Group 1 (G1): Syllogisms Requiring No Existential Premises

These syllogisms are provable in FOL without any existential assumptions. They remain valid even when all terms are empty. There are five syllogisms in this group:

Figure 1: *Barbara*, *Celarent*

Figure 2: *Cesare*, *Camestres*

Figure 4: *Camenes*

Group 2 (G2): Syllogisms with Existential Commitments Satisfied by Particular Premise

These syllogisms have particular premises (I or O propositions) that provide the necessary existential commitments. The particular premises explicitly assert the existence of at least one individual satisfying certain conditions, and this is sufficient for the conclusion. There are ten syllogisms in this group:

Figure 1: *Darii*, *Ferio*

Figure 2: *Festino*, *Baroco*

Figure 3: *Disamis*, *Datisi*, *Bocardo*, *Ferison*

Figure 4: *Dimaris*, *Fresison*

Group 3 (G3): Syllogisms Requiring Additional Existential Premise

These syllogisms have only universal premises but particular conclusions. They require an explicit existential premise asserting that at least one instance of a specific term exists. There are nine syllogisms in this group, three of them being moods historically recognised as legitimate (*Bramantip*, *Felapton*, *Darapti*), and five of them being the subalternated moods in the sense outlined in Chapter 2 (*Barbari*, *Celaront*, *Cesaro*, *Camestros*, *Camenos*):

Figure 1: *Barbari*, *Celaront*

Figure 2: *Cesaro*, *Camestros*

Figure 3: *Darapti*, *Felapton*

Figure 4: *Bramantip*, *Camenos*, *Fesapo*

This three-group classification provides a complete and systematic answer to the question of what existential premises are needed for traditional syllogisms in first order predicate calculus. We should note that the identification of the nine G3 syllogisms as requiring existential supplementation is not itself new. The standard textbook treatment, codified in Hurley's *A Concise Introduction to Logic* (2018) and Copi and Cohen's *Introduction to Logic* (Copi et al., 2019), distinguishes fifteen "unconditionally valid" syllogisms (valid from the Boolean standpoint without additional existential premises) from nine 'conditionally valid' syllogisms (valid only from the Aristotelian standpoint, on the condition that a specific term denotes existing things). This two-group classification is relatively well known (Burgess, 2009), and the identification of which specific term must be non-empty for each conditionally valid form also appears in the pedagogical literature (see, e.g., Eshing (2022)). However, the standard terminology obscures an important structural difference within the group of the fifteen moods. To say that all fifteen are valid "without existential import" conflates two distinct situations: syllogisms whose premises and conclusion involve no existential content at all, and syllogisms whose premises already carry the existential content the conclusion demands. What has not, to our knowledge, been explicitly articulated is the further subdivision that this distinction requires and that our analysis will show: G1, whose syllogisms have exclusively universal premises and universal conclusions and are therefore valid even when all terms are empty – they involve no existential import at either the propositional or the inferential level; and G2, whose syllogisms contain at least one particular premise that already carries the existential import (propositional sense) required by the particular conclusion – they need no existential supplementation (inferential sense) precisely because their premises already supply it. The tripartite G1-G2-G3 classification thus refines the standard binary partition by making explicit *why* the fifteen unconditionally valid syllogisms need no existential supplement – either because no existential content is demanded at all (G1), or because the premises themselves supply it (G2).

Furthermore, our analysis enables deeper reflection on which term has to be non-empty for each G3 syllogism. For the syllogisms in the third group, only one extra existential premise is necessary to prove them in first order predicate calculus – namely, to demonstrate these syllogisms, we only need to assume the non-emptiness of one of the three terms that make up the syllogism, with the exact term (major, minor, or middle) varying depending on the figure. For our formal analysis and for the simplicity of illustrating these findings, we shall use analytic tableaux as our proof method. Analytic tableaux provide a systematic way to test whether a conclusion follows from a set of premises by attempting to construct a counterexample. If all attempts to construct a counterexample fail (all branches close; we indicate the fact that a branch is closed by $\times$), the argument is valid. If at least one branch remains open, the argument is invalid, and the open branch describes a countermodel (Smullyan, 1995).

For each syllogism, we can test validity by constructing a tableau with the premises and the negation of the conclusion in the root. If the tableau closes, the syllogism is valid. If a branch remains open, the syllogism is invalid without additional premises, and we can determine what existential premise would close the branch.

4.2 Analytic tableaux

4.2.1 Group 1: syllogisms requiring no existential premises

Group 1 syllogisms are valid in first order predicate calculus without any existential assumptions. They remain valid even when all terms are empty because their logical structure does not depend on the existence of instances.

As an example let us consider the syllogism *Barbara*: from $\forall x(M(x) \rightarrow P(x))$, $\forall x(S(x) \rightarrow M(x))$ to derive $\forall x(S(x) \rightarrow P(x))$

T1: *Barbara*

$$\forall x(Mx \rightarrow Px)$$

$$\forall x(Sx \rightarrow Mx)$$

$$\exists x(Sx \wedge \neg Px)$$

$$Sa \wedge \neg Pa$$

$$Sa$$

$$\neg Pa$$

$$Ma \rightarrow Pa$$

$$Sa \rightarrow Ma$$

$$\begin{array}{cc} \neg Ma & Pa \\ \begin{array}{cc} \neg Sa & Ma \\ \times & \times \end{array} & \times \end{array}$$

All branches close, so *Barbara* is valid without existential premises. The proof works entirely through the logical structure of conditionals and does not require any term to have instances.

The key feature of G1 syllogisms is that they have universal conclusions. Since universal propositions in first order predicate calculus are vacuously true when their subject terms are empty, these syllogisms remain valid even when no instances exist. The logical structure of the argument – the chain of conditionals – ensures validity regardless of whether any terms have instances.

4.2.2 Group 2: syllogisms with existential commitments satisfied by particular premise

Group 2 syllogisms have particular premises (I or O propositions) that explicitly assert the existence of at least one individual satisfying certain conditions. These existential commitments are sufficient for the validity of the syllogism; no additional existential premises are needed. Here let us consider *Darii*: from $\forall x(M(x) \to P(x))$, $\exists x(S(x) \wedge M(x))$ to derive $\exists x(S(x) \wedge P(x))$.

T2: *Darii*

$$\forall x(Mx \to Px)$$

$$\exists x(Sx \wedge Mx)$$

$$\forall x(Sx \to \neg Px)$$

$$Sa \wedge Ma$$

$$Sa$$

$$Ma$$

$$Ma \to Pa$$

$$Sa \to \neg Pa$$

$\neg Ma$	Pa
$\times$	

$\neg Sa$	$\neg Pa$
$\times$	$\times$

All branches close, so *Darii* is valid. The particular minor premise (line 2) provides the existential commitment needed for the particular conclusion. No additional existential premise is required.

G2 syllogisms are valid because their particular premises explicitly assert existence. When a premise states "Some *S* is *M*" ($\exists x(S(x) \wedge M(x))$), it commits to the existence of at least one individual that is both S and M. This existential commitment is then "carried through" the argument to the conclusion via the universal premise.

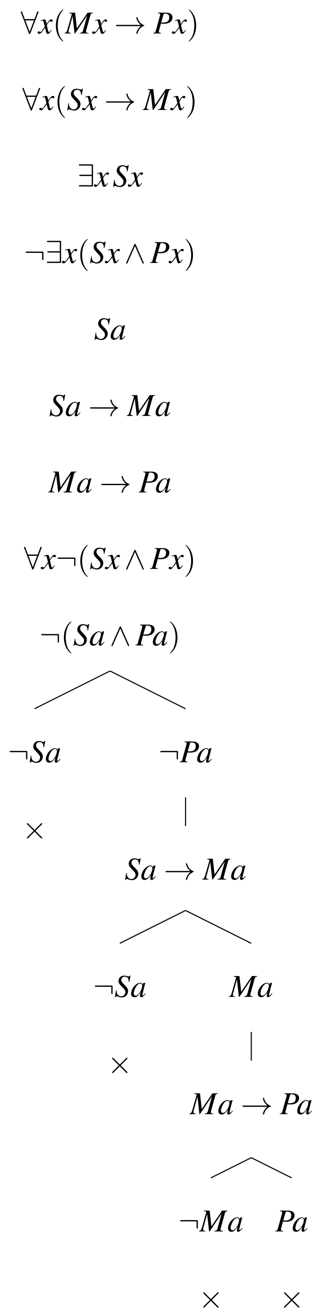
The key insight is that particular propositions in first order predicate calculus have their existential commitments built-in. Unlike universal propositions, which can be vacuously true, particular propositions cannot be true unless at least one instance exists. Therefore, any syllogism with a particular premise automatically has the existential resources needed for a particular conclusion, provided the logical structure of the argument is valid.

All ten G2 syllogisms can be proven valid using similar tableau constructions. The particular premises provide the existential commitments, and the universal premises provide the logical connections needed to derive the conclusion.

4.2.3 Group 3: syllogisms requiring additional existential premise

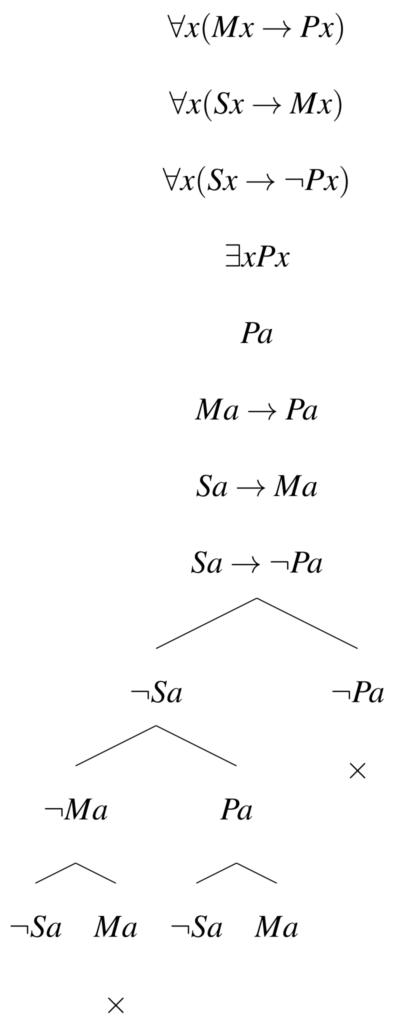
Group 3 syllogisms have only universal premises but particular conclusions. Since universal premises can be vacuously true when their terms are empty, and particular conclusions require the existence of at least one instance, these syllogisms cannot be proven valid without an additional existential premise. Here let us use as an example the syllogism *Barbari*: from $\forall x(M(x) \to P(x))$, $\forall x(S(x) \to M(x))$ to derive $\exists x(S(x) \wedge P(x))$. T3 is an analytic tableau for *Barbari* with the correct additional existential premise, $\exists xSx$, employed.

T3: *Barbari* with $\exists x S(x)$

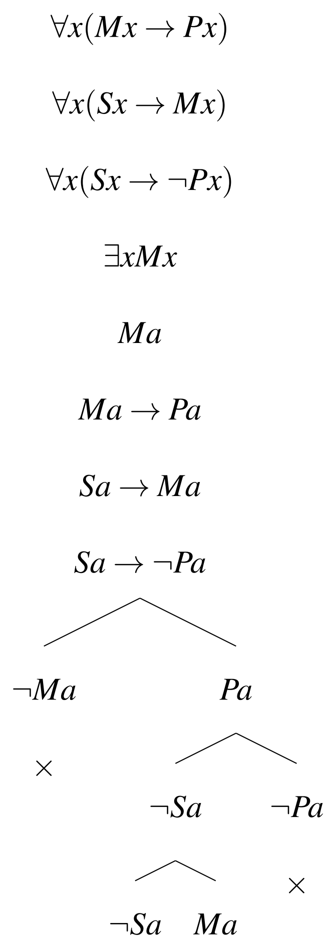


All branches close, so Barbari is valid with the existential premise $\exists x S(x)$. It is easily seen that without it, the tableau will not close, while with it the tableau does close, thus forming a proof of the syllogism in question. If we add to *Barbari*'s original premises any other existential premise, the tableau will not close. T4 and T5, instead, present examples in which the tableaux do not close: T4 is an example involving the premise $\exists x Px$, T5 is an example involving $\exists x Mx$.

T4: *Barbari* with $\exists x Px$

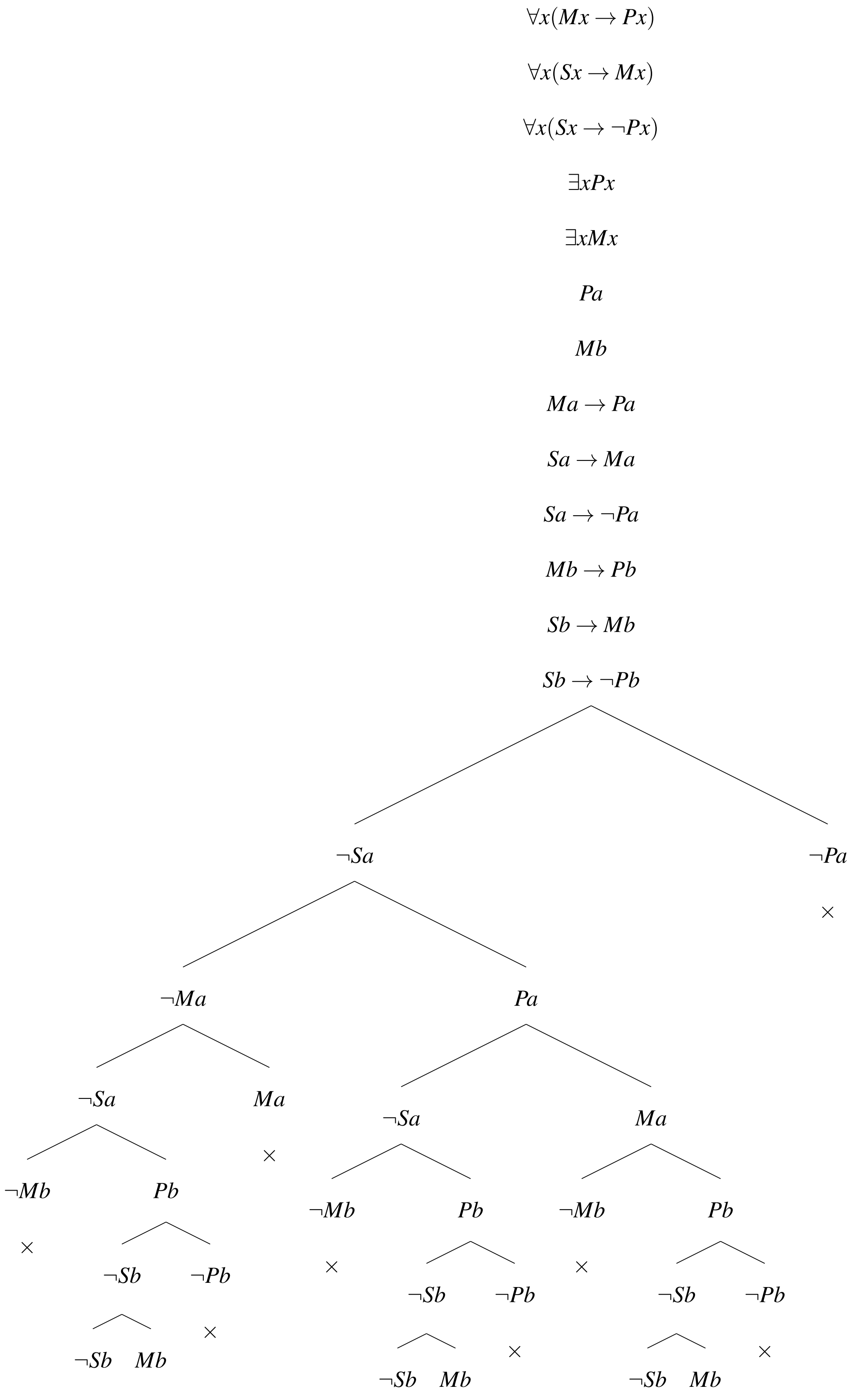


T5: *Barbari* with $\exists x Mx$



Moreover, only addition of the $\exists x Sx$ premise allows to prove *Barbari* in first order predicate calculus. Consider the following unsuccessful attempt, T6, with two existential premises added ($\exists x Px$, $\exists x Mx$):

T6: *Barbari* with $\exists x Px$ and $\exists x Mx$



The same holds for all the other syllogisms in the G3 group. In first order predicate calculus, they require explicit existential premises because their premises are all universal and can be vacuously true when terms are empty, while their conclusions are particular and require at least one instance to exist. Thus there is no logical connection between vacuously true universals and non-vacuously true particulars without an existential bridge. However, it is not always the minor term that needs to be non-empty to prove a G3 syllogism: this depends on the figure. The general principle governing which existential premise is needed is the following: the existential premise must supply a witness that the universal premises can propagate into both conjuncts of the particular conclusion. Since a G3 conclusion always has the form $\exists x(\phi(x) \wedge \psi(x))$, validating it requires producing an individual a such that both $\phi(a)$ and $\psi(a)$ can be derived. The witness introduced by the existential premise must therefore be one that the chain of universal conditionals in the premises can carry to both the subject term and the predicate term of the conclusion. Which term occupies this bridging position depends on how the middle term (M), minor term (S), and major term (P) are arranged in the premises – that is, on the figure.

As another example consider *Darapti*: All M are P; All M are S; therefore Some S are P. The premises have the form $\forall x(M(x) \rightarrow P(x))$ and $\forall x(M(x) \rightarrow S(x))$, with M as the common antecedent. A witness to M propagates in both directions at once: $M(a) \rightarrow P(a)$ and $M(a) \rightarrow S(a)$, immediately yielding both conjuncts of the conclusion. The required existential premise is therefore $\exists x M(x)$ – the middle term must be non-empty. Neither $\exists x S(x)$ nor $\exists x P(x)$ suffices, since a witness to S or P alone cannot be driven back to M to activate both conditional chains.

This pattern holds systematically across all figures, though which specific term must be non-empty varies: in figures 1 and 2 the minor term must be non-empty, in figure 3 the middle one and in figure 4 this varies by mood (major term in *Bramantip*, middle term in *Fesapo*, minor term in *Camenos*). The complete figure-by-figure analysis demonstrating why each specific term is required for each G3 syllogism appears in Appendix A.

G2 syllogisms differ from G3 in that their particular premises themselves introduce the required existential witnesses. When a premise states “Some S is M” ($\exists x(S(x) \wedge M(x))$), it commits to the existence of at least one individual that is both S and M. This existential commitment is then “carried through” the argument to the conclusion via the universal premise. Unlike G3 syllogisms, no external existential premise is needed because the particular premise itself introduces a witness into the proof.

The general principle governing why this carrying-through succeeds is the following: the particular premise always introduces a witness connected to the middle term M. Since a G2 conclusion, like a G3 conclusion, has the form $\exists x(\phi(x) \wedge \psi(x))$, validating it requires producing an individual a such that both $\phi(a)$ and $\psi(a)$ can be derived. In a G2 syllogism, the particular premise directly certifies one (or both) of these components, while the universal premise supplies the conditional link needed for the remaining component. The reason this always works is structural: the particular premise contains the middle term M, which is the term that connects the two premises.

Consider *Datisi*: All M are P; Some M are S; therefore Some S are P. The particular minor premise $\exists x(M(x) \wedge S(x))$ introduces a witness a with $M(a) \wedge S(a)$. The conjunct Sa directly provides the S-component of the conclusion. The conjunct Ma triggers the universal major premise $M(a) \to P(a)$, yielding the P-component. The existential content of the particular premise is thus sufficient: it supplies both the witness and the connection point ($M(a)$) that the universal major needs to complete the conclusion. No additional existential premise is required.

A structural observation unifies all ten G2 cases: the middle term M always appears in the particular premise. This is not coincidental but necessary. The middle term is, by definition, the term that appears in both premises but not in the conclusion; it is the logical bridge connecting the major and minor terms. For the particular premise’s witness to be “carried through” to both components of the conclusion – one involving S and one involving P (or $\neg P$) – it must be connected to the term that links both premises. A particular premise witnessing only at S or only at P, without any connection to M, could not activate the universal premise's conditional, and the proof would fail.

The G2 mechanism thus mirrors the G3 mechanism in reverse: where G3 syllogisms require an external existential premise at the right point in the conditional chain, G2 syllogisms have a particular premise already positioned at that point – namely, at M – so that no supplementation is needed.

These analyses justify Burgess's (Burgess, 2009, p. 28) claim that "the details are tedious". Our principle-based explanation shows, however, that the details are not merely tedious but structurally revealing: the required existential premise (in G3) or witness propagation path (in G2) is always determined by which term can serve as a witness that the universal premises can propagate to both components of the particular conclusion. Moreover, Burgess treats all fifteen unconditionally valid syllogisms uniformly, whereas our G1/G2 distinction reveals a fundamental structural difference: G1 syllogisms succeed because conditionals compose, while G2 syllogisms succeed because particular premises introduce witnesses that universal premises then propagate.

4.2.4 Conclusions: implications for history and philosophy of logic, and for contemporary logic

The historical and formal analysis presented above has several implications for contemporary logic as well as for history and philosophy of logic.

First, it clarifies the relationship between traditional and modern logic. The common view that modern logic "corrected" traditional logic by allowing empty terms is misleading. As seen in the previous Chapter, traditional logic recognized the problem of empty terms and developed ways to handle them. Modern first order predicate calculus makes these insights explicit through formal semantics, but the underlying conceptual issues were already understood by historical figures. In this regard, our considerations demonstrate the value of combining historical scholarship with formal analysis. Understanding the development of logical thought requires both historical sensitivity (to understand what past logicians actually said and what problems they addressed) and formal competence (to make their insights precise and compare different systems rigorously). Neither of these approaches suffices on its own.

Second, it highlights the importance of making existential commitments explicit. One of the key advances of modern logic is the explicit representation of existential commitments through existential quantifiers and existence predicates. This explicitness allows for greater precision and clarity about what a logical system presupposes. Simultaneously, it suggests that the three-group classification has pedagogical value. When teaching the relationship between traditional and modern logic, the three-group classification provides a clear and systematic way to explain which syllogisms require existential premises and why. It avoids both the oversimplification of Łukasiewicz's interpretation (all terms non-empty) and the confusion of treating each syllogism as a separate case.

Finally, the analysis suggests directions for future research. The three-group classification could be extended to other logical systems (modal logic, intuitionistic logic, free logic) to see how existential commitments are handled in different frameworks. The historical development could be traced in more detail, examining how specific logicians addressed specific problems. This way, the philosophical implications could be explored more fully, examining what the problem of existential import reveals about the nature of logical consequence and ontological commitment in general. This, however, is beyond the point of this dissertation.

Chapter 5

The search for foundations of syllogistic reasoning

Introduction

The purpose of this Chapter is to present a novel view of syllogistic reduction and to contest it with the view of Aristotle, medieval Scholastics, and other authorities such as Leibniz, referring our own findings to the discussions described earlier. Contrary to the previous Chapter, we will not be using the framework of modern first order predicate calculus, but instead use only the internal toolkit of syllogistic, such as the Square of Opposition relations, conversion, and syllogistic moods themselves.

The principal aim is to present different possible minimal sets of inference rules sufficient to prove all 24 valid Aristotelian syllogistic moods using the *indirect-proof method*, that is, by showing that assuming the conclusion of a syllogism to be false leads to a contradiction between the negated conclusion and one of the syllogism's premises. While the history of the search for a minimal set of inference rules for syllogistic reasoning starts already with Aristotle and is inaugurated by the Aristotelian statement that from the universal moods of first figure the rest of syllogistic moods can be derived. The *reductio per impossibile* as a proof method was confined by Aristotle only to moods *Baroco* and *Bocardo*, which are not provable in the direct way.

We, in turn, shall be applying the indirect method to all syllogisms. The enterprise most closely resembling ours was carried out by Leibniz, who has applied the indirect method to all syllogisms in order to prove that certain reductions hold (see (Besler, 2018)). We, however, will not only be proving all syllogism by some available reduction *per impossibile*, but by every possible reduction that is valid in a specific case. From that, we will obtain a clear picture of the internal structure of syllogistic when proved indirectly, and will be able to answer our main research question, which is: *what are the minimal sets of inference rules needed to prove all the syllogistic moods indirectly?* While answering this, we will also touch questions such as (1) *whether, when proved indirectly, the relations between respective syllogistic moods are the same* and (2) whether Aristotle was right and the reduction-scheme he proposed while using direct proofs holds when syllogisms are proved indirectly as well?

The Chapter will start with the explanation of the status of *reductio per impossibile* as a part of syllogistic system and proceed to a description of the Leibnizian method. Then, we shall present an example proof-procedure using the mood *Cesaro*. To answer our question, we shall identify four different scenarios: three in which a specific combination of single-premise inference rules is allowed, and one in which no single-premise inference rule is used. Thus, in the respective scenarios we will use: (1) subalternation and conversion, (2) conversion-only, (3) subalternation-only, (4) no single-premise inference rules. The results for each scenario will then be shown as a 25x25 chart containing every possible pair of moods, with the cases in which one mood can be proved by another (i.e., reduced to it) being marked in color.

Considering the charts, we shall then draw conclusions regarding both differences in the results between different sets of single-premise inference rules and the results of the subalternation and conversion set, being the most interesting one. The results will show that in some scenarios, the Aristotelian reduction indeed *does not hold*, at least in a pure form.

Moreover, we shall see that all the syllogisms (respectively, syllogistic moods) can be divided into few main groups based on the fact that they can prove and be proved by the same set of moods. Apart from those, minor groups containing two or three moods exhibiting no ontological differences when it comes to indirect proofs shall be identified. Finally, we shall combine the results obtained by the indirect method with the results present in Aristotle (thus considering all direct and indirect proof-cases), and see that in this scenario, the regularities are even more frequent, and that all 24 syllogistic moods can be accordingly divided into four groups different from the traditional figures. Then, various relationships between those groups will be shown and the minimal set of inference rules identified.

5.1 Aristotle's *Dictum de omni et nullo*

As was noted in the first Chapter of this work, in Chapter 7 of *Prior Analytics*, Aristotle gives a proof that all syllogistic moods ultimately reduce to two "most evident" moods of the first figure, that is, *Barbara* and *Celarent*. 29b1 reads: "It is possible also to reduce all deductions to the universal deductions in the first figure". Although Aristotle never refers to this fact in a particular way (Patterson, 1993), this doctrine, which was later to be termed *Dictum de omni et nullo* was held even by the commentators most close in time to Aristotle (Gili, 2013) and then consequently throughout the Middle Ages by pretty much every author (Gili, 2015). The *Dictum*, which in some interpretations is said to be an intuitive and obviously valid reasoning present in every mind, conveys the fact that everything (not) predicated of all of some group is also (not) predicated of everything which this group contains (Patterson, 1993, p. 359). This, essentially, is the content of *Barbara* and *Celarent* combined.[81]

[81] We leave out here the discussions on whether this identification of *Dictum* with the first two moods is historically legitimate. In some interpretations, the *Dictum* is to be regarded as a meta-principle that allows to establish the validity of *Barbara* and *Celarent* in the first place (Striker, 2022) – essentially, the rule due to which a specific chain of reasoning is 'obviously' true to a human mind. For heuristic purposes, we will refer to the *Dictum* as equal to the moods *Barbara* and *Celarent* throughout this Chapter.

A slightly less strict version of this claim, the notion that all the syllogistic moods are provable by (or: reducible to) the first-figure ones[82] is then consequently held by all medieval authors we have encountered in the previous Chapters. We have it in William of Sherwood (1966, p. 67), Roger Bacon (de Libera, 1987, p. 205), Lambert of Auxerre (2015, p. 143-145), and, most importantly, Peter of Spain (2014, beginning at p. 171). Virtually every scholar who used the mnemonic device described in Chapter 2 must have also admitted the first-figure reducibility, as the proofs, respectively, reductions, were encoded there. Thus, although Aristotle does not speak of this reduction in the absolute terms, the medieval account is categorical, that is, no other sets of fundamental rules are admitted. From the time of Peter, since, as we have noted, his work served as a standard for few generations, this categorical account has become widespread, and the *Dictum de omni et nullo*, together with all first-figure moods, was regarded as the basis of syllogistic.

As far as we are concerned, two things connected with the *Dictum* are important: first, that *Barbara* and *Celarent* play the most important role in the whole system; second, that the first figure plays the most important role among the other figures. Thus, first, we have the notion that all the other figures are ultimately reducible to the first one (i.e., (alternatively), their validity stems from the validity of the first); secondly, we have the notion that among the first-figure moods, *Barbara* and *Celarent* are those which are most essential and serve as the foundation for all syllogistic.

In the further Sections of this work, we shall try to contend, or at least to challenge this view, that is, we shall first check whether the first-figure reduction scenario holds with indirect

[82] The original Aristotelian formulation was that 1) all moods can be reduced to first-figure ones, 2) *Darii* and *Ferio* may be then reduced to *Barbara* and *Celarent*. In the Middle Ages, as was already said here, it was a custom to treat first figure as the basic one. The details of the relationship between first figure as a whole and *Barbara* and *Celarent* as even more 'basic' moods, as well as historical discussions about this issue, are beyond the scope of this dissertation. What can be further noted here is that two different senses of reduction can be distinguished (see Striker (2022) and Morison (2015)): one is to analyze an argument in terms of its canonical syllogistic form (by reformulation), the other is to trace its validity to a previously established scheme. In what follows, we will be concerned with the latter: justifying moods by having their validity traced to other moods.

proofs as well, and second, try to provide alternative minimal sets which are enough to then prove the validity of all the rest of the system.[83,84]

5.2 Aristotelian-style direct proofs

The indirect proof, or proof by contradiction, is done by showing that assuming the proposition to be false leads to a contradiction between the (as we shall call it) indirect premise and one of the other premises. In the case of syllogisms, it consists of 1) assuming that a conclusion of a given syllogism is false, 2) taking its contradictory (which must be true), 3) showing that this contradictory is itself contradictory to one of the syllogism's premises, 4) inferring that hence, it must be false, and the initial conclusion must be true.

Although Aristotle uses the *per impossibile* method principally to prove only *Baroco* and *Bocardo* which are not provable otherwise (Aristotle, 1989),[85] the method allows for a much greater number of possible proof scenarios:[86] whereas when proving directly, only two (direct)

[83] It is also worth saying that the precise interpretation of what syllogism is or of what the rules in syllogistic are is not of much methodological relevance to us. Thus, for our arguments concerning the metastructure of syllogistic, it is not relevant whether we consider moods and immediate inferences to be rules of deduction or to be the axioms of the system, as in either case the essential component of giving an account alternative to the one of Aristotle is preserved. The rest is interpretation, but that can be applied to the Aristotelian position in the same way as to ours.

[84] A shortened version of this Chapter has been submitted as an article to *History and Philosophy of Logic* (Wapniarski and Urbański, 2026).

[85] The indirect method is used also do show the reducibility of *Darii* and *Ferio* to *Barbara* and *Celarent* through second-figure moods (and due to this fact, the indirect proofs can even be compounded, so as to give a nested indirect-proof scenarios (Antonelli and von Plato, 2026)). What concerns us here, however, is that when showing the mutual dependencies between the moods, the indirect-proof method is not systematically applied – it is reserved only for these scenarios in which the direct method is not applicable.

[86] Such alternative proof scenarios were historically recognized (as we shall see in the next Section on Leibniz), although only to some degree. As an example we may take Buridan, who in his *Summulae...* admits that beside *Baroco* and *Bocardo*, all other moods from figures two and three can be reduced to their counterparts from the first figure by the reduction to impossibility as well (Buridan, 2001, p. 331, 335); however, he gives example of only one possibility of such reduction for each mood – he thus recognizes the alternative of **an** indirect proof, but does not recognizes the possible alternative cases **of** the indirect proofs. Another example is again connected to the use of syllogistic diagrams mentioned in Chapter 3: there have been cases where Byzantine scholars, while representing various proofs or reductions on the Byzantine diagrams, supplied their diagrammatic representations with parts encoding alternative indirect proofs (Agiotis, 2022, p. 65).

premises can be modified in order to then apply a certain syllogistic mood on them and get the desired conclusion, in the indirect-style proof, both of the direct premises can be combined with the *indirect premise*, amounting to (at least) two times more possible cases. The indirect-proof method allows us then to get a much richer sense of the internal structure of syllogistic.

All proofs possible by the direct-proof method are summarized in the charts below. Successive rows indicate moods which are proved, and successive columns moods which are used in a proof:

	Barbara	Celarent	Darii	Ferio	Barbari	Celaront
Barbara						
Celarent						
Darii						
Ferio						
Barbari	■		■			
Celaront		■		■		
Cesare		■				
Camestres		■				
Festino				■		
Baroco						
Cesaro		■		■		■
Camestros		■				
Darapti			■			
Disamis			■			
Datisi			■			
Felapton				■		
Bocardo						
Ferison				■		
Bramantip	■		■		■	
Camenes		■				

Dimaris			■			
Fesapo				■		
Fresison				■		
Camenos		■				

Table 5.1 Direct: first figure

	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros
Barbara						
Celarent	■	■				
Darii						
Ferio			■			
Barbari						
Celaront	■		■		■	
Cesare		■				
Camestres	■					
Festino						
Baroco						
Cesaro	■	■	■			
Camestros	■	■		■		
Darapti						
Disamis						
Datisi						
Felapton			■			
Bocardo						
Ferison			■			
Bramantip						
Camenes	■	■				

Dimaris						
Fesapo			■			
Fresison			■			
Camenos	■	■		■		■

Table 5.2 Direct: second figure

	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison
Barbara						
Celarent						
Darii		■	■			
Ferio						■
Barbari		■	■			
Celaront						■
Cesare						
Camestres						
Festino						■
Baroco						
Cesaro						■
Camestros						
Darapti		■	■			
Disamis			■			
Datisi		■				
Felapton					■	■
Bocardo						
Ferison						
Bramantip		■	■			
Camenes						

Dimaris		■	■			
Fesapo				■	■	■
Fresison						■
Camenos						

Table 5.3 Direct: third figure

	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara						
Celarent		■				
Darii			■			
Ferio					■	
Barbari	■		■			
Celaront		■			■	
Cesare		■				
Camestres		■				
Festino					■	
Baroco						
Cesaro		■			■	
Camestros		■				■
Darapti			■			
Disamis			■			
Datisi			■			
Felapton				■	■	
Bocardo						
Ferison					■	
Bramantip			■			
Camenes						

Dimaris						
Fesapo					■	
Fresison						
Camenos		■				

Table 5.4 Direct: fourth figure

Here, we can note that 1) many moods can be proved only mutually by each other, i.e., those starting with C only by others starting with C, etc., 2) *Baroco* and *Bocardo* do not have any proof. In fact, those moods, being unprovable in the direct way, are the only two for which Aristotle in *Prior Analytics* employs the indirect method (see 29a30-b26). As the indirect method is, in turn, applicable to all the moods, what we shall do in the Sections that follow is to broaden the scope of its use and actually apply it to all of them.

5.3 Leibnizian reduction

An enterprise closest to the one we want to accomplish in this work is the one attempted by Leibniz in his *Nouveaux Essais* (1996) and *De Formis Syllogismorum Mathematicae Definiendis* (1966).[87] There, Leibniz is trying to replace the Aristotelian proof-reduction schema by his own, justifying single-premise inferences by valid syllogistic moods from the first figure. In this way, his attempt is methodologically opposite to ours (as we are, in essence, interested in obtaining a reduction schema containing as few moods as possible). It is, however, similar technically, as Leibniz bases his consideration on the use of the proof (or reduction) *per impossibile*.

The use which Aristotle makes of the indirect method, apart from reducing *Baroco* and *Bocardo* which are not reducible in the direct way, is the so called *conversio syllogismi*, presented in the second book of *Prior Analytics*. The *conversio* is based on the behavior of

[87] For more on Leibnizian reduction see (Besler, 2018).

the indirect proof, which can be applied not only to reduce imperfect moods to perfect (i.e., first-figure) ones, but also to transform any valid mood into some another.

The idea of such a transformation, having only limited presence in Aristotle (for more on this issue see (Patzig, 1968)), was adopted and fully developed by Leibniz. Considering a scenario with no single-premise inference rules ("immediate" inferences) allowed, i.e., using neither subalternation nor conversions (but making use of the law of contradiction taken from the Square of Opposition), he structures all the moods into triples, consisting only of those moods which are indirectly reducible to each other. To illustrate, for an example mood *Celarent*, we have:

1.MeP	(major premise)
2.SaM	(minor premise)
3.SeP	(conclusion)
4.SiP	(indirect premise (law of contradiction))
5.SoM	(1, 4, **Festino**)
6.SaM	(5, law of contradiction)
7.MiP	(4, 6, **Disamis**)
8.MeP	(7, law of contradiction)
9.SeP	(8, 6, **Celarent**)

And so on...[88] If we were to start by using *Disamis*, it and *Festino* would get switched:

1.*MeP*	(major premise)
2.*SaM*	(minor premise)
3.*SeP*	(conclusion)
4.*SiP*	(indirect premise (law of contradiction))
5.*MiP*	(4, 2, **Disamis**)
6.*MeP*	(5, law of contradiction)
7.*SoM*	(6, 4, **Festino**)
8.*SaM*	(7, law of contradiction)
9.*SeP*	(6, 8, **Celarent**)

And thus, *Celarent*, *Festino*, and *Disamis* form a triple. If we would like to reduce *Festino*, we would get *Celarent* and *Disamis*, and so on. In this way, considering also the subalternated moods, Leibniz obtains six such triples:

1. *Barbara*, *Baroco*, *Bocardo*
2. *Celarent*, *Festino*, *Disamis*
3. *Darii*, *Camestres*, *Ferison*
4. *Ferio*, *Cesare*, *Datisi*
5. *Barbari*, *Camestros*, *Felapton*

[88] With the “law of contradiction” we refer here to the fact that the indirect premise, for the purposes of the indirect proof, has to be contradictory to the conclusion. It is therefore derivable from the conclusion by applying the “law of contradiction” from the Logical Square, but not as a mere rule of inference, but due to the principles of constructing the indirect proof. We can thus infer *SaM* from *SoM* (step 6.), as with this step we begin the second indirect proof within the chain – one for which *SoM* is the conclusion of the second syllogism being proved (i.e., *Festino*).

6. *Celaront*, *Cesaro*, *Darapti*

Each containing a mood from the first, second, and third figure respectively. In this part of schema, Leibniz is not applying *conversio syllogismi* to fourth-figure moods (although he accounts for them in his system, as we have noted in Section 2.4.), but we shall later see that it in fact can be done. What is to note is that already here we can draw some conclusions which will be important for our own results, namely, that in the syllogistic structure obtained by the indirect-proof method, *Felapton*, *Darapti*, and (extending to the fourth figure) *Bramantip* with *Fesapo*, behave equally to the **subalternated** moods. Thus, the system contains 15 main or "normal" moods and 9 subalternated ones.

5.3.1 *More apagogico*, or the indirect-style proofs

The Leibnizian reduction and the triples-schema shall be, effectively, equated with the first part of our work, that is, the scenario when we will not be using any single-premise inference rule apart from the Logical Square's contradiction; apart from that, our work will be more extensive, and we shall consider also scenarios when application of conversion and/or subalternation is allowed. Methodologically, we will not be interested in devising a particular procedure which allows to reduce syllogistic into a set of rules (as it is the case with Leibniz), but in reducing (respectively, proving) every mood in every possible way *and then* devising a minimal set (or sets) of inference rules. In this way, apart from the set mentioned above, we shall get a clear structure (and some variations of this structure) of syllogistic when proved indirectly, from which a greater number of interesting conclusions shall follow.

5.3.2 Sets

To do it, we shall use four different sets of single-premise inference rules as our basic set, thus considering four basic scenarios. Single-premise rules available to us (see Sections 1.1.2 and 1.1.3 from Chapter 1) are subalternation and two types of conversion. For the sake of clarity, we shall treat both types of conversion equal. Then, the proofs shall be made using the specified set of single-premise rules *and* various syllogisms as multi-premise inference rules. The results will be four charts pertaining to the four different scenarios. The sets we will consider shall be:

1. subalternation and conversion
2. conversion-only
3. subalternation-only
4. no single premise inference rule

5.3.3 Example mood: *Cesaro*

The example mood we shall use for demonstrating the subsequent parts of proving procedure shall be *Cesaro*.[89] We shall begin from the last of our identified scenarios, that is, by trying to prove *Cesaro* without using any single-premise inference rule. The structure of the mood *Cesaro* looks like:

1. No P is M (PeM) (major premise)
2. All S is M (SaM) (minor premise)
3. Some S is not P (SoP) (conclusion)

And our premises for proving it indirectly look like:

1.*PeM* (major premise)
2.*SaM* (minor premise)
3.*SaP* (indirect premise from the negation of SoP)

[89] Here, we shall present only the different *valid* cases for four sets respectively. For an illustration showing that we indeed run through all the possible cases, see Appendix B.

No single-premise inference rules

We shall thus begin by proving the mood combining the premises in four most obvious ways, without using either subalternation or conversion. For *Cesaro*, the premises can be combined as follows:

1.

1.*PeM*
2.*SaM*
3.*SaP*
4.*SeM* (1, 3, Celarent)
5.*SoM* (1, 3, Celaront)

2.

1.*PeM*
2.*SaM*
3.*SaP*
4.*MeS* (3, 1, Camenes)

3.

1.*PeM*
2.*SaM*
3.*SaP*
4.*PiM* (2, 3, Darapti)

4.

1.PeM	
2.SaM	
3.SaP	
4.MiP	(3, 2, Darapti)

For which the proofs can now be finished by juxtaposing the remaining inference premise with the acquired proposition. In this way, *Cesaro* can be proved by *Celaront* and *Darapti*:

1.

1.PeM	
2.SaM	
3.SaP	
4.SoM	(1, 3, Celaront)
5.$\perp$	(2, 4)

2.

1.PeM	
2.SaM	
3.SaP	
4.PiM	(2, 3, Darapti)
5.$\perp$	(1, 4)

However, if we are to note that *Celaront* itself is a subalternated mood, then the single syllogism by which *Cesaro* can be proved in the no single-premise inference rules scenario is *Darapti*.

Conversion-only

Now, we shall check what are the possible ways of proving if conversion is to be applied (either on the main premises before the inference using syllogism or on the main premises or the conclusion after the use of it). For the second case, we shall resume the combinations done in the last Section:

1.

1.PeM	
2.SaM	
3.SaP	
4.SeM	(1, 3, Celarent)
5.MiS	(2, p-conversion)
6.SiM	(5, s-conversion)
7.$\bot$	(4, 6)

2.

1.PeM	
2.SaM	
3.SaP	
4.MeS	(3, 1, Camenes)
5.MiS	(2, p-conversion)

6.*SiM*	(5, s-conversion)
7.*SeM*	(4, s-conversion)
8.⊥	(6, 7)

And see that *Cesaro* can be thus proved additionally by *Celarent* and *Camenes*.

For the first scenario, we can convert one of the main premises (major or minor), the indirect premise, or both of them at the same time. Moreover, if either of the premises is a universal affirmative proposition, we can first perform a p-conversion, thus getting a particular affirmative proposition, and then perform an s-conversion, switching the terms in the obtained particular affirmative proposition. Thus, for *Cesaro* we get:

1.*PeM*	
2.*SaM*	
3.*SaP*	
4.*MeP*	(1, s-conversion)
5.*PiS*	(3, p-conversion)
6.*SiP*	(5, s-conversion)
7.*MiS*	(2, p-conversion)
8.*SiM*	(7, s-conversion)

And the proofs are as follows:

9.1.*SeM*	(4, 3, Cesare)
10.1.⊥	(8, 9.1)
9.2.*SoM*	(4, 3, Cesaro)

10.2.$\perp$	(2, 10.1)
9.3.*MeS*	(3, 4, Camestres)
10.3.$\perp$	(7, 11.1)
9.4.*SoM*	(1, 5, Ferison)
10.4.$\perp$	(2, 12.1)
9.5.*SoM*	(1, 6, Ferio)
10.5.$\perp$	(2, 13.1)
9.6.*SoM*	(4, 5, Fresison)
10.6.$\perp$	(2, 14.1)
9.7.*SoM*	(4, 6, Festino)
10.7.$\perp$	(2, 15.1)
9.8.*PiM*	(7, 3, Dimaris)
10.8.$\perp$	(1, 16.1)
9.9.*MiP*	(3, 7, Darii)
10.9.$\perp$	(4, 17.1)
9.10.*PiM*	(8, 3, Disamis)
10.10.$\perp$	(1, 18.1)
9.11.*MiP*	(3, 8, Datisi)
10.11.$\perp$	(4, 19.1)

We can then see that adding conversion to our inference rule set makes it possible to prove *Cesaro* by *Cesare*, *Cesaro*,[90] *Camestres*, *Ferison*, *Ferio*, *Fresison*, *Festino*, *Dimaris*, *Darii*,

[90] Note that whereas in the case of direct proofs, each mood can be proved by itself, this is not so obvious in the case of indirect proofs. In fact, while inspecting the tables, one can spot many moods which are not indirectly provable by themselves.

Disamis and *Datisi*, which together with earlier *Celaront* and *Darapti* amounts to a total of 15 moods (note, that if we agree to dismiss the subalternated moods whenever subalternation is not included as a rule, that number gets reduced to 13).

Subalternation-only

When using subalternation as an inference rule, from every universal statement we can get a corresponding particular one. In the case of *Cesaro*, we can get:

1.*PeM*	
2.*SaM*	
3.*SaP*	
4.*PoM*	(1, subaltern.)
5.*SiM*	(3, subaltern.)
6.*SiP*	(5, subaltern.)

For which the proofs are as follows:

7.1.*SeM*	(1, 3, Celarent)
7.2.*SoM*	(7.1, subaltern.)
7.3.$\perp$	(2, 7.2)
8.1.*SoM*	(1, 6, Ferio)
8.2.$\perp$	(2, 8.1)
9.1.*PiM*	(5, 3, Disamis)
9.2.$\perp$	(1, 9.1)
10.1.*PiM*	(2, 6, Datisi)

10.2.$\perp$ (1, 10.1)

Thus, adding subalternation to the inference rule set makes it possible to prove *Cesaro* additionally with *Celarent*, *Ferio*, *Disamis* and *Datisi*, amounting to six proofs in total.

Subalternation and conversion

In the case of *Cesaro*, adding both subalternation and conversion to the set of rules does not give results different from adding just conversion, which is due to the fact that all moods working with conversion and subalternation set are working also with the conversion-only set. For example, SiP can be achieved either by applying a p-conversion and an s-conversion successively or by applying subalternation to SaP. Thus, in this case the set of moods by which one can prove *Cesaro* amounts to 15, as in the case of conversion-only.

There are, however, some cases when a proof which is done with subalternation cannot be repeated with conversion-only, for example, when the minor premise is a universal negative statement, as in the case of *Barbari*:

1.*MaP*

2.*SaM*

3.*SeP*

4.*SoP* (3, subaltern.)

5.*SoM* (1, 4, Baroco)

6.$\perp$ (2, 5)

Where the a-o premise alignment cannot be achieved without using subalternation.

5.3.4 The charts

Below, we present the results obtained after going through every syllogistic mood in the way described above. The charts are split, so that a single figure fits on one chart. Thus, every Subsection has four charts corresponding to four figures. Again, successive rows indicate moods which are proved, and successive columns moods which are used in a proof (or, when talking in terms of reduction: rows can be reduced to columns, with colors marking possible reduction cases):

5.3.5 No single-premise inference rules

	Barbara	Celarent	Darii	Ferio	Barbari	Celaront
Barbara						
Celarent						
Darii						
Ferio						
Barbari						
Celaront						
Cesare				■		
Camestres			■			
Festino		■				
Baroco	■					
Cesaro						■
Camestros					■	
Darapti						■
Disamis		■				
Datisi				■		
Felapton					■	
Bocardo	■					

Ferison						
Bramantip						
Camenes						
Dimaris						
Fesapo						
Fresison						
Camenos						

Table 5.5 No single-premise inference rules: first figure

	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros
Barbara						
Celarent						
Darii						
Ferio						
Barbari						
Celaront						
Cesare						
Camestres						
Festino						
Baroco						
Cesaro						
Camestros						
Darapti						
Disamis						
Datisi						
Felapton						
Bocardo						

Ferison		■				
Bramantip						
Camenes						
Dimaris						
Fesapo						
Fresison						
Camenos						

Table 5.6 No single-premise inference rules: second figure

	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison
Barbara					■	
Celarent		■				
Darii						■
Ferio			■			
Barbari				■		
Celaront	■					
Cesare			■			
Camestres						■
Festino		■				
Baroco					■	
Cesaro	■					
Camestros				■		
Darapti						
Disamis						
Datisi						
Felapton						
Bocardo						

Ferison						
Bramantip						
Camenes						
Dimaris						
Fesapo						
Fresison						
Camenos						

Table 5.7 No single-premise inference rules: third figure

	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara						
Celarent						
Darii						
Ferio						
Barbari						
Celaront						
Cesare						
Camestres						
Festino						
Baroco						
Cesaro						
Camestros						
Darapti						
Disamis						
Datisi						
Felapton						
Bocardo						

Ferison						
Bramantip				■		■
Camenes			■		■	
Dimaris		■			■	
Fesapo	■					■
Fresison		■	■			
Camenos	■			■		

Table 5.8 No single-premise inference rules: fourth figure

From here, we can see several interesting things: first, every mood has exactly two ways in which it can be proved - it cannot have more, as that would require either modifying the set of available premises or converting the result of the syllogistic inference. What follows is the already known Leibnizian triples-schema, which is easily deducible from the chart: *Cesare* is reducible to *Ferio* and *Datisi*, *Camestres* to *Darii* and *Ferison*, and so on. We can also add that each of the first three figures is reducible (or: provable) to other two: the first to the second and the third, the second to the first and the third, and the third to the first and the second. Thus every Leibnizian triple contains one mood from the first figure, one from the second, and one from the third.

Secondly, while the first three figures can be mutually proved, fourth figure can be proved only "through itself". However, while Leibniz did not consider this scenario, maintaining that fourth-figure moods are to be reduced to the first-figure ones by the means of conversion, it is indeed the case that the fourth figure itself can be *described* using the *conversio syllogismi* method. Looking at the table, we would get two more triples:

1. *Camenes*, *Fresison*, *Dimaris*

2. *Camenos*, *Fesapo*, *Bramantip*

Which are, respectively, one main and one subalternated triple. The fourth figure thus stands alone and is not reducible to either of the other three ones by the means of pure *per impossibile* reduction when no other immediate inference apart from the contradiction is admitted. The only way to make it reducible is, as Leibniz have already noted, to use the usual and the *per accidens* conversion which would equal the fourth-figure moods with the first-figure ones. It is, however, describable using the *conversio* and the triples terminology.

5.3.6 Subalternation-only

We shall note here that the gray color marks those cases which become valid proofs after adding subalternation as an inference rule (so as to differentiate them from the pure no single-premise inference rules cases).

	Barbara	Celarent	Darii	Ferio	Barbari	Celaront
Barbara						
Celarent						
Darii						
Ferio						
Barbari						
Celaront						
Cesare						
Camestres						
Festino						
Baroco						
Cesaro						
Camestros						
Darapti						
Disamis						
Datisi						

Felapton						
Bocardo						
Ferison						
Bramantip						
Camenes						
Dimaris						
Fesapo						
Fresison						
Camenos						

Table 5.9 Subalternation-only: first figure

	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros
Barbara						
Celarent						
Darii						
Ferio						
Barbari						
Celaront						
Cesare						
Camestres						
Festino						
Baroco						
Cesaro						
Camestros						
Darapti						
Disamis						
Datisi						

Felapton						
Bocardo						
Ferison						
Bramantip						
Camenes						
Dimaris						
Fesapo						
Fresison						
Camenos						

Table 5.10 Subalternation-only: second figure

	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison
Barbara						
Celarent						
Darii						
Ferio						
Barbari						
Celaront						
Cesare						
Camestres						
Festino						
Baroco						
Cesaro						
Camestros						
Darapti						
Disamis						
Datisi						

Felapton						
Bocardo						
Ferison						
Bramantip						
Camenes						
Dimaris						
Fesapo						
Fresison						
Camenos						

Table 5.11 Subalternation-only: third figure

	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara						
Celarent						
Darii						
Ferio						
Barbari						
Celaront						
Cesare						
Camestres						
Festino						
Baroco						
Cesaro						
Camestros						
Darapti						
Disamis						
Datisi						

Felapton						
Bocardo						
Ferison						
Bramantip						
Camenes						
Dimaris						
Fesapo						
Fresison						
Camenos						

Table 5.12 Subalternation-only: fourth figure

Here, we can observe that by adding subalternation, certain syllogisms, but only those which have a particular statement as a conclusion, can be proved in more ways. Essentially, the three subalternated Leibnizian triples: *Barbari-Camestros-Felapton*, *Celaront-Cesaro-Darapti*, and *Camenos-Fesapo-Bramantip* (from the fourth figure) get more reduction options, amounting to four per mood. What is to note is that those additional reductions (or proofs) are not arbitrary: in every case (quite obviously) at least two of the moods are superalternations of the previously-valid reductions, and two remaining are always from the same figure as those. Thus for an example case of *Celaront* we get *Cesare* (second figure) as a superalternation of *Cesaro* and *Datisi* as a superalternation of *Darapti*,[91] and then also *Festino* (from the group of *Cesare*) and *Disamis* (from the group of *Datisi*).

5.3.7 Conversion-only

[91] Note that if we are to treat *Darapti*, *Felapton*, *Bramantip*, and *Fesapo* as subalternated moods, superalternations of those would be (respectively): *Datisi* and/or *Disamis* (by superalternating one of the premises, for it would yield a subalternated mood in the same way as subalternating the conclusion); *Ferison* (superalternating minor); *Dimaris* (superalternating major); *Fresison* (superalternating minor).

	Barbara	Celarent	Darii	Ferio	Barbari	Celaront
Barbara						
Celarent			■	■		
Darii		■		■		
Ferio		■	■			
Barbari		■		■		
Celaront		■	■	■		■
Cesare			■	■		
Camestres			■	■		
Festino		■	■			
Baroco	■					
Cesaro		■	■	■		■
Camestros	■		■	■	■	
Darapti		■		■		■
Disamis		■		■		
Datisi		■		■		
Felapton	■	■	■		■	
Bocardo	■					
Ferison		■	■			
Bramantip		■		■		
Camenes			■	■		
Dimaris		■		■		
Fesapo	■	■	■		■	
Fresison		■	■			
Camenos	■		■	■	■	

Table 5.13 Conversion-only: first figure

	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros
Barbara				■		
Celarent			■			
Darii	■	■	■			
Ferio	■	■				
Barbari	■	■	■			■
Celaront	■	■	■		■	
Cesare			■			
Camestres			■			
Festino	■	■				
Baroco						
Cesaro	■	■	■		■	
Camestros			■			
Darapti	■	■	■		■	
Disamis	■	■	■			
Datisi	■	■	■			
Felapton	■	■				■
Bocardo				■		
Ferison	■	■				
Bramantip	■	■	■			■
Camenes			■			
Dimaris	■	■	■			
Fesapo	■	■				■
Fresison	■	■				
Camenos			■			

Table 5.14 Conversion-only: second figure

	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison
Barbara					■	
Celarent		■	■			■
Darii						■
Ferio		■	■			
Barbari				■		■
Celaront	■	■	■			■
Cesare		■	■			■
Camestres		■	■			■
Festino		■	■			
Baroco					■	
Cesaro	■	■	■			■
Camestros		■	■	■		■
Darapti						■
Disamis						■
Datisi						■
Felapton		■	■			
Bocardo						
Ferison		■	■			
Bramantip				■		■
Camenes		■	■			■
Dimaris						■
Fesapo		■	■			
Fresison		■	■			
Camenos		■	■	■		■

Table 5.15 Conversion-only: third figure

	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara						
Celarent			■		■	
Darii		■			■	
Ferio		■	■			
Barbari		■		■	■	■
Celaront		■	■		■	
Cesare			■		■	
Camestres			■		■	
Festino		■	■			
Baroco						
Cesaro		■	■		■	
Camestros	■		■	■	■	
Darapti		■			■	
Disamis		■			■	
Datisi		■			■	
Felapton	■	■	■			■
Bocardo						
Ferison		■	■			
Bramantip		■		■	■	■
Camenes			■		■	
Dimaris		■			■	
Fesapo	■	■	■			■
Fresison		■	■			
Camenos	■		■	■	■	

Table 5.16 Conversion-only: fourth figure

5.3.8 Subalternation and conversion

Here, we shall note that gray color marks cases which become valid after adding subalternation to conversion-only scenario (i.e., they are subalternation and conversion minus conversion-only)

	Barbara	Celarent	Darii	Ferio	Barbari	Celaront
Barbara						
Celarent			■	■		
Darii		■		■		
Ferio		■	■			
Barbari		■		■		
Celaront		■	■	■		■
Cesare			■	■		
Camestres			■	■		
Festino		■	■			
Baroco	■					
Cesaro		■	■	■		■
Camestros	■		■	■	■	
Darapti		■		■		■
Disamis		■		■		
Datisi		■		■		
Felapton	■	■	■		■	
Bocardo	■					
Ferison		■	■			
Bramantip		■		■		
Camenes			■	■		
Dimaris		■		■		
Fesapo	■	■	■		■	
Fresison		■	■			
Camenos	■		■	■	■	

Table 5.17 Subalternation and conversion: first figure

	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros
Barbara						
Celarent						
Darii						
Ferio						
Barbari						
Celaront						
Cesare						
Camestres						
Festino						
Baroco						
Cesaro						
Camestros						
Darapti						
Disamis						
Datisi						
Felapton						
Bocardo						
Ferison						
Bramantip						
Camenes						
Dimaris						
Fesapo						
Fresison						
Camenos						

Table 5.18 Subalternation and conversion: second figure

	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison
Barbara					■	
Celarent		■	■			■
Darii						■
Ferio		■	■			
Barbari				■	▒	■
Celaront	■	■	■			■
Cesare		■	■			■
Camestres		■	■			■
Festino		■	■			
Baroco					■	
Cesaro	■	■	■			■
Camestros		■	■	■	▒	■
Darapti						■
Disamis						■
Datisi						■
Felapton		■	■			
Bocardo						
Ferison		■	■			
Bramantip				■	▒	■
Camenes		■	■			■
Dimaris						■
Fesapo		■	■			
Fresison		■	■			
Camenos		■	■	■	▒	■

Table 5.19 Subalternation and conversion: third figure

	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara						■
Celarent			■		■	
Darii		■			■	
Ferio		■	■			
Barbari		■		■	■	■
Celaront		■	■		■	
Cesare			■		■	
Camestres			■		■	
Festino			■			
Baroco						
Cesaro		■	■		■	
Camestros	■		■	■	■	
Darapti		■			■	
Disamis		■			■	
Datisi		■			■	
Felapton	■	■	■			■
Bocardo						
Ferison		■	■			
Bramantip		■		■	■	■
Camenes			■		■	
Dimaris		■			■	
Fesapo	■	■	■			■
Fresison		■	■			
Camenos	■		■	■	■	

Table 5.20 Subalternation and conversion: fourth figure

5.4 Results

Looking at the tables for conversion and conversion with subalternation, we can see that there are certain syllogistic moods which behave exactly the same when it comes to which moods they prove and by which they can be proved; those are, most importantly:

(1) The C group, containing *Celarent*, *Cesare*, *Camestres* and *Camenes*

(2) The D group, containing *Darii*, *Disamis*, *Datisi* and *Dimaris*

(3) The F group, containing *Ferio*, *Festino*, *Ferison* and *Fresison*

Which, moreover, are proved only mutually by themselves; i. e. moods from the C group can be proved by the moods from the D and the F group **and only by those**. Apart from them, similarly we have:

(4) *Barbari-Baroco-Bocardo*

Where *Barbara* can be proved only by *Baroco* and *Bocardo*, and so on;[92] and:

(5) *Bramantip-Barbari*

(6) *Fesapo-Felapton*

(7) *Celaront-Cesaro*

(8) *Camestros-Camenos*

[92] *Barbara-Bocardo-Baroco* can be thus said to form a special Leibnizian triple, the moods of which, apart from being reducible to each other, *cannot be reduced into any other mood*.

Which, in pairs, appear to exhibit no difference when it comes to the proofs, as it is the case with the C, D and F group above. Finally, we have:

(9) *Darapti*

Which proves only *Cesaro* and *Camestros*, and, apart from being proved additionally by exactly *Cesaro* and *Camestros*, exhibits no differences from the D group.

If we shall thus divide all the valid moods not into figures, but into groups containing ontologically equivalent (with respect to indirect reduction) moods, the chart we would get would look like:

	Barbara	Baroco	Bocardo	Celarent	Cesare	Camestres	Camenes
Barbara		■	■				
Baroco	■		■				
Bocardo	■	■					
Celarent							
Cesare							
Camestres							
Camenes							
Darii				■	■	■	■
Disamis				■	■	■	■
Datisi				■	■	■	■
Dimaris				■	■	■	■
Ferio				■	■	■	■
Festino				■	■	■	■
Ferison				■	■	■	■
Fresison				■	■	■	■
Bramantip		■	■	■	■	■	■
Barbari		■	■	■	■	■	■
Fesapo	■	■		■	■	■	■

Felapton	■	■		■	■	■	■
Celaront				■	■	■	■
Cesaro				■	■	■	■
Camestros	■		■				
Camenos	■		■				
Darapti				■	■	■	■

Table 5.21 Group-division, I

	Darii	Disamis	Datisi	Dimaris	Ferio	Festino	Ferison	Fresison
Barbara								
Baroco								
Bocardo								
Celarent	■	■	■	■	■	■	■	■
Cesare	■	■	■	■	■	■	■	■
Camestres	■	■	■	■	■	■	■	■
Camenes	■	■	■	■	■	■	■	■
Darii					■	■	■	■
Disamis					■	■	■	■
Datisi					■	■	■	■
Dimaris					■	■	■	■
Ferio	■	■	■	■				
Festino	■	■	■	■				
Ferison	■	■	■	■				
Fresison	■	■	■	■				
Bramantip					■	■	■	■
Barbari					■	■	■	■
Fesapo	■	■	■	■				

Felapton								
Celaront								
Cesaro								
Camestros								
Camenos								
Darapti								

Table 5.22 Group-division, II

	Bramantip	Barbari	Fesapo	Felapton
Barbara				
Baroco				
Bocardo				
Celarent				
Cesare				
Camestres				
Camenes				
Darii				
Disamis				
Datisi				
Dimaris				
Ferio				
Festino				
Ferison				
Fresison				
Bramantip				
Barbari				
Fesapo				

Felapton				
Celaront				
Cesaro				
Camestros				
Camenos				
Darapti				

Table 5.23 Group-division, III

	Celaront	Cesaro	Camestros	Camenos	Darapti
Barbara					
Baroco					
Bocardo					
Celarent					
Cesare					
Camestres					
Camenes					
Darii					
Disamis					
Datisi					
Dimaris					
Ferio					
Festino					
Ferison					
Fresison					
Bramantip					
Barbari					
Fesapo					

Felapton			■	■	
Celaront	■	■			■
Cesaro	■	■			■
Camestros					
Camenos					
Darapti	■	■			

Table 5.24 Group-division, IV

If we are then to equate all the ontologically equal moods and consider only the groups, the chart would be:

	Barbara	Baroco	Bocardo	Group **C**	Group **D**	Group **F**
Barbara		■	■			
Baroco	■		■			
Bocardo	■	■				
Group **C**					■	■
Group **D**				■		■
Group **F**				■	■	
Bramantip-Barbari		■	■	■		■
Fesapo-Felapton	■	■		■	■	
Celaront-Cesaro				■	■	■
Camestros-Camenos	■		■		■	■
Darapti				■		■

Table 5.25 Groups, I

	Bram.-Barb.	Fesap.-Felap.	Celar.-Cesar.	Cames.-Camen.	Darapti
Barbara					
Baroco					
Bocardo					
Group **C**					
Group **D**					
Group **F**					
Bramantip-Barbari		■		■	
Fesapo-Felapton	■			■	
Celaront-Cesaro			■		■
Camestros-Camenos	■	■			
Darapti			■		

Table 5.26 Groups, II

From which we can easily see that the internal relationships between the groups C, D, and F are the same as between *Barbara*, *Baroco*, and *Bocardo*, which is additionally reinforced by the fact that moods from those groups cannot be proved by any other mood.

All possible cases

If we are now to sum up all our findings irrespective of the proving method, that is, consider results of both direct and indirect reductions at the same time (which is nothing less than a complete internal structure of syllogistic), our findings would look like:[93]

	Barbara	Celarent	Darii	Ferio	Barbari	Celaront
Barbara	■					
Celarent		■	■	■		
Darii		■	■	■		

[93] Note that here we are also marking the self-reduction scenario, i.e., that each mood is ultimately "reducible" also to itself.

Ferio		■	■	■		
Barbari	■	■	■	■	■	
Celaront		■	■	■		■
Cesare		■	■	■		
Camestres		■	■	■		
Festino		■	■	■		
Baroco	■					
Cesaro		■	■	■		■
Camestros	■	■	■	■	■	
Darapti		■	■	■		■
Disamis		■	■	■		
Datisi		■	■	■		
Felapton	■	■	■	■	■	
Bocardo	■					
Ferison		■	■	■		
Bramantip	■	■	■	■	■	
Camenes		■	■	■		
Dimaris		■	■	■		
Fesapo	■	■	■	■	■	
Fresison		■	■	■		
Camenos	■	■	■	■	■	

Table 5.27 Complete, I

	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros
Barbara				■		
Celarent	■	■	■			
Darii	■	■	■			

Ferio	■	■	■			
Barbari	■	■	■	■		■
Celaront	■	■	■		■	
Cesare	■	■	■			
Camestres	■	■	■			
Festino	■	■	■			
Baroco				■		
Cesaro	■	■	■		■	
Camestros	■	■	■	■		■
Darapti	■	■	■		■	
Disamis	■	■	■			
Datisi	■	■	■			
Felapton	■	■	■	■		■
Bocardo				■		
Ferison	■	■	■			
Bramantip	■	■	■	■		■
Camenes	■	■	■			
Dimaris	■	■	■			
Fesapo	■	■	■	■		■
Fresison	■	■	■			
Camenos	■	■	■	■		■

Table 5.28 Complete, II

	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison
Barbara					■	
Celarent		■	■			■
Darii		■	■			■

Ferio		■	■			■
Barbari		■	■	■	■	■
Celaront	■	■	■			■
Cesare		■	■			■
Camestres		■	■			■
Festino		■	■			■
Baroco					■	
Cesaro	■	■	■			■
Camestros		■	■	■	■	■
Darapti	■	■	■			■
Disamis		■	■			■
Datisi		■	■			■
Felapton		■	■	■	■	■
Bocardo					■	
Ferison		■	■			■
Bramantip		■	■	■	■	■
Camenes		■	■			■
Dimaris		■	■			■
Fesapo		■	■	■	■	■
Fresison		■	■			■
Camenos		■	■	■	■	■

Table 5.29 Complete, III

	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara						
Celarent		■	■		■	
Darii		■	■		■	

Ferio						
Barbari						
Celaront						
Cesare						
Camestres						
Festino						
Baroco						
Cesaro						
Camestros						
Darapti						
Disamis						
Datisi						
Felapton						
Bocardo						
Ferison						
Bramantip						
Camenes						
Dimaris						
Fesapo						
Fresison						
Camenos						

Table 5.30 Complete, IV

After rearranging these results similarly to the previous ones, we obtain what follows (notice that the moods *Camestros* and *Camenos* are being moved before *Celaront* and *Cesaro* and are displayed in the Table 5.33):

	Barbara	Baroco	Bocardo	Celarent	Cesare	Camestres	Camenes

Barbara							
Baroco							
Bocardo							
Celarent							
Cesare							
Camestres							
Camenes							
Darii							
Disamis							
Datisi							
Dimaris							
Ferio							
Festino							
Ferison							
Fresison							
Bramantip							
Barbari							
Fesapo							
Felapton							
Camestros							
Camenos							
Celaront							
Cesaro							
Darapti							

Table 5.31 Complete, Group-division, I

	Darii	Disamis	Datisi	Dimaris	Ferio	Festino	Ferison	Fresison

Barbara								
Baroco								
Bocardo								
Celarent								
Cesare								
Camestres								
Camenes								
Darii								
Disamis								
Datisi								
Dimaris								
Ferio								
Festino								
Ferison								
Fresison								
Bramantip								
Barbari								
Fesapo								
Felapton								
Camestros								
Camenos								
Celaront								
Cesaro								
Darapti								

Table 5.32 Complete, Group-division, II

	Bramantip	Barbari	Fesapo	Felapton	Camestros	Camenos

Barbara						
Baroco						
Bocardo						
Celarent						
Cesare						
Camestres						
Camenes						
Darii						
Disamis						
Datisi						
Dimaris						
Ferio						
Festino						
Ferison						
Fresison						
Bramantip						
Barbari						
Fesapo						
Felapton						
Camestros						
Camenos						
Celaront						
Cesaro						
Darapti						

Table 5.33 Complete, Group-division, III

	Celaront	Cesaro	Darapti

Barbara			
Baroco			
Bocardo			
Celarent			
Cesare			
Camestres			
Camenes			
Darii			
Disamis			
Datisi			
Dimaris			
Ferio			
Festino			
Ferison			
Fresison			
Bramantip			
Barbari			
Fesapo			
Felapton			
Camestros			
Camenos			
Celaront			
Cesaro			
Darapti			

Table 5.34 Complete, Group-division, IV

From which one major interesting thing is deducible, namely: when it comes to reduction, there are no ontological differences between moods contained in each Leibnizian triple. Be it

Celaront-Cesaro-Darapti or *Barbara-Baroco-Bocardo*, all of them are ultimately reducible and can be used to reduce exactly the same set of moods. Moreover, four of the Leibnizian triples, i.e.:

1. *Celarent*, *Festino*, *Disamis*,
2. *Darii*, *Camestres*, *Ferison*,
3. *Ferio*, *Cesare*, *Datisi*, and
4. *Camenes*, *Fresison*, *Dimaris*

can be ultimately equated with each other, as there is no ontological difference between them, thus forming a special set. The structure of the syllogistic could be then said to have:

1. The F, C, D Group, containing 12 syllogisms in total
2. The *Barbara-Baroco-Bocardo* Group, provable only mutually
3. The subalternated group (S Group), containing 9 Leibnizian subalternated moods, from which
4. The *Celaront-Cesaro-Darapti* (SII) Group can be further distinguished

The table summarizing this last division is:

	B Group	C, D, F Group	S Group	SII Group
B Group	■			
C, D, F Group		■		
S Group	■	■	■	
SII Group		■		■

Table 5.35 Final reduction

When it comes to the minimal sets of inference rules in case the indirect method is applied, the results are:

(1) In the no single-premise inference rules and subalternation-only cases, neither the first nor the second and third figure is enough to prove all the moods by itself, since the moods from the fourth figure stand alone and can be proved only mutually. This can even shed some additional light on why Aristotle has not admitted the fourth figure and why it was considered superfluous by many authors. The usual reason was that the fourth is nothing more than the first with the premises in the moods converted – here, we additionally see that without conversion as a rule of inference, the fourth figure not derivable from either of the first three figures, which makes it structurally distinct.

(2) In the case of the conversion and conversion with subalternation scenarios, Aristotle's reduction seems to hold for *Celarent* and *Barbara* (including the fourth figure). There is, however, no ontological difference between taking *Celarent* and any other mood from the C group described above. Furthermore, the same result (i. e. possibility of proving every other mood) can be obtained by combining *Barbara* with *Ferio* (and, further, any mood from the F group). It cannot be obtained from *Barbara* + a D syllogism, as the combination cannot prove *Darapti*. It is further worth to note that, **if subalternation is included**, the same result would be obtained by combining any of the F group syllogisms with *Baroco*. If we decide to omit the subalternation, in turn, it does not work for the C syllogisms only due to *Baroco's* inability to prove *Camestros* and *Camenos*. Thus, if we agree that when subalternation does not hold, the additional moods with particular conclusions (such as *Camestros* and *Cesaro*) should not work as well, then there is no difference between *Barbara* + a C/F syllogism and *Baroco* + a C syllogism taken as a minimal set.

If we decide to further combine the original, Aristotelian direct proofs with the indirect ones to consider all possible valid proof (or reduction) scenarios for all the moods, it can be seen that even greater amount of moods can be grouped together according to the reduction one can make by the use of them. If we are to next reduce the chart, we shall obtain four different possible reduction behaviors: the *Barbara-Baroco-Bocardo* Group, the C, D, and F Groups further grouped together, the Subalternated Group, and the *Celaront-Cesaro-Darapti* Group, essentially dividing all the moods into four different sets, entirely different from the usual syllogistic figures. In this scenario, as the last two of the identified groups can be reduced to the first two, the ultimately irreducible moods (i.e., the counterpart of Aristotelian *Dictum*) can be any pair combining one mood from the B Group and one mood from the combined C, D, F Group.

We can also comment on the status of the fourth figure taking into account our results. In this respect, we have to admit that the fourth figure indeed behaves differently from the first three. In the no single-premise inference rules and subalternation-only scenarios, this is the group that cannot be proved by any mood from any other group and thus undermines the Aristotelian *Dictum* schema (note, however, that since Aristotle does not admit the fourth figure, this argument cannot be seen as a direct critique of Aristotle). Its validity is then not provable by the validity of the first figure by the use of pure indirect method, and one is forced to the Leibnizian step of applying conversions directly to the premises.

The last thing worth noting is the precise status of the Leibnizian triples. Here, we have seen that the complete indirect-reduction(s) schema reveals that their reduction possibilities are exactly the same across the moods they contain. It can be, however, further seen that all the subalternated Leibnizian triples, as well as those pertaining to our C, D, and F groups, can be ultimately equated with each other (when we decide to combine the indirect cases with the direct ones). Here, we are not left with the question of what is the precise status of the fourth figure, but with the question of *why the Celaront-Cesaro-Darapti Group stands as a separate one?*

5.5 The computational approach: connecting alternative proofs to the non-emptiness assumptions

This final question points us to other direction: besides the calculation, arrangement, and interpretation of the proofs, what remains to be done is answering (or at least attempting to answer) the question of *why the proofs are as they are?*, i.e., why certain moods have more proofs than others, why some moods exhibit exactly the same proof-pattern, while others are in this term unique, etc. To attempt answering this question, we need to do some calculations. If we count all the alternative **direct** proofs as they are present in Charts 5.1 to 5.4, as well as all the **indirect** ones present in Charts 5.17 to 5.20 and the number of all the possibilities taken together as present in Charts 5.27 to 5.30, we obtain the following:

Mood	**Number of *direct proofs***	**Number of *indirect proofs***	**Number of *direct* plus *indirect* proofs**
		First figure	
Barbara	0	2	2
Celarent	3	8	11
Darii	3	8	11
Ferio	3	8	11
Barbari	6	14	20
Celaront	8	15	**15**
		Second figure	
Cesare	3	8	11
Camestres	3	8	11
Festino	3	8	11
Baroco	0	2	2
Cesaro	9	15	**15**

Camestros	6	14	20
	Third figure		
Darapti	4	10	14
Disamis	3	8	11
Datisi	3	8	11
Felapton	6	14	20
Bocardo	0	2	2
Ferison	3	8	11
	Fourth figure		
Bramantip	6	14	20
Camenes	3	8	11
Dimaris	3	8	11
Fesapo	6	14	20
Fresison	3	8	11
Camenos	6	14	20

Table 5.36 Moods proof-count

The patterns discernible in the Table 5.32. are, of course, parallel to those already read off from the Charts, and the moods can be seen to be grouped somewhat accordingly. However, considering the quantitative and not qualitative aspects of the computed alternative proofs can bring some interesting results as well.

Starting from the number of direct proofs, we can note that with the basic amount of proofs being three, all of the "anomalies" (notwithstanding *Barbara*, *Baroco*, and *Bocardo*, which have the number zero as expected) are due to the moods subalternated in the sense of Leibniz, or, more generally, those having two general sentences as their premises and a particular statement as their conclusion. These moods, moreover, correspond exactly to the moods which validity necessitates the non-emptiness assumptions as showed in Chapter 4, and this correspondence is not a trivial one, as it appears that the need for assuming non-emptiness of one of the terms allows for the proofs to include more tools in their proof-toolkit, namely the conversion *per accidens* and two subalternations – single-premise inference rules which by themselves need the assumption of non-emptiness to be valid.

This correspondence can be spotted in the third column (pertaining to the *indirect* proofs) of the Table 5.32 as well: *Barbara*, *Baroco*, and *Bocardo* are all proved mutually and hence have the number two; all the moods from Groups C, D, and F as defined earlier, have the number eight; the number of proofs for Leibniz-subalternated moods, however, is again significantly higher and varies between 10 and 15. This is once again due to the use of subalternations and the third conversion, which allow for more proof-scenarios (this time *indirect* proof-scenarios) to be conclusive. Interestingly, the Leibnizian-subalternate with the least amount of proofs is *Darapti*, and the two ones with the most amount of them are *Celaront* and *Cesaro*, which, together with *Darapti*, form the previously recognized **SII** Group. The most frequent amount of proofs for L-subalternated moods is 14, for 6 of them grouped in the previously-defined **SI** Group.

Finally, note that if we consider sequences of the three numbers, we obtain the same grouping as before: the F, C, D Group has pattern 3–8–11, the *Barbara-Baroco-Bocardo* Group has 0–2–2, the S Group (minus the SII Group) has 6–4–20, and, finally, the SII Group is not only standing as separate, but do not have a consistent number pattern, with all the three moods exhibiting different behavior.

5.5.1 Organizing the proofs through the lenses of Buridan and Leibniz

At the end of Chapter 2, we have described proposals of Buridan and Leibniz which consisted of either creating new syllogistic moods and their names, or renaming several classical syllogistic moods posing names different than in the medieval custom. Here we further note that organizing the proofs computed in this Chapter according to these proposals allows to look at them from a different angle and spot several accuracies.

First, we note that the two subalternated moods showing the highest degree of anomalous behavior, *Celaront* and *Cesaro*, are exactly those two renamed by Leibniz to begin with **L**: ***Legano*** and ***Lesaro***, which, together with *Darapti*, constitute the **SII** Group (and the sixth Leibnizian triple as defined in Section 5.3); the other two moods renamed by Leibniz, this time to begin with **G**, are *Barbari* (***Gabali***) and *Camestros* (***Gaceno***), which instead, together with *Felapton*, constitute the core of the **SI** Group (i.e., the whole of the SI Group without the fourth figure; and the fifth "Leibnizian triple"). Note also that all three of them exhibit exactly the same "proof-behavior", with number of direct proof being 6, number of indirect proofs being 14, and number of direct and indirect proofs combined being 20. The ***Legano***-***Lesaro***-*Darapti* triple, instead, has it that all three moods have varying numbers of proofs.

When it comes to answering the question why Leibniz did not renamed *Darapti* and *Felapton* to begin with **L** and **G** accordingly, the answer is most likely that he was coining new names only for those syllogistic moods which were not commonly accepted at the time, i.e., *Celaront*, *Camestros*, etc., and was not looking to alter syllogistic names already established. Other point is that the **L** and **G** were primarily supposed to indicate the common proof method to be used ("G syllogisms" reduced to "G syllogisms" etc.) – in the case of newly introduced moods, such proof procedure might have been possible to establish, but in the case of already known moods which had their proof methods indicated by Aristotle and upheld by a long tradition, it could not be the case. That being said, we might speculate that if he was governed more by such metatheoretical considerations as the number of alternative proofs and behavior of moods with respect to *conversio syllogismi* and the indirect-proof method, he might have altered other names as well.

5.5.2 Viewing the proofs through the lenses of additional non-emptiness assumptions

In Chapter 4, we have divided all the moods into three groups: in group one, no additional existential premise was needed, as no ontological requirements were necessary; in group two, no additional existential premise was needed, as the ontological requirements were addressed by a particular premise of a given mood; in group three, finally, an additional existential premise was needed, as the ontological requirements were not addressed by any of the premises, both of them being universal.

These three groups, with the moods they contain listed in Chapter 4, do not overlap strictly with the groups of moods defined in this chapter. However, what they do have in common is that the first two groups defined in Chapter 4 (G1 and G2) overlap with the first two groups defined here: the F, C, D Group and the *Barbara-Baroco-Bocardo* Group; and, most importantly, the subalternated group (S Group) overlaps with the G3 group from Chapter 4, both of them representing the subalternated moods, i.e., those which necessitate the adoption of an additional existential premise.

Now, we may connect these results to the fact that the subalternated moods also exhibit different behavior when it comes to number of respective proofs. What is happening is that, since subalternated moods are necessitating adoption of an additional existential premise, their proofs require the use of tools which necessitate such an adoption themselves, i.e., conversion with limitation, either of the two subalternations, or, which is the case with a significant amount of proofs done in this chapter, other subalternated mood used as an inference rule. The ultimate reason, then, of the S Group standing out quantitatively as shown in Table 5.36, is the fact that since they admit more inference rules to be used in their proofs, they also get combinatorially more proof-options.[94] Moreover, we also have a clear computational evidence that the necessity

[94] That holds for both indirect (consult proofs in Section 5.3.3 of this Chapter) and direct proofs (consult lists of alternative direct proofs present in the next chapter).

of adoption of additional existential premises may be traced back to the fundamental syllogistic relations of subalternation(s) and conversion with limitation, which both consist of inferring a universal premise from a particular one.

Chapter 6

The formalization of assertoric syllogistic in Isabelle/HOL

6.1 Introduction: theorem provers

Automated proof assistants, known also as theorem provers, are interactive tools developed since the second half of the XX century and widely used for formalization of mathematics. Their primary role being formal verification, they are being used for verifying mathematical proofs (via formalization) and checking the correctness of programming languages or computer software. In the case of formal verification of mathematics (and logic), mathematical formulas, being expressed in the formal language of the prover, are checked using various types of logical calculi via the prover's automated tools.

Isabelle is a software developed since almost forty years by scientists from the Technical University of Munich and the University of Cambridge and is classified among the most important theorem provers (Gordon, 2008). Its most widely used instance and simultaneously one of the most widely used automated provers for formalization of mathematics is the Isabelle/HOL, based on higher order logic. In the years 2017 to 2023, within the framework of the ALEXANDRIA ERC project, it was used to formalize a wide array of mathematical theories.[95] Additionally, a formalization of the foundations of the Aristotelian syllogistic has been made (Koutsoukou-Argyraki, 2019, 2022, 2024).

The present Chapter has been written on the basis of an article written in collaboration with Dr Angeliki Koutsoukou-Argyraki from the University of Cambridge (Koutsoukou-Argyraki and Wapniarski, 2025), and is aimed at extending the formalization of the foundations of syllogistic reasoning done by Dr Angeliki Koutsoukou-Argyraki (Koutsoukou-Argyraki, 2019, 2022, 2024), using this extension in order to gain further insights on the observations made in the preceding Chapters (most notably Chapter 3 and Chapter 5), and, commenting on the philosophical interpretations and implications of the use of a theorem prover to formalize the traditional logic. The plan is, accordingly, to present the foundational formalization, to present its extension, and to discuss its various properties. As Aristotle's logic is still a source of inspiration for developments of AI, especially in computational argumentation and cognitive modeling (Saldanha and Kakas, 2019), the proposed formalization can be treated as a pilot study, enabling seamless use of the Aristotelian system in more advanced future research projects (Koutsoukou-Argyraki and Wapniarski, 2025, p. 2).

Although the initial choice of Isabelle was predicated upon the expertise of the formalization's author, this choice proved to be beneficial for both the obtained results and their potential for providing philosophical insights. For example, the user guided nature of the Isabelle system (interactivity) turned out to be crucial for spotting the behavior of QuickCheck (Isabelle's tool for auto-correction) regarding the non-emptiness of sets assumption (see Section 6.4.2).

6.2 The formalization: basics

The basics of the formalization are built following the analysis present in (Smith, 2022) and correspond roughly to the system of syllogistic as present in writings of Aristotle and

[95] The results of the project can be found on he Archive of Formal Proofs (https://www.isa-afp.org).

presented in the first Chapter of this work. The underlying basis for the formalization is the set-theoretic formulation, which, in turn, corresponds to the strategy taken in Chapter 5 and applied to determining the necessity of assuming non-emptiness of terms. The Logical Square is formalized as presented in Table 6.1 (with indefinite sentences, mentioned by Aristotle but not of a much use thereafter – see Section 1.1.1, being also included).

	Affirmations	**Denials**
Universal	P affirmed of all of S (All S is/are P)	P denied of all of S (No S is P)
Particular	P affirmed of some of S (Some S is/are P)	P denied of some of S (Some S is not P)
Indefinite	P affirmed of S (S is P)	P denied of S (S is not P)

Table 6.1 Types of affirmations and denials

The set-theoretic formulation, together with the abbreviations used in the Isabelle/HOL code, are as presented in Table 6.2. Note that the abbreviations for a universal affirmation and particular denial have been altered from **A** to **Q** and from **O** to **Z**, respectively, due to Isabelle's notation already using the A and O letters.

	Abbreviations	**Sentences**
AQB	A belongs to all B (Every B is A)	Universal Affirmation
AEB	A belongs to no B (No B is A)	Universal Denial
AIB	A belongs to some B (Some B is A)	Particular Affirmation
AZB	A does not belong to all B (Some B is not A)	Particular Denial
A(QI)B	B is A	Indefinite Affirmation
A(EZ)B	B is not A	Indefinite Denial

Table 6.2 Logical abbreviations and their meanings

The sentences are then formalized in the Isabelle/HOL code as presented in Figure 6.1.

The simple inferences, including two classical simple conversions (following the notation established in Isabelle): AEB $\rightarrow$ BEA, AIB $\rightarrow$ BIA, and the conversion with limitation: AQB

```
text‹The Square of Oppositions›

definition universal_affirmation :: "'a set  ⇒'a set  ⇒ bool"  (infixr "Q" 80)
  where "A Q B ≡ ∀ b ∈ B . b ∈ A "

definition universal_denial ::  "'a set  ⇒'a set   ⇒ bool"  (infixr "E" 80)
  where "A E B ≡ ∀ b ∈ B. ( b ∉ A)  "

definition particular_affirmation ::  " 'a set  ⇒'a set  ⇒ bool"  (infixr "I" 80)
  where "A I B ≡ ∃ b ∈ B. ( b ∈ A) "

definition particular_denial ::  "'a set  ⇒'a set ⇒ bool"  (infixr "Z" 80)
  where "A Z B ≡ ∃ b ∈ B. ( b ∉ A) "

text‹The indefinite sentences›

definition indefinite_affirmation ::  " 'a set ⇒'a set ⇒ bool"  (infixr "QI" 80)
  where "A QI B ≡(( ∀ b ∈ B. (b ∈ A)) ∨  (∃ b ∈ B. (b ∈ A)))"

definition indefinite_denial ::  "'a set  ⇒'a set ⇒ bool"  (infixr "EZ" 80)
  where "A EZ  B ≡ (( ∀ b ∈ B. (b ∉ A)) ∨ (∃ b ∈ B. (b ∉ A)))"
```

Fig. 6.1 Formalization of the Square of Oppositions' sentences

→ BIA, together with the subalternations: AQB → AIB and AEB → AZB, are then formalized as follows in Figure 6.2.

```
text‹The conversions›

lemma simple_conversion1:
  assumes "A E B" shows "B E A"
  using assms universal_denial_def by blast

lemma simple_conversion2:
  assumes "A I B" shows "B I A"
  using assms unfolding  particular_affirmation_def by blast

lemma per_accidens_conversion3:
  assumes "A Q B" and "B ≠{}" shows "B I A"
  using assms  unfolding universal_affirmation_def particular_affirmation_def by blast

text‹The subalternations›

lemma subalternation1:
  assumes "A Q B" and "B≠{}" shows "A I B"
  using assms(1) assms(2) simple_conversion2 per_accidens_conversion3 by auto

lemma subalternation2:
  assumes "A E B" and "B ≠{}" shows "A Z B"
  by (meson assms(1) assms(2) particular_affirmation_def particular_denial_def subalternation1
universal_affirmation_def universal_denial_def)
```

Fig. 6.2 Formalization of the conversions and subalternations and proved in Isabelle/HOL using various methods (`blast`, `auto`, `meson`).[96] The proofs being found automatically by Sledgehammer, Isabelle's primary automation tool,[97] making use of relevant previously introduced definitions as assumptions (noted by `assms`). All three conversions are then proved by unfolding the definitions of categorical statements (i.e., only definitions of these are needed for the proof), while the proofs of the two subalternations are slightly more elaborate: the first subalternation (AQB $\rightarrow$ AIB) is proved by a combination of the *per accidens* conversion (AQB $\rightarrow$ BIA) and the second simple conversion (AIB $\rightarrow$ BIA) (which corresponds to how it would be proved by hand); the second subalternation, in turn, is proved by Isabelle using the first subalternation and, once more, the definitions of the categorical statements, making the exact proof hard to reproduce (for a comment on about explainability of the prover see Section 6.4.3).

6.2.1 Syllogistic figures

With the categorical sentences and conversions formalized, the four syllogistic figures in their purely Aristotelian form (i.e., without the subalternated moods, which will be introduced later) can be formalized as well. The first figure is formalized as seen in the Figure 6.3, with the medieval mnemonic names covered in Chapter 2 being used:

---

[96] Explaining the exact nature of these methods is beyond the scope of this work (for an overwiev of them, consult (Nipkow et al., 2002)) and would not be of much use either, as the exact proofs carried out by Isabelle remain unknown for the user – what is known is only the final result (i.e., that a theorem is provable or not) and the information about what assumptions/lemmas were used to carry out the proof. Although it can sound unsatisfactory for someone seeking deep understanding of whats happening, such *modus operandi* lies in the nature of theorem provers. Although the so-called explainability has been recently a hot topic when it comes to AI and Large Language Models, literature on this subject with respect to theorem provers is still scarce. One good reference may be (Ayers, 2021).

[97] For an introduction to Sledgehammer see (Blanchette et al., 2024).

```
subsubsection‹First Figure›

lemma Barbara:
  assumes "A Q B " and "B Q C" shows "A Q C"
  by (meson assms universal_affirmation_def)

lemma Celarent:
  assumes "A E B " and "B Q C" shows "A E C"
  by (meson assms universal_affirmation_def universal_denial_def)

lemma Darii:
  assumes  "A Q B" and "B I C" shows "A I C"
  by (meson assms particular_affirmation_def universal_affirmation_def)

lemma Ferio:
  assumes  "A E B" and "B I C" shows "A Z C"
  by (meson assms particular_affirmation_def particular_denial_def universal_denial_def)
```

Fig. 6.3 Formalization of the First-Figure basic moods

In the code, we see that only the definitions of categorical sentences were used to proof the four respective basic moods, which aligns with the Aristotelian notion of the first figure serving as a basis for the others. Isabelle's proof method employed by Sledgehammer is `meson`, but other methods could have been implemented successfully as well, while still reasoning solely on the definitions of the categorical sentences.

Using the formalization done so far, we can also formalize example deductions both with individual and with general predication. These formalizations are presented in Figure 6.4. The first formalization uses mood Barbara as its proof, while the second one is being proved only by the definition of the universal affirmation (and constitutes a nice example of ecthesis – see Section 1.3 in Chapter 1).

```
text‹Example of a deduction with general predication.›

lemma GreekMortal :
  assumes  "Mortal Q Human" and "Human Q Greek "
  shows " Mortal Q Greek "
using assms Barbara by auto

text‹Example of a deduction with individual predication.›

lemma SocratesMortal:
  assumes "Socrates ∈ Human " and "Mortal Q Human"
  shows "Socrates ∈ Mortal "
using assms by (simp add: universal_affirmation_def)
```

Fig. 6.4 Example deductions with individual and general predication

In the next two figures (the second and the third), the proofs are slightly more complicated, as instead of using definitions of the categorical statements, Isabelle's automation tool finds proofs which use the already-proved moods from the first figure. This again aligns seamlessly with the Aristotelian claim that all the moods which do not belong to the first figure should be reduced to the moods that do. The code for figures two and three is presented in Figures 6.5 and 6.6.

```
subsubsection‹Second Figure›

lemma Cesare:
  assumes  "A E B " and "A Q C" shows "B E C"
  using Celarent simple_conversion1 assms by blast

lemma Camestres:
  assumes  "A Q B " and "A E C" shows "B E C "
  using Cesare simple_conversion1 assms by blast

lemma Festino:
  assumes  "A E B " and "A I C" shows "B Z C "
  using Ferio simple_conversion1 assms by blast

lemma Baroco:
  assumes  "A Q B " and "A Z C" shows "B Z C    "
  by (meson assms particular_denial_def universal_affirmation_def)
```

Fig. 6.5 Formalization of the second-figure basic moods

In the Figures, we see that most of the proofs found by Isabelle reflect those described by Aristotle. The second- and third-figure moods are being successfully proved by using their basic moods from the first figure and various types of conversions. This is the case with *Cesare* (proved by *Celarent*), *Festino* (proved by *Ferio*), *Darapti* and *Disamis* (proved by *Darii*), and *Ferison* (proved by *Ferio*). We can also observe that this is not the case for all of the moods, and there are examples where a mood is proved not by its first-figure counterpart, but by another mood from its or other figure – this is the case with *Camestres* (proved by *Cesare*), *Felapton* (proved by *Festino*), and *Datisi* (proved by *Disamis*). The reason for that is most likely that Isabelle's automation tool, viewing these two kinds of proofs as equal in length and complexity, treats them equally and does not distinguish between a proof which uses a first-figure moods and a one that uses second- or third-figure ones (for a full discussion concerning finding alternative direct proofs with Isabelle/HOL see Section 6.3).

```
subsubsection‹Third Figure›

lemma Darapti:
  assumes  "A Q C " and "B Q C" and "C ≠{}"   shows "A I B "
  using Darii assms unfolding  universal_affirmation_def particular_affirmation_def
  by blast

lemma Felapton:
  assumes  "A E C" and "B Q C"  and  "C ≠{}"   shows "A Z B"
  using Festino simple_conversion1 per_accidens_conversion3 assms by blast

lemma Disamis:
  assumes  "A I C" and "B Q C" shows "A I B"
  using Darii simple_conversion2 assms by blast

lemma Datisi:
  assumes  "A Q C" and "B I C" shows "A I B"
  using Disamis simple_conversion2 assms by blast

lemma Bocardo:
  assumes  "A Z C" and "B Q C" shows "A Z B"
  by (meson assms particular_denial_def universal_affirmation_def)

lemma Ferison:
  assumes  "A E C " and "B I C" shows "A Z B   "
  using Ferio simple_conversion2 assms by blast
```

Fig. 6.6 Formalization of the third-figure basic moods

The two remaining moods which are not proved in either way are *Baroco* and *Bocardo*. We have noted many times in Chapter 5 that these two moods do not have any direct proofs. In this case, as an indirect proof of either of these moods would be more complex, Isabelle finds a simpler proof instead, using solely the definitions of the categorical sentences and applying the automatic proof method `meson`. In this regard, the respective proofs for *Baroco* and *Bocardo* resemble the proofs found by Isabelle for the four basic first-figure moods. We can therefore say that *Baroco* and *Bocardo* are treated *as if* they were “basic” moods as well.

The remaining fourth figure can be formalized in Isabelle in both of its historical forms: as the fourth and as the indirect-first figure. This formalization is presented in Figures 6.7 and 6.8. Note that since Isabelle’s automation tools do not distinguish the order of premises (i.e., BQA $\wedge$ CQB $\rightarrow$ AIC is treated equally with CQB $\wedge$ BQA $\rightarrow$ AIB), the indirect-first figure, although written using different using different names for moods and different order of premises, will be treated by the program equally with the fourth figure.

```
text‹Fourth Figure›

lemma Bramantip:
  assumes "B Q A" and "C Q B" and "A≠{}" shows "A I C"
  using Barbara assms(1) assms(2) assms(3) per_accidens_conversion3 by blast

lemma Camenes:
  assumes "B Q A" and "C E B" shows "A E C"
  using Celarent assms(1) assms(2) simple_conversion1 by blast

lemma Dimaris:
  assumes "B I A" and "C Q B" shows "A I C"
  using Darii assms(1) assms(2) simple_conversion2 by blast

lemma Fesapo:
  assumes "B E A" and "C Q B" and "B≠{}" shows "A Z C"
  using Festino assms(1) assms(2) assms(3) per_accidens_conversion3 by blast

lemma Fresison:
  assumes "B E A" and "C I B" shows "A Z C"
  using Ferio assms(1) assms(2) simple_conversion1 simple_conversion2 by blast
```

Fig. 6.7 Formalization of the fourth-figure basic moods

```
text‹Fourth Figure as indirect First Figure›

lemma Baralipton:
  assumes "C Q B" and "B Q A" and "A≠{}" shows "A I C"
  using Barbara assms(1) assms(2) assms(3) per_accidens_conversion3 by blast

lemma Celantes:
  assumes "C E B" and "B Q A" shows "A E C"
  using Cesare assms(1) assms(2) simple_conversion1 by blast

lemma Dabitis:
  assumes "C Q B" and "B I A" shows "A I C"
  using Disamis assms(1) assms(2) simple_conversion2 by blast

lemma Fapesmo:
  assumes "C Q B" and "B E A" and "B≠{}" shows "A Z C"
  using Ferio assms(1) assms(2) assms(3) per_accidens_conversion3 simple_conversion1 by blast

lemma Fresisomorum:
  assumes "C I B" and "B E A" shows "A Z C"
  by (meson Festino assms(1) assms(2) particular_affirmation_def)
```

Fig. 6.8 Formalization of the indirect first-figure moods

6.2.2 Subalternated moods

Using the formalization of subalternations presented earlier (Figure 6.2), the subalternated moods can be formalized as well (Figure 6.9). What follows is that the first figure gets additional *Barbari* derived from *Barbara* and *Celaront* derived from *Celarent*. The second figure is augmented by *Cesaro* from *Cesare* and *Camestros* from *Camestres*, and the fourth figure gets additional *Camenos* from *Camenes*.

```
text‹Additions to the First Figure with subalternations›

lemma Barbari:
  assumes "A Q B" and "B Q C" and "C≠{}" shows "A I C"
  using Barbara assms(1) assms(2) assms(3) subalternation1 by blast

lemma Celaront:
  assumes "A E B" and "B Q C" and "C≠{}" shows "A Z C"
  using Celarent assms(1) assms(2) assms(3) subalternation2 by blast

text‹Additions to the Second Figure with subalternations›

lemma Cesaro:
  assumes "A E B" and "A Q C" and "C≠{}" shows "B Z C"
  using Celarent assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast

lemma Camestros:
  assumes "A Q B" and "A E C" and "C≠{}" shows "B Z C"
  using Celarent assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast

text‹Additions to the Fourth Figure with subalternations›

lemma Camenos:
  assumes "B Q A" and "C E B" and "C≠{}" shows "A Z C"
  using Cesare assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
```

Fig. 6.9 Formalization of the subalternated moods

The added moods are easily proved in Isabelle by automation – *Barbari* and *Celaront* are proved by using *Barbara* and *Celarent* respectively together with the relevant type of subalternation as one would expect. *Cesaro*, *Camestros*, and *Camenos* are, in turn, not proved directly by their “superalternate” moods, but different proofs by *Celarent* and *Cesare* were found instead.

With the four figures and all the subalternated moods added, we now have a complete formalization of all 24 classically recognized syllogistic moods. This formalization may be now used in order to verify our results and statements from previous Chapters concerning both alternative syllogistic proofs and the necessity of using non-empty terms.

6.3 The formalization: alternative proofs

The possibility of finding alternative proofs using Isabelle's automation has already emerged earlier, when we have seen that in some cases, moods from the figures II-IV are proved not by their "mother" first-figure moods, but by other moods from figures II-IV already proved by Isabelle and thus treated as legitimate lemmas for further proofs.

Now, we intentionally apply Isabelle's automation tools in order to replicate the study of direct syllogistic proofs done in Chapter 5, i.e., to find **all** the possible alternative direct proofs using the prover. All the possible alternative direct syllogistic proofs found by Isabelle for figures I-IV are shown in Figures 6.10, 6.11, 6.12, and 6.13 respectively.

```
subsubsection‹First Figure iterated›

lemma Barbara_iterated:
  assumes "A Q B " and "B Q C" shows "A Q C"
  by (meson assms universal_affirmation_def)
  (*no alternative direct proofs*)

lemma Celarent_iterated:
  assumes "A E B" and "B Q C" shows "A E C"
  by (meson assms universal_affirmation_def universal_denial_def)
  (*using Cesare assms(1) assms(2) simple_conversion1 by blast
  using Camestres assms(1) assms(2) simple_conversion1 by blast
  using Camenes assms(1) assms(2) simple_conversion1 by blast*)

lemma Darii_iterated:
  assumes "A Q B" and "B I C" shows "A I C"
  by (meson assms particular_affirmation_def universal_affirmation_def)
  (*using Disamis assms(1) assms(2) simple_conversion2 by blast
  using Datisi assms(1) assms(2) simple_conversion2 by blast
  using Dimaris assms(1) assms(2) simple_conversion2 by blast*)

lemma Ferio_iterated:
  assumes "A E B" and "B I C" shows "A Z C"
  by (meson assms particular_affirmation_def particular_denial_def universal_denial_def)
  (*using Festino assms(1) assms(2) simple_conversion1 by blast
  using Ferison assms(1) assms(2) simple_conversion2 by blast
  using Fresison assms(1) assms(2) simple_conversion1 simple_conversion2 by blast*)

lemma Barbari_iterated:
  assumes "A Q B" and "B Q C" and "C≠{}" shows "A I C"
  using Barbara assms(1) assms(2) assms(3) subalternation1 by blast
  (*using Darii assms(1) assms(2) assms(3) subalternation1 by blast
  using Darii assms(1) assms(2) assms(3) simple_conversion2 per_accidens_conversion3 by blast
  using Disamis assms(1) assms(2) assms(3)  simple_conversion2 per_accidens_conversion3 by blast
  using Datisi assms(1) assms(2) assms(3)  per_accidens_conversion3 by blast
  using Bramantip assms(1) assms(2) assms(3)  simple_conversion2 by blast
  using Dimaris assms(1) assms(2) assms(3)  simple_conversion2 subalternation1 by blast*)

lemma Celaront_iterated:
  assumes "A E B" and "B Q C" and "C≠{}" shows "A Z C"
  using Celarent assms(1) assms(2) assms(3) subalternation2 by blast
  (*using Ferio assms(1) assms(2) assms(3) simple_conversion2 per_accidens_conversion3 by blast
  using Cesare assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  using Festino assms(1) assms(2) assms(3) simple_conversion1 subalternation1 by blast
  using Cesaro assms(1) assms(2) assms(3) simple_conversion1 by blast
  using Ferison assms(1) assms(2) assms(3) per_accidens_conversion3 by blast
  using Camenes assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  using Fresison assms(1) assms(2) assms(3) simple_conversion1 per_accidens_conversion3 by blast*)
```

Fig. 6.10 Formalization of the first-figure alternative direct proofs

All of the proofs were found by pure automation – just by calling the Sledgehammer, and the search was initiated only after all 24 moods (note that the indirect first-figure moods are not included here) were first available to Isabelle as lemmas. In cases where not many alternative proofs were possible for a mood (for example, in the case of *Festino*), a simple call was enough to generate all the alternatives. In those cases where more proofs were possible, after hitting the generated proofs, the lemmas (i.e., the moods) which were already used in them were commented out in the code, and the Sledgehammer was called again.

```
subsubsection‹Second Figure iterated›

lemma Cesare_iterated:
  assumes "A E B" and "A Q C" shows "B E C"
 using Celarent simple_conversion1 assms by blast
  (*using Camestres assms(1) assms(2) simple_conversion1 by blast
  using Camenes assms(1) assms(2) simple_conversion1 by blast*)

lemma Camestres_iterated:
  assumes "A Q B" and "A E C" shows "B E C"
  using Cesare simple_conversion1 assms by blast
  (*using Celarent assms(1) assms(2) simple_conversion1 by blast
  using Camenes assms(1) assms(2) simple_conversion1 by blast*)

lemma Festino_iterated:
  assumes "A E B" and "A I C" shows "B Z C"
  using Ferio simple_conversion1 assms by blast
  (*using Ferison assms(1) assms(2) simple_conversion1 simple_conversion2 by blast
  using Fresison assms(1) assms(2) simple_conversion2 by blast*)

lemma Baroco_iterated:
  assumes "A Q B" and "A Z C" shows "B Z C"
  by (meson assms particular_denial_def universal_affirmation_def)
  (*no alternative direct proofs*)
lemma Cesaro_iterated:
  assumes "A E B" and "A Q C" and "C≠{}" shows "B Z C"
  using Celarent assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  (*using Celaront assms(1) assms(2) assms(3) simple_conversion1 by blast
  using Ferio assms(1) assms(2) assms(3) simple_conversion1 subalternation1 by blast
  using Cesare assms(1) assms(2) assms(3) subalternation2 by blast
  using Camestres assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast)
  using Festino assms(1) assms(2) assms(3) subalternation1 by blast
  using Ferison assms(1) assms(2) assms(3) simple_conversion1 per_accidens_conversion3 by blast
  using Camenes assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  using Fresison assms(1) assms(2) assms(3) per_accidens_conversion3 by blast*)

lemma Camestros_iterated:
  assumes "A Q B" and "A E C" and "C≠{}" shows "B Z C"
  using Celarent assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  (*using Cesare assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast)
  using Camestres assms(1) assms(2) assms(3) subalternation2 by blast
  using Baroco assms(1) assms(2) assms(3) subalternation2 by blast
  using Camenes assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  using Camenos assms(1) assms(2) assms(3) simple_conversion1 by blast*)
```

Fig. 6.11 Formalization of the second-figure alternative direct proofs

This procedure was then reapplied until the Sledgehammer was not able to find any other proofs. In both cases, the procedure was finished when the “no proof found” message was displayed – meaning that there were no lemmas left in the code by which a mood in question can be proved. In each case, Sledgehammer was able to find all the possible alternative proofs without having been provided with any hints with respect to the proof method or assumptions (which can be easily reproduced).

```
subsubsection‹Third Figure iterated›

lemma Darapti_iterated:
  assumes "A Q C" and "B Q C" and "C≠{}" shows "A I B"
  using Darii assms unfolding  universal_affirmation_def particular_affirmation_def by blast
  (*by (meson Darii assms(1) assms(2) assms(3) per_accidens_conversion3)
  using Disamis assms(1) assms(2) assms(3) subalternation1 by blast
  using Datisi assms(1) assms(2) assms(3) subalternation1 by blast
  using Dimaris assms(1) assms(2) assms(3) per_accidens_conversion3 by blast*)

lemma Disamis_iterated:
  assumes "A I C" and "B Q C" shows "A I B"
  using Darii simple_conversion2 assms by blast
  (*using Datisi assms(1) assms(2) simple_conversion2 by blast
  using Dimaris assms(1) assms(2) simple_conversion2 by blast*)

lemma Datisi_iterated:
  assumes "A Q C" and "B I C" shows "A I B"
  using Disamis simple_conversion2 assms by blast
  (*using Darii assms(1) assms(2) simple_conversion2 by blast
  using Dimaris assms(1) assms(2) simple_conversion2 by blast*)

lemma Felapton_iterated:
  assumes "A E C" and "B Q C" and "C≠{}" shows "A Z B"
  using Festino simple_conversion1 per_accidens_conversion3 assms by blast
  (*using Ferio assms(1) assms(2) assms(3) per_accidens_conversion3 by blast
  using Ferison assms(1) assms(2) assms(3) subalternation1 by blast
  using Bocardo assms(1) assms(2) assms(3) subalternation2 by blast
  using Fesapo assms(1) assms(2) assms(3) simple_conversion1 by blast
  using Fresison assms(1) assms(2) assms(3) simple_conversion1 subalternation1 by blast*)

lemma Bocardo_iterated:
  assumes "A Z C" and "B Q C" shows "A Z B"
  by (meson assms particular_denial_def universal_affirmation_def)
  (*no alternative direct proofs*)

lemma Ferison_iterated:
  assumes "A E C" and "B I C" shows "A Z B"
  using Ferio simple_conversion2 assms by blast
  (*using Festino assms(1) assms(2) simple_conversion1 simple_conversion2 by blast
  using Fresison assms(1) assms(2) simple_conversion1 by blast*)
```

Fig. 6.12 Formalization of the third-figure alternative direct proofs

All the alternative proofs found by Sledgehammer correspond to those presented in Chapter 5 and reported in (Wapniarski and Urbański, 2026). The results obtained using Isabelle's automation in this regard can thus provide a twofold reassurance for the correctness of the identified proofs: first, we can rest assured that the alternative proofs presented in Chapter 5 are indeed **all** the possible proofs; this, in turn, can further reassure us that the method developed for finding them enables one to accurately spot all the valid proof cases; and as a similar method has been used to determine the set of all possible *indirect* proofs, we can also more confidently state that in the case of the indirect proofs none has been missed as well.

```
subsubsection‹Fourth Figure iterated›

lemma Bramantip_iterated:
  assumes "B Q A" and "C Q B" and "A≠{}" shows "A I C"
  using Barbara assms(1) assms(2) assms(3) per_accidens_conversion3 by blast
  (*using Darii assms(1) assms(2) assms(3) simple_conversion2 subalternation1 by blast
  using Barbari assms(1) assms(2) assms(3) simple_conversion2 by blast
  using Disamis assms(1) assms(2) assms(3) per_accidens_conversion3 by blast
  by (meson Datisi assms(1) assms(2) assms(3) simple_conversion2 per_accidens_conversion3)
  using Dimaris assms(1) assms(2) assms(3) subalternation1 by blast*)

lemma Camenes_iterated:
  assumes "B Q A" and "C E B" shows "A E C"
  using Celarent assms(1) assms(2) simple_conversion1 by blast
  (*using Cesare assms(1) assms(2) simple_conversion1 by blast
  using Camestres assms(1) assms(2) simple_conversion1 by blast*)

lemma Dimaris_iterated:
  assumes "B I A" and "C Q B" shows "A I C"
  using Darii assms(1) assms(2) simple_conversion2 by blast
  (*by (metis Disamis assms(1) assms(2) simple_conversion2)
  using Datisi assms(1) assms(2) simple_conversion2 by blast*)

lemma Fesapo_iterated:
  assumes "B E A" and "C Q B" and "B≠{}" shows "A Z C"
  using Ferio assms(1) assms(2) assms(3) simple_conversion1 per_accidens_conversion3 by blast
  (*using Festino assms(1) assms(2) assms(3) per_accidens_conversion3 by blast
  using Bocardo assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  using Ferison assms(1) assms(2) assms(3)  simple_conversion1 subalternation1 by blast
  using Fresison assms(1) assms(2) assms(3) subalternation1 by blast
  using Felapton assms(1) assms(2) assms(3) simple_conversion1 by blast*)

lemma Fresison_iterated:
  assumes "B E A" and "C I B" shows "A Z C"
  using Ferio assms(1) assms(2) simple_conversion1 simple_conversion2 by blast
  (*using Ferison assms(1) assms(2) simple_conversion1 by blast
  by (meson Festino assms(1) assms(2) particular_affirmation_def)
  using Festino assms(1) assms(2) simple_conversion2 by blast*)

lemma Camenos_iterated:
  assumes "B Q A" and "C E B" and "C≠{}" shows "A Z C"
  using Celarent assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  (*using Cesare assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  using Camestres assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  using Camestros assms(1) assms(2) assms(3) simple_conversion1 by blast
  using Baroco assms(1) assms(2) assms(3) simple_conversion1 subalternation2 by blast
  using Camenes assms(1) assms(2) assms(3) subalternation2 by blast*)
```

Fig. 6.13 Formalization of the fourth-figure alternative direct proofs

Second, the correspondence between results obtained by hand and the results obtained using Isabelle shows that the prover can be successfully used for formal verification of exercises in traditional logic. It is important to note that, as mentioned earlier, Isabelle's automation has found alternative proofs even when it was not "asked" for it. It is as if the program, not having any culturally, traditionally or epistemologically[98] driven preference for the proofs which involve first-figure moods, treated them on equal means with those involving second-, third-, and fourth-figure ones. At this point, one may speculate about the connection between a tendency to formalize syllogistic and render it combinatorically and a proclivity of recognizing proofs (and moods) other than the traditionally recognized ones. The previously described account of Leibniz (as he was the one to propose *some* of the alternative proofs presented in Chapter 5) may exemplify this connection, as he was also one of the proponents of both the algebraization of syllogistic and of combinatorically determining its valid moods. The reason for it being the case might be that, once syllogistic is formalized using some external means (e.g., the framework of Isabelle or the Leibnizian algebraic notation), one is not propelled to follow many of the Aristotelian and scholastic guidelines which are intertwined with the natural language formulation of syllogisms (e.g., the primacy of the first figure), and is free to manipulate the system more freely. As an example of this, recall that although the axiomatic base of Isabelle's formalization hereby presented was chosen to be the first figure (following the Aristotelian guidelines), it is not the case that it had to be so – we might have had as well chosen a different axiomatic base and, provided it contained moods sufficient to prove all other moods, it would have provided analogous results. ...

It is, moreover, noticeable that moods which have the highest number of alternative direct proofs are precisely the subalternated moods and those needing an additional existential assumption according to the schema developed in Chapter 4. In each alternative proof-case of these moods, one needs to employ means that in themselves necessitate the addition of existential assumptions: either one of the subalternations, the conversion *per accidens*, or one of the subalternated moods which in itself is proved using one of the first two means.

[98] Note that the main Aristotelian argument for the primacy of first-figure moods rests on the epistemological claim of their necessity being "evident" at a first sight. See Section 1.1.6.

6.4 Discussion

6.4.1 Reducing deductions: *Dictum de omni et nullo* revisited

As is evident from Figures 6.3, 6.5, 6.6, and 6.7, as well as from the comments already made upon them, the formal proofs discovered by Isabelle (without forcing Sledgehammer into finding alternatives) do correspond to the Aristotelian comment about the ultimate reducibility of all of the syllogistic moods to the *Dictum de omni et nullo*, i.e., to *Barbara* and *Celarent*. When the alternative proofs found later are taken into consideration, it is even more obvious that the Aristotelian reduction metatheorem holds in Isabelle's formalization as well, and the extent of direct proofs is equal to what we have seen in Chapter 5. What is shown below is that the indirect proofs or the Aristotelian proofs by impossibility (i.e., indirect proofs) used to reduce, first, *Baroco* and *Bocardo* to *Barbara* (Figure 6.14); second, *Darii* and *Ferio* to *Camestres* and *Cesare* (respectively – Figure 6.15), can be replicated by Isabelle's automation as well.

```
lemma Baroco_reducedto_Barbara :
  assumes "A Q B " and " A Z C  " and " B Q C "
  shows "B Z C" (*assms , concl. of Baroco and  B Q C *)
proof-
  have "A Q C" using  ‹A Q B ›  ‹ B Q C › Barbara by blast
  show ?thesis using  ‹A Q C›  ‹ A Z C ›
    by (simp add: particular_denial_def universal_affirmation_def)
qed

lemma Bocardo_reducedto_Barbara :
  assumes " A Z C" and "B Q C" and "A Q B"
  shows "A Z B" (*assms, concl of Bocardo and A Q B *)
proof-
  have "A Q C" using  ‹B Q C›  ‹ A Q B› using Barbara by blast
  show ?thesis using  ‹A Q C› ‹ A Z C›
    by (simp add: particular_denial_def universal_affirmation_def)
qed
```

Fig. 6.14 *Per impossibile* reductions of *Baroco* and *Bocardo*

```
lemma Darii_reducedto_Camestres:
  assumes "A Q B " and "B I C" and "A E C  " (*assms, concl. of Darii  and A E C *)
  shows "A I C"
proof-
  have "B E C" using Camestres ‹ A Q B   ›  ‹A E C›    by blast
  show ?thesis using ‹ B I C ›   ‹B E C›
    by (simp add: particular_affirmation_def universal_denial_def)
qed

lemma Ferio_reducedto_Cesare:
  assumes  "A E B " and "B I C" and "A Q C  " (*assms, concl. of Ferio  and A Q C *)
shows "A Z C"
 proof-
  have "B E C" using Cesare ‹A E B ›  ‹A Q C›  by blast
  show ?thesis using  ‹B I C ›  ‹B E C›
    by (simp add: particular_affirmation_def universal_denial_def)
qed
```

Fig. 6.15 *Per impossibile* reductions of *Darii* and *Ferio*

6.4.2 The assumptions of non-empty sets

In Chapter 5, we have already discussed the issue of non-empty terms and have seen that the only moods which need this assumption, according to modern first order predicate calculus, are those consisting of two universal statements as premises and a particular statement as a conclusion. Apart from all the subalternated moods, those include *Darapti* and *Felapton* from the third figure and *Bramantip* and *Fesapo* from the fourth. We have also seen that among these moods, different groups may be identified, necessitating different terms to be non-empty.

In the Isabelle code, we may also investigate this issue and notice that the term that needs to be non-empty is dependent on the way in which a particular mood is proved: in the case of first-figure subalternated moods, this term is the subject of the conclusion, as they are proved through subalternations applied on conclusions of the moods by which they are proved (*Barbari* by *Barbara*, *Celarent* by *Celaront*).

This is true also for second-figure subalternated moods, as they require subalternating the conclusion of the deduction used in the proof as well. In the third figure, the non-empty term is the subject of the premises, as the moods that require it are proved through applying the conversion *per accidens* (which necessitates non-emptiness of its subject) on either one of these. *Felapton*, for example, is proved by using *Festino* and by applying the third conversion on the second premise. In the fourth figure, the term which must be non-empty varies between deductions, which is also determined by their respective proofs. *Bramantip* which is proved by *Barbari* requires the same non-empty term as *Barbari*, but since its premises are switched (note the construction of the fourth figure as described in Chapter 1) that term is not C but A. In the case of *Fesapo*, the term is the subject of the second premise, as this mood is proved by applying the third conversion rule on this premise. An interesting anomaly is *Camenos*, which needs subalternating a universal negative conclusion in the proof. Apparently, it is the only mood in which the non-empty set is not the subject of any of the premises, but the predicate.

One may ask how these observations regarding the proof-procedure correspond to the fact that many moods do have alternative proofs as showed in Chapter 5. The answer is that all the alternative proofs, regardless of the procedure, require the same term to be non-empty.

In the Isabelle/HOL formalization, for the proofs provided by Isabelle's automation for the moods in the first three figures, we can see that *Darapti* and *Felapton* are the only moods that require an assumption that the subject is a non-empty set, as it can be seen from Figures 6.3, 6.5 and 6.6. *Felapton* relies on the conversion *per accidens* rule that requires this assumption (as seen in Figure 6.2). The proof of *Darapti* displayed there does not explicitly use the conversion *per accidens*, but it uses the same two definitions (universal affirmation and particular affirmation) that are used for the proof of the third conversion.

As is shown in Figure 6.12, different alternative direct proofs for *Darapti* have been found using the formalization as well, and all of them do make use either of the conversion *per accidens* or of one of the subalternations, both of which require non-emptiness. As in order to prove both the conversion *per accidens* and *Darapti*, Isabelle uses the definitions of the universal affirmation and the particular affirmation combined, this appears to be the ultimate source of the non-emptiness-of-sets assumption in this case, which further agrees with the aforementioned findings present in Chapter 5 and in Wapniarski and Urbański (2026). In the case of the fourth figure, both *Bramantip* and *Fesapo* were recognized as requiring the existential assumptions, where *Bramantip* is proved using the subalternated *Barbari* and *Fesapo* using the conversion *per accidens* (see Figure 6.7). Furthermore, all the subalternated moods also do require the assumption since they are proved using the subalternation which requires it, as can be seen from Figure 6.9.

It can be therefore seen that Isabelle/HOL has indeed recognized exactly all the cases where existential assumption is needed according to our findings in Chapter 5 and present in (Wapniarski and Urbański, 2026). The rest of the deductions are provable without making use of this assumption, as Isabelle can also verify.

What is of a special interest is that in the cases where the assumption that a certain set is not an empty set was necessary, Isabelle's automation has immediately alerted the user that the deduction is not valid without the non-empty set assumption via its counterexample-finding tool Auto Quickcheck, which gives an explicit counterexample where the set is the empty set.

This procedure is illustrated in Figures 6.16, 6.17, 6.18, 6.19, and 6.20, where the non-empty set assumption for term C (compare with the proof of *Darapti* in Figure 6.6) has been removed. In Figure 6.16, we see that after removing this assumption and having all the terms empty, Sledgehammer is not able to find any proof for *Darapti*. In Figure 6.17, we further uncover the source of it being a counterexample found by Quickcheck, and may try to correct the proof by adding the non-emptiness assumptions of some of the terms.

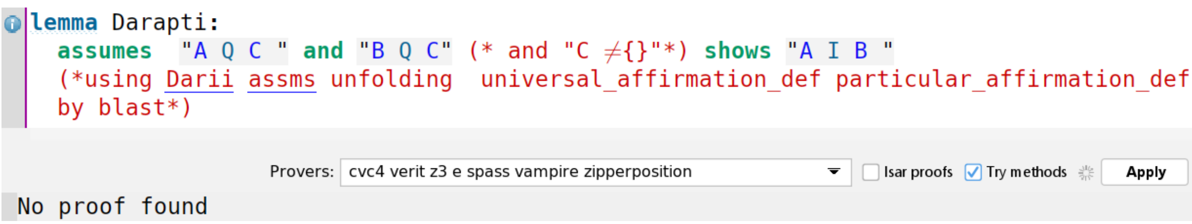


Fig. 6.16 Darapti without the non-emptiness assumption

```
lemma Darapti:
  assumes  "A Q C " and "B Q C" (* and "C ≠{}"*) shows "A I B "
  (*using Darii assms unfolding  universal_affirmation_def particular_affirmation_def
  by blast*)

Proof state  Auto update  Update  Search:

Auto Quickcheck found a counterexample:
  A = {}
  C = {}
  B = {}
```

Fig. 6.17 Darapti – a counterexample found by Quickcheck

Auto Quickcheck does not give us the information on which of the terms needs to be non-empty, but if we proceed by attempting to add the assumption that A (instead of the "correct" C) is non-empty, it still gives a counterexample, this with the non-emptiness of A term accounted for (Figure 6.18); an analogous situation results from attempting to add the assumption that B (instead of C) is non-empty (Figure 6.19). Finally, if the correct term is assumed to be non-empty (Figure 6.20), Sledgehammer is again able to find various direct proofs.

```
lemma Darapti:
  assumes  "A Q C " and "B Q C" and "A≠{}" shows "A I B "
  (*using Darii assms unfolding  universal_affirmation_def particular_affirmation_def
  by blast*)

Proof state  Auto update  Update  Search:

Auto Quickcheck found a counterexample:
  A = {a₁}
  C = {}
  B = {}
```

Fig. 6.18 *Darapti* – a counterexample found by Quickcheck after assuming the non-emptiness of a wrong term

```
lemma Darapti:
  assumes  "A Q C " and "B Q C" and "B≠{}" shows "A I B "
  (*using Darii assms unfolding  universal_affirmation_def particular_affirmation_def
  by blast*)

Proof state  Auto update  Update  Search:

Auto Quickcheck found a counterexample:
  A = {}
  C = {}
  B = {a₁}
```

Fig. 6.19 *Darapti* – a counterexample found by Quickcheck after again assuming the non-emptiness of a wrong term

```
lemma Darapti:
  assumes  "A Q C " and "B Q C" and "C≠{}" shows "A I B "
  (*using Darii assms unfolding  universal_affirmation_def particular_affirmation_def
  by blast*)

Provers: cvc4 verit z3 e spass vampire zipperposition   Isar proofs  Try methods  Apply  Cancel  Locate
verit found a proof...
cvc4 found a proof...
vampire found a proof...
verit: Try this: using Darii assms(1) assms(2) assms(3) per_accidens_conversion3 by blast (15 ms)
cvc4: Duplicate proof
vampire: Try this: by (meson Darii assms(1) assms(2) assms(3) per_accidens_conversion3) (0.0 ms)
```

Fig. 6.20 *Darapti* with the correct non-emptiness assumptions and the proofs displayed again

What has been shown here with *Darapti* can be replicated with every mood requiring the non-emptiness assumption. This was the case with all the other moods which need the non-empty set assumption. Throughout the entire Isabelle formalization, in all cases in which the non-empty set assumption has been added, its necessity had been automatically discovered by Auto Quickcheck as in the above described case, with Auto Quickcheck automatically generating counterexamples and the user attempting to add non-empty set assumptions for various sets until the correct non-empty set assumption was found. This shows how Isabelle's interactive nature and automation can be of substantial help for the user in real time, by providing immediate feedback during the process of formalization, and thus guiding the user.

6.4.3 Philosophical implications of the use of theorem prover

When we look upon the whole history of syllogistic and of logic in general, theorem provers are a relatively new phenomenon which is not fully understood yet. Isabelle/HOL itself is used mainly to formalize mathematics, as a tool providing formal and automatic verification of theorems through its built-in methods of step-by-step reasoning. Such idea of formalization of mathematics using logic stems – surprisingly – from the XVI-century attempts of applying precisely syllogistic reasoning to Euclid's *Elements* in order to render them more formally coherent and provide a clear step-by-step structure (modeled upon the syllogisms, in this case) to all the Euclidean theorems, as it was thought of by contemporary scholars (see (Bertato and

Basti, 2018)). The contemporary formal verification methods provided by theorem provers have their advantages and disadvantages. The main advantage is the automation and verification theorem provers provide together with a significant reduction of time needed for obtaining and correcting a proof once it is coded in the system. The main disadvantage, which can be particularly important for philosophers (and logicians alike), is the so-called *explainability*, or rather lack thereof. Isabelle, as we have seen even on a relatively straightforward example-formalization of syllogistic, does not provide its users with a full chain of reasoning it has performed in any particular case. Rather, such step-by-step reasoning is packed within Isabelle's automatic proof methods and remains unknown for the user. What is being "told" to us by Isabelle is only the final information that the proof in question exists and can be done using such and such definitions/lemmas. One can be unsatisfied with such a result, especially if understanding is what we are primarily concerned with. However, formal proofs performed by Isabelle can still a) serve as a reassurance that a certain argument is provable, b) provide a clear indication about the "ingredients" which are needed for a proof (Koutsoukou-Argyraki and Wapniarski, 2025). As it was made clear by the extended formalization carried out in this Chapter, it can also be a useful tool for verifying metatheoretical results about a particular system which were earlier obtained through other means (e.g., as was done here with alternative direct proofs and cases where non-emptiness had to be assumed).

Chapter 7

End notes

7.1 Summary

The first part of the dissertation consisted of primarily historical issues and addressing misconceptions about them. In Chapter 2, we have surveyed the historical development of the syllogistic mnemonic devices and established that their role was not merely to aid memorization of syllogistic moods, but also to structure and thereafter *extend* (the boundaries of) the system. In Chapter 3, we covered wide range of standpoints about the admission of empty terms to syllogistic and about its ontological (or existential) assumptions, starting with Aristotle himself and going through Arabic and medieval European logic up until the times of Leibniz, and thereafter Boole, Venn and the development of modern formal logic. Moreover, in this part we touched upon several side (but nonetheless important) issues, such as the transition from rules to verses in the mnemonic tradition, the "new names" of syllogistic moods proposed by Buridan and Leibniz, the importance of Leibnizian *Difficulatates quaedam logicae*, and the supposed mutual influences between the development of logical diagrams and views on empty terms.

In the second part, we have extended our search to encompass formal considerations as well. In Chapter 4, having in mind what was discovered in Chapter 3, we showed what additional existential assumptions must be made so that all syllogistic moods are provable in the framework of first order logic. In this way, we identified three classes of syllogisms: 1) moods that do not need any existential assumptions, 2) moods in which a required existential assumption is guaranteed by one of the premises, 3) moods which need an additional existential assumption. In Chapter 5, we extended these formal considerations and developed a framework for computing all alternative direct and indirect proofs for all syllogistic moods, which provided us with a yet another possible arrangement of moods into different groups. There, we saw that alternative sets of "basic" syllogistic moods (i.e., moods to which all the others can be ultimately reduced), different from the Aristotelian first figure syllogisms, can in fact be identified, and moreover, that the necessity of adopting additional existential assumptions for a moods to be valid results in a far greater amount of possible alternative indirect proofs.

In this way, we established a connection between the admission of empty terms on the one hand and the reducibility of syllogistic moods on the other; next, we established that some additional patterns can be spotted when the mnemonic schema of Leibniz (identified in Chapter 2) is applied to the results obtained in Chapter 5. This, in turn, prompted us to speculate that the Leibnizian recognition of empty terms might have been influenced by the attention he gave to indirect proofs and his attempts to structure syllogistic differently than Aristotle.

In the third part (Chapter 6), we have made a formalization of Aristotelian syllogistic using a modern theorem-prover Isabelle/HOL and verified a great portion of findings from the second part of the dissertation while using the mnemonic framework and historical knowledge covered in the first. The prover proved to be useful not only for formalizing the basics of syllogistic, but also its historical extensions and features like the non-emptiness of sets assumption and alternative direct proofs.

Already at the end of Chapter 2, we concluded that it is unwise to simply reduce Aristotelian syllogistic to a small part of first order modern formal logic and treat it as such; rather, we should regard it on its own terms as a legitimate logical system standing next to modern systems and try to understand it taking into account its rich history. In Chapter 3, that has once again proved to be true, as we have seen that our modern logical intuitions are often not suitable enough to grasp the nuances of doctrines held by ancient, Arabic, medieval, and early-modern commentators and scholars, and that the discussions concerning the non-emptiness of terms were far more rich than we may have presupposed.

Moreover, appreciating this historical richness and having a deeper knowledge of how the syllogistic was understood without modern formal logic as an intermediary are necessary preconditions to understanding the transition or shift from syllogistic to modern formal logic that happened at the turn of XIX and XX centuries; in this regard, we hope that this dissertation, with its analysis of mnemonic devices and discussions about the non-emptiness of terms, makes a small but not insignificant contribution.

Throughout the second part of the dissertation, in Chapter 4 and Chapter 5, we tried to bridge this gap between modern formal logic with modern conceptualizations on the one hand, and traditional logic with its own conceptualizations on the other. The two parallel formal reconstructions of syllogistic proofs, one done with tools of modern formal logic, the other with those of syllogistic, and the subsequent superimposition of the first upon the second, allowed us to connect the two and better understand their mutual relationship. Taking into account especially the many misconceptions regarding the non-emptiness of terms assumption and the consequences of its adoption, we also hope that what has been accomplished in this work will serve as a useful basis for future research.

Future research

Besides what has been stated above, many themes which have only briefly been touched upon in the present work demand more detailed, future studies, and many things accomplished are welcoming future research building upon them. In the first group, we may count studies explaining Leibniz's influence upon the "fathers" of modern formal logic (e.g., Frege, Russell), more detailed analyses of relationship between diagrammatic representations and discussions about empty terms, an in-depth exposition of mnemonic names propositions posed by Leibniz and Buridan (and maybe also by other logicians, if one shall find such?), etc. In the second, one can place further formal studies building upon the ones presented here and aimed to uncover even more regularities and patterns among various groups of syllogistic moods, connecting properties of syllogistic which are seemingly unconnected, and further explaining the complex relationship between traditional and modern formal logic. To these we may also add explorations within the realm of modal syllogistic, particularly developing a way to systematically find alternative proofs for modal moods, as well as formalizing modal syllogistic using theorem provers (this, however, requires rewriting modal syllogistic using sequent calculus in the first place, as modal operators can usually be implemented in theorem provers only using these). An area of research that may build upon the studies hereby presented is also psychology of reasoning – a promising topic of study being checking whether any of the logical regularities discovered in this dissertation are reflected in studies on the cognitive difficulty of syllogistic moods. These endeavors, however, all surpass the scope of this dissertation and demand separate, detailed, and hopefully fruitful studies.

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Appendix A

Figure-by-figure analysis of existential requirements

This appendix provides complete figure-by-figure analysis of existential requirements for all G3 syllogisms (those requiring additional existential premises beyond what their universal premises provide). The main text (Chapter 4) presents the general principles and representative examples, while this appendix provides verification of claims therein contained.

G3 syllogisms: requiring additional existential premises

G3 syllogisms have only universal premises but particular conclusions. Since universal premises can be vacuously true when their terms are empty, and particular conclusions require the existence of at least one instance, these syllogisms require an explicit existential premise. The general principle is that *the existential premise must supply a witness that the universal premises can propagate into both conjuncts of the particular conclusion*.

Figure 1 (M–P, S–M)

In Figure 1, the conditional chain flows from S through M to P. A witness must be introduced at S to propagate through both conditionals.

***Barbari*:** All M are P; All S are $M \vdash$ Some S are P

Major: $\forall x(M(x) \to P(x))$

Minor: $\forall x(S(x) \to M(x))$

Conclusion: $\exists x(S(x) \land P(x))$

A witness a satisfying $S(a)$ propagates forward: $S(a) \to M(a)$ by the minor premise, then $M(a) \to P(a)$ by the major premise, yielding both $S(a)$ and $P(a)$.

Required existential premise: $\exists x\, S(x)$

***Celaront*:** No M are P; All S are $M \vdash$ Some S are not P

Major: $\forall x(M(x) \to \neg P(x))$

Minor: $\forall x(S(x) \to M(x))$

Conclusion: $\exists x(S(x) \land \neg P(x))$

A witness a satisfying $S(a)$ propagates: $S(a) \to M(a)$ by the minor premise, then $M(a) \to \neg P(a)$ by the major premise, yielding both $S(a)$ and $\neg P(a)$.

Required existential premise: $\exists x\, S(x)$

Figure 2 (P–M, S–M)

In Figure 2, M appears as the consequent of both premises. A witness at S propagates to M via the minor premise, and contraposition of the major premise connects M to P.

***Cesaro*:** No P are M; All S are $M \vdash$ Some S are not P

Major: $\forall x(P(x) \to \neg M(x))$

Minor: $\forall x(S(x) \to M(x))$

Conclusion: $\exists x(S(x) \land \neg P(x))$

A witness a satisfying $S(a)$ propagates: $S(a) \to M(a)$ by the minor premise. The major premise contraposes to $\forall x(M(x) \to \neg P(x))$, so $M(a) \to \neg P(a)$, yielding both $S(a)$ and $\neg P(a)$.

Required existential premise: $\exists x\, S(x)$

***Camestros*:** All P are M; No S are $M \vdash$ Some S are not P

Major: $\forall x(P(x) \to M(x))$

Minor: $\forall x(S(x) \to \neg M(x))$

Conclusion: $\exists x(S(x) \wedge \neg P(x))$

A witness a satisfying $S(a)$ yields: $S(a) \to \neg M(a)$ by the minor premise. The major premise contraposes to $\forall x(\neg M(x) \to \neg P(x))$, so $\neg M(a) \to \neg P(a)$, yielding both $S(a)$ and $\neg P(a)$.

Required existential premise: $\exists x\, S(x)$

Figure 3 (M–P, M–S)

In Figure 3, M is the common antecedent of both conditionals. A witness at M propagates simultaneously in both directions.

***Darapti*:** All M are P; All M are $S \vdash$ Some S are P

Major: $\forall x(M(x) \to P(x))$

Minor: $\forall x(M(x) \to S(x))$

Conclusion: $\exists x(S(x) \wedge P(x))$

A witness a satisfying $M(a)$ propagates in both directions at once: $M(a) \to P(a)$ by the major premise and $M(a) \to S(a)$ by the minor premise, immediately yielding both $S(a)$ and $P(a)$.

Required existential premise: $\exists x\, M(x)$

***Felapton*:** No M are P; All M are $S \vdash$ Some S are not P

Major: $\forall x(M(x) \to \neg P(x))$

Minor: $\forall x(M(x) \to S(x))$

Conclusion: $\exists x(S(x) \wedge \neg P(x))$

A witness a satisfying $M(a)$ propagates: $M(a) \to \neg P(a)$ by the major premise and $M(a) \to S(a)$ by the minor premise, yielding both $S(a)$ and $\neg P(a)$.

Required existential premise: $\exists x M(x)$

Figure 4 (P–M, M–S)

In Figure 4, the conditional chains have different structures, so the required existential premise varies by mood.

Bramantip**:** All P are M; All M are $S \vdash$ Some S are P

Major: $\forall x(P(x) \to M(x))$

Minor: $\forall x(M(x) \to S(x))$

Conclusion: $\exists x(S(x) \wedge P(x))$

A witness a satisfying $P(a)$ propagates through the chain: $P(a) \to M(a)$ by the major premise, then $M(a) \to S(a)$ by the minor premise, yielding both $P(a)$ and $S(a)$.

Required existential premise: $\exists x P(x)$

Fesapo**:** No P are M; All M are $S \vdash$ Some S are not P

Major: $\forall x(P(x) \to \neg M(x))$

Minor: $\forall x(M(x) \to S(x))$

Conclusion: $\exists x(S(x) \wedge \neg P(x))$

A witness a satisfying $M(a)$ propagates: $M(a) \to S(a)$ by the minor premise. The major premise contraposes to $\forall x(M(x) \to \neg P(x))$, so $M(a) \to \neg P(a)$, yielding both $S(a)$ and $\neg P(a)$.

Required existential premise: $\exists x M(x)$

Camenos**:** All P are M; No M are $S \vdash$ Some S are not P

Major: $\forall x(P(x) \to M(x))$

Minor: $\forall x(M(x) \rightarrow \neg S(x))$

Conclusion: $\exists x(S(x) \wedge \neg P(x))$

A witness a satisfying $S(a)$ yields: the minor premise contraposes to $\forall x(S(x) \rightarrow \neg M(x))$, so $S(a) \rightarrow \neg M(a)$. The major premise contraposes to $\forall x(\neg M(x) \rightarrow \neg P(x))$, so $\neg M(a) \rightarrow \neg P(a)$, yielding both $S(a)$ and $\neg P(a)$.

Required existential premise: $\exists x\, S(x)$

Summary: G3 existential requirements by figure

- **Figure 1:** Minor term (S) must be non-empty – 2 syllogisms (Barbari, Celaront)
- **Figure 2:** Minor term (S) must be non-empty – 2 syllogisms (Cesaro, Camestros)
- **Figure 3:** Middle term (M) must be non-empty – 2 syllogisms (Darapti, Felapton)
- **Figure 4:** Varies by mood – 3 syllogisms
  - Bramantip: Major term (P) must be non-empty
  - Fesapo: Middle term (M) must be non-empty
  - Camenos: Minor term (S) must be non-empty

The figure-sensitivity follows from a uniform principle: the witness must be introduced at the point in the conditional chain from which the universal premises can propagate it to both components of the particular conclusion.

Appendix B

A complete proof-procedure for *Felapton*

To illustrate all the different possible cases of how a certain syllogism can be indirectly reduced to other moods, we shall consider an example of *Felapton*. In the case of *Felapton*, our premises are:

1.*MeP*	(major premise)
2.*MaS*	(minor premise)
3.*SaP*	(indirect premise from contradiction to SoP)

If we want our indirect conclusion to be contradiction of MeP, we need to compile SaP with MaS, in case we want it to be contradiction of MaS, we are left with SaP and MeP. If we include subalternation, every universal negative proposition can be subalternated to a particular negative one (thus MeP to MoP), and every universal affirmative to a particular affirmative (thus MaS to MiS and SaP to SiP). If we include conversion(s), every universal negative proposition can be converted simply (thus MeP to PeM, note that PeM can give then an additional particular negative, namely PoM), every particular affirmative can be converted simply as well (thus an example SiP obtained from the subalternated SaP to PiS), and every universal affirmative can be converted with limitation (thus MaS to SiM and SaP to PiS) and then simply (thus SiM to

MiS and PiS to SiP). In the case of *Felapton*, the set of possible premises is as follows:

1.*MeP*	(major premise)
2.*MaS*	(minor premise)
3.*SaP*	(indirect premise)
4.*MoP*	(MeP, subalternation)
5.*PeM*	(MeP, s-conversion)
6.*PoM*	(PeM, subalternation)
7.*SiM*	(MaS, p-conversion)
8.*MiS*	(SiM, s-conversion *or* MaS, subalternation)
9.*PiS*	(SaP, p-conversion)
10.*SiP*	(PiS, s-conversion *or* SaP, subalternation)

Each pair of thus obtained premises can be then combined in two ways, respecting the restraint that we are always combining one direct premise with one indirect one, so that derivatives of two direct premises are not to be combined. In the case of *Felapton*, we have four versions of the major premise, three versions of the minor premise, and three versions of the indirect premise. The combination of major and indirect premise thus gives us $4 \times 3 \times 2$ (arrangement of premises) possible cases, the combination of minor and indirect $3 \times 3 \times 2$, amounting to a total of $24 + 18 = 42$ combinations.

If we are to treat the problem of case (or proof) validity in a systematic way, we should discard all cases where a particular affirmative proposition is the first one as well as those consisting of two particular propositions. Thus, again in the case of *Felapton*, we get 18 cases less, giving a total of 24 possibilities, the valid ones among which are (after completion of the proofs): *Barbara*, *Celarent*, *Darii*, *Barbari*, *Cesare*, *Camestres*, *Baroco*, *Camestros*, *Disamis*, *Datisi*, *Bramantip*, *Camenes*, *Dimaris*, *Camenos*.